VA0248

Adult Basic Education Intermediate Level MATHEMATICS

Module 1 Arithmetic and Estimation

Revised 2000 BRITISH COLUMBIA

Ministry of Advanced Education, Training and Technology

CENTRE FOR CURRICULUM, TRANSFER & TECHNOLOGY

Canadian Cataloguing in Publication Data

Main entry under title: Intermediate level mathematics

(Adult basic education)

ISBN 0-7718-9544-5 (set)

 Mathematics – Study and teaching (Continuing education) – British Columbia. I. Brodie, Sarah. II. British Columbia. Ministry of Advanced Education, Training and Technology. III. Centre for Curriculum, Transfer and Technology. IV. Series.

QA14.C32B742 1999 510'.71'5

Program Information

Coordinator, Adult Basic Education Programs Ministry of Advanced Education, Training and Technology Post Secondary Education Division PO Box 9877, Stn Prov Govt Victoria, British Columbia Canada V8W 9T6 Web: http://www.aett.gov.bc.ca/

Curriculum Information

C99-960118-0

Curriculum Development Team Centre for Curriculum, Transfer and Technology 6th Floor, 1483 Douglas Street Victoria, British Columbia Canada V8W 3K4 Telephone: 250-413-4409 Fax: 250-413-4403 Web: http://www.ctt.bc.ca/curric

Ordering	ABE Intermediate Level Mathematics	Order No.	ISBN
Marketing Department	Module 1: Arithmetic and Estimation	VA0248	0-7719-1242-0
Open Learning Agency	Module 2: Measurement	VA0249	0-7719-1243-9
4355 Mathissi Place	Module 3: Perimeter, Area and Volume	VA0250	0-7719-1244-7
Burnaby, British Columbia	Module 4: Ratio and Proportion	VA0251	0-7719-1245-5
Canada V5G 4S8	Module 5: Percent	VA0252	0-7719-1246-3
Telephone: 604-431-3210 or	Module 6: Geometry: Introduction	VA0253	0-7719-1247-1
1-800-663-1653	Module 7: Geometry: Construction	VA0254	0-7719-1248-X
Fax: 604-431-3381	Module 8: Statistics	VA0255	0-7719-1249-8
Email: catalogue@ola.bc.ca	Module 9: Signed (Rational) Numbers	VA0256	0-7719-1250-1
Web: http://www.ola.bc.ca	Module 10: Algebra: Equations	VA0257	0-7719-1251-X
	Module 11: Algebra: Polynomials	VA0258	0-7719-1252-8
	Module 12: Powers, Roots and Scientific	VA0259	0-7719-1253-6
	Notation		
	Module 13: Graphing	VA0260	0-7719-1254-4
	Module 14: Trigonometry	VA0261	0-7719-1255-2
	Instructor Package	VA0262	0-7719-1256-0

Copyright © 1999 Province of British Columbia Ministry of Advanced Education, Training and Technology This document may not be reproduced in any form without permission of the Centre for Curriculum, Transfer and Technology.

Adult Basic Education Intermediate Level Mathematics

Module 1 Arithmetic and Estimation

Prepared by Sarah Brodie, North Island College Pat Corbett, North Island College Paul Grinder, Okanagan University College Peter Robbins, Kwantlen University College Ada Sarsiat, Northwest Community College

for the Province of British Columbia Ministry of Advanced Education, Training and Technology and the Centre for Curriculum, Transfer and Technology 1999 Revised 2000

Contents

Learning outcomes	2
Glossary	3
Introduction	4
Unit 1: Place value and rounding numbers	5
Exercise 1	7
Unit 2: Comparing numbers	8
Exercise 2	10
Unit 3: Rounding numbers	11
Exercise 3	13
Unit 4: Operations with decimals	14
Exercise 4	15
Unit 5: Multiplying whole numbers and decimal fractions	17
Exercise 5	
Unit 6: Dividing whole numbers and decimal fractions	
Exercise 6	
Unit 7: Order of operations	25
Exercise 7	
Unit 8: Operations with fractions	
Exercise 8	30
Unit 9: Equivalent fractions	31
Exercise 9	33
Unit 10: Adding and subtracting common fractions	34
Exercise 10	37
Unit 11: Multiplying common fractions	39
Exercise 11	41
Unit 12: Dividing common fractions	
Exercise 12	44
Unit 13: Conversions between common fractions and	
decimal fractions	45
Exercise 13	47
Unit 14: Conversions between decimal fractions and	
common fractions	
Exercise 14	50
Unit 15: Estimation	
Exercise 15	52
Practice Test	53
Answers	56

Learning outcomes

Being able to work with whole numbers, decimals and fractions is important in everyday life. Estimating an answer to a problem helps put the problem in context. This process should be part of any problem solving activity. Checking answers and solutions to problems is equally important. The calculator is a useful tool, not only for obtaining accurate results, but also for checking answers.

When you have completed this Module, you should be able to:

- round numbers and estimate answers to problems
- use a scientific calculator to calculate and solve problems involving adding, subtracting, multiplying and dividing whole numbers, decimals and fractions
- check that answers and solutions to problems are reasonable in the context of the given questions

Requirements

You will be required to complete mathematics labs and a written Post Test relating to this Module. You will require a scientific calculator to check your calculations throughout this Module.

Procedure for independent study

- 1. Read each of the units in order and complete all of the exercises. If you need assistance, contact your instructor.
- 2. Complete the Practice Test and check your answers.
- 3. Study the terminology in the Glossary to become familiar with the definitions.
- 4. Contact your instructor to complete the Math Labs.
- 5. If recommended by your instructor, complete additional problem sets.
- 6. Complete the Post Test for this Module.

Glossary

Difference

The answer to a subtraction question is called the difference.

Equivalent

Equivalent quantities are equal.

Factor

A number that divides evenly into another number. E.g., 2 and 3 are factors of 6.

Improper fraction

An improper fraction has a numerator larger than the denominator.

Mixed number

A mixed number is a combination of a whole number and a proper fraction.

Prime number

A prime number is divisible only by itself and one. The number one is not a prime number. The first few prime numbers are 2, 3, 5, 7, 11, ...

Product

The answer to a multiplication question is called the product.

Proper fraction

A proper fraction has a numerator smaller than the denominator.

Quotient

The answer to a division question is called the quotient.

Sum

The answer to an addition question is called the sum.

Introduction

Estimation can be used to calculate the approximate cost of something.

Example

You and a friend are having dinner at a restaurant. As you look at the menu, you try to estimate the cost of your meal.

Main Course	
Roast Beef	\$13.95
Chicken	\$11.50
Spaghetti	\$ 9.75
Vegetarian Lasagne	\$10.99
Desserts	
Cheesecake	\$ 4.25
Apple Pie	\$ 3.90
Ice Cream	\$ 2.75
Beverages	
Milk	\$ 1.50
Coffee	\$ 0.99
Tea	\$ 0.99
Soft Drinks	\$ 1.55

You choose chicken, apple pie and coffee. Your friend chooses lasagne, ice cream and a soft drink.

To estimate the cost of each person's meal, you can round each price up or down. In this example, if your round up, your estimate will be higher than the actual cost.

Your meal		Your friend	
Chicken	\$12	Lasagne	\$11
Apple pie	\$4	Ice cream	\$3
Coffee	\$1	Soft drink	\$2
Total estimated cost	\$17		\$16

Note that estimates are not exact answers and may vary depending on whether you round up or down.

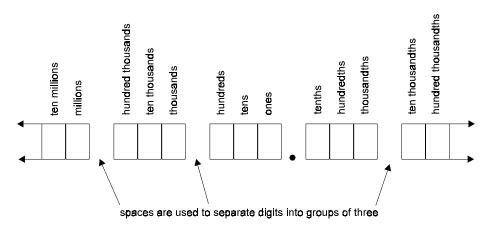
Unit 1: Place value and rounding numbers

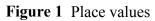
Place value

Words are formed with letters. Numbers are formed with digits. The ten digits of our number system are:

0, 1, 2, 3, 4, 5, 6, 7, 8 and 9

By stringing these digits together we can form numbers. The position that a digit occupies in a number tells us how many ones, tens, hundreds, thousands (and so on) that the number contains. If the number has a decimal point, then the position of the digits after the decimal point indicates how many tenths, hundredths, thousandths (and so on) the number contains. The place values or position values in a number are shown in Figure 1.





If a number contains a decimal point it is called a decimal fraction. Otherwise, the number is called a whole number. For example, 25.8 is a decimal fraction while 63 is a whole number.

Example 1

Write the place value for each underlined digit.

a. 43 2<u>0</u>9 b. 6.05<u>3</u> 21 c. <u>6</u>35 096.58 Solution

a. tens b. thousandths c. hundred thousands

Arithmetic and Estimation

Example 2

Write 23 805.90 in expanded form.

Solution

 $23\ 805.90 = 20\ 000 + 3\ 000 + 800 + 5 + \frac{9}{10} + \frac{0}{100}$

Example 3

Write three million in standard notation.

Solution

To write a number in standard notation means to write it using only digits. Three million in standard notation is 3 000 000.



Now complete Exercise 1 and check your answers.

1. Write the place value for each underlined digit.

a. 85 <u>2</u>	f. 4 <u>2</u> 806 900
b. 43.92 <u>1</u>	g. 8. <u>4</u> 2
c. 0.0 <u>5</u> 6	h. 9 <u>4</u> .06
d. 1 <u>6</u> 3 940	i. 3 698.5 <u>1</u>
e. 0.000 5 <u>0</u> 2	j. 7 <u>3</u> 480

2. Underline the digit for the place value named.

a.	tens	842.56
b.	tenths	842.56
c.	hundredths	68 923.05
d.	millions	843 907 000
e.	ten thousandths	63 521.009 842

- 3. Write in expanded form.
 - a. 823 b. 8.23
 - c. 9 605
 - d. 0.087
 - e. 3 842.65
- 4. Luna had 6 one hundred dollar bills, 3 tens and 8 loonies. How much money did she have?
- 5. Bill had 5 ten dollar bills, 3 dimes and 2 pennies. How much money did Bill have?
- 6. Ida found 4 pennies and 1 loonie. Together with her ten dollar bill, how much money does Ida have?
- 7. Dan wants \$203.41. How many hundred dollar bills, ten dollar bills, loonies, dimes and pennies should he receive?
- 8. A certain company lost 18.3 million dollars last year. Write this number in standard notation.

Answers are on page 56.

Unit 2: Comparing numbers

There are only three possibilities when comparing numbers. Either one number is equal to (=) another number, greater than (>) another number or less than (<) another number.

When comparing decimal fractions, it is often a good idea to write both numbers with the same number of digits after the decimal point. This is possible because placing zeros after the last decimal digit will not change the value of the number. For example:

63.5 = 63.50 63.5 = 63.500 63.5 = 63.5000 and so on.

We can do the same with whole numbers, but only after placing a decimal point at the end of the whole number. For example:

804 = 804 804 = 804.0 804 = 804.00 804 = 804.000and so on.

Example 1

Use >, < or = to compare each pair of numbers.

a. 0.002	0.01	b. 69	69.00	c. 305	30.5
----------	------	-------	-------	--------	------

Solution

a. Since 0.002 has three decimal digits, write 0.01 with three decimal digits, by putting another 0 after the 1. Comparing:

0.002 < 0.010

b. 69 and 69.00 have the same value. They are said to be equivalent:

69 = 69.00

c. 305 is greater than 30.5. Notice that a decimal could be placed at the end of 305.

305 > 30.5



Notice how greater than or less than signs always point to the smaller number, such as 13 > 5 or 5 < 13.

Example 2

Find the smallest of the three numbers below:

5.0 5.01 4.999

Solution

The equivalent numbers with three decimal digits are:

5.000 5.010 4.999

The smallest number is 4.999.



Now complete Exercise 2 and check your answers.

1. Use >, < or = to compare the numbers.

a. 9 9199 991	f. 6 9246.924
b. 0.50.50	g. 0.050 90.49
c. 6.2 <u>6.02</u>	h. 0.000 650.001
d. 84.00984.1	i. 3.03.000 000
e. 31 28031 280.0	j. 1.010.999

- 2. Find the smallest number in each set of numbers.
 - a. 16 1.6 16.0 160
 - b. 0.040 20 0.040 18 0.041
 - c. 8.70 8.700 000 8.07 8.7
 - d. 13 562 13 625 13 265 16 235
 - e. 199.0 99.999 0.999 999 999
- 3. Use an "is equal to" (=) or an "is not equal to" (\neq) symbol to compare the numbers.

a. 0.02	0.20	d. 842	8420.0
b. 6.3	63	e. 107	1.07
c. 9.00	9	f. 98 761	98.761

4. Jill bought gas for 49.9 cents per litre. Jack said he bought gas for \$0.499 per litre. Who paid the most for gas?

Answers are on page 57.

Unit 3: Rounding numbers

No one would ever report their age as being 30.859 years old. Knowing that the person is approximately 30 years old is usually sufficient. Most values like age can never be known exactly. To make things simple, we often round numbers to a convenient place value.

Example 1

The population of Canada is 28 463 900 people. Round to the nearest million.

Solution

Step 1: Underline the millions digit

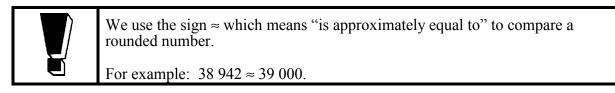
2<u>8</u> 463 900

- Step 2: If the next digit after the underlined digit is a 0, 1, 2, 3, or 4, then do not change the underlined digit. If the next digit is a 5, 6, 7, 8, or 9, then add 1 to the underlined digit. Since the next digit is a 4, the number is still 28 463 900.
- Step 3: Change all the digits after the underlined digit to zeros. Then remove the underline. The population of Canada is 28 000 000.

The steps required to round any decimal fraction are as follows:

- Step 1: Underline the digit to which the number is being rounded.
- Step 2: If the next digit after the underlined digit is a 0, 1, 2, 3, or 4, then do not change the underlined digit. If the next digit is a 5, 6, 7, 8, or 9, then add 1 to the underlined digit.
- Step 3: Change all the digits after the underlined digit to zeros. If any of the digits after the underlined digit follow a decimal point, then drop them from the number.

Step 4: Remove the underline from your answer.



Example 2

Round the following to the nearest tenth.

a. 13.57 b. 0.034 c. 18.962 487 Arithmetic and Estimation

Solution

a. Step 1: the digit in the tenths place is a 5. Underline it.

13.<u>5</u>7

Step 2: the digit after the 5 is a 7. Add 1 to the 5.

13.<u>6</u>7

Step 3: change the 7 to a zero. Since this zero comes after the decimal point, drop it.

 $13.\underline{6}0 = 13.\underline{6}$

Step 4: 13.57 ≈ 13.6

b. Step 1: the digit in the tenths place is a 0. Underline it.

```
0.<u>0</u>34
```

Step 2: the digit after the 0 is a 3. The 0 remains unchanged.

0.<u>0</u>34

Step 3: replace the 3 and 4 with zeros and drop them.

 $0.\underline{0}00 = 0.\underline{0}$

- Step 4: $0.034 \approx 0.0$
- c. Step 1: the digit in the tenths place is a 9. Underline it.

18.<u>9</u>62 487

Step 2: the digit after the 9 is a 6. Adding 1 to 9 = 10, which is carried to the ones place.

19.<u>0</u>62 487

Step 3: replace all the numbers after the underlined number with zeros.

19.<u>0</u>00 000

Step 4: 18.962 487 ≈ 19.0



Now complete Exercise 3 and check your answers

1. Round to the nearest hundred.

a. 8 355	d. 47
b. 203	e. 1 342 969
c. 85	f. 433.789

2. Round to the nearest one (also called rounding to the nearest whole number).

a. 6.3	f. 3.141 59
b. 18.7	g. 499.503
c. 49.6	h. 2.499 999
d. 55.55	i. 109.5
e. 0.807	j. 0.389 642

3. Round as indicated.

- a. 43 826 984 to the nearest ten thousand
- b. 0.0352 to the nearest tenth
- c. 0.499 to the nearest hundredth
- d. 0.438 206 to the nearest ten thousandth
- e. 0.83 to the nearest whole number
- f. 645.96 to the nearest ten

4. Round to the nearest hundredth.

a. 0.909	f. 10.698
b. 16.583	g. 45.0249
c. 0.123 456	h. 5.3062
d. 968.155	i. 0.096 12
e. 0.000 016	j. 11.682

5. Mike had \$19.50, Mary had \$84.05 and Sandra had \$43.88. Round their amounts to the nearest dollar.

Answers are on page 57.

Unit 4: Operations with decimals

When adding or subtracting whole numbers of decimal fractions it is important that we add only ones digits to ones digits, tens digits to tens digits, hundreds digits to hundreds digits, tenths digits to tenths digits and so on. To accomplish this, always make sure that the decimal points are lined in a column and each decimal fraction has the same number of digits after the decimal point.

Example 1

Add the following:

1.56 + 38 + 0.009 + 0.7

Solution

Arrange the numbers as shown. Note that a decimal point is included in the number 38 and every number has been written with three digits after the decimal point.

 $1.560 \\ 38.000 \\ 0.009 \\ 0.700 \\ 40.269$

Example 2

Subtract the following:

54 - 9.89

Solution

Align the decimal points. Rewrite 54 as 54.00. Notice that we must borrow from the 4 and again from the 5.

	13
	4 🕉 9 10
54.00	\$4.\$ 0
<u>-9.89</u>	- 9.89
	44.11

When numbers are added, the answer is called a sum. The sum of 8 and 5 is 13. When numbers are subtracted, the answer is called a difference. The difference of 10 and 4 is 6.



Now complete Exercise 4 and check your answers.

1. Find the sum.

a. 846	b. 13	c. 692
+95	74	5
	812	84
	+195	1387
		+42966

- 2. Find the sum.
 - a. 64 + 31 + 942 + 7
 - b. 88 943 + 369 + 1 003
 - c. 109 864 + 9 980 + 49 877 + 4 698
- 3. Find the sum.

a.	84.26	b. 16.5	с.	1249.0
	+12.09	0.03		70.16
		47.2		+8.297
		+ 92.06		

- 4. Find the sum.
 - a. 84 + 1.2 + 0.96 + 107
 - b. 29.093 + 14.08 + 9.635
 - c. 0.005 + 0.94 + 0.01258 + 0.00092

5. Find the difference.

a.	$\frac{182}{-39}$	b.	$-\frac{4\ 602}{-\ 4\ 583}$	C.	116 058 - 37 069
d.	27.03 - 5.84	e.	$\frac{0.029}{-0.00843}$	f.	96 -0.74
g.	11.023 - 7.2	h.	69 003 - 8 527.81	i.	$\frac{0.006837}{-0.000938}$

Arithmetic and Estimation

6. Find the difference.

a. 83 - 0.83	b. 29 - 6.52
c. 0.5 - 0.003	d. 74.3 - 8.930 2

7. Mom found three dimes, three quarters, one nickel, seven pennies and two loonies in Billy's pocket. How much money did Billy have?

- 8. Nadine had \$50 before she bought a calculator for \$19.95 and a geometry set for \$2.98. How much did she have after these purchases?
- 9. Calculate the balance values.

Date	Item	Withdrawal	Deposit	Balance
June 1				480.51
June 4	cash withdrawal	200.00		а
June 8	salary deposit		891.45	b
June 12	BC Tel	33.69		С
June 20	car repair cheque	146.35		d
June 30	rent cheque	450.00		е

10. Round each of the numbers to the nearest ten and estimate the sum or difference.

a. 462	b. 8 299
95.6	-315
0.25	
+79.1	

Answers are on page 57.

Unit 5: Multiplying whole numbers and decimal fractions

When two numbers are multiplied together, the number is called a product. The numbers being multiplied together are called factors. For example, when $2 \times 3 = 6$, we say that 2 and 3 are factors while 6 is the product.

Example 1

Find the product of 537 and 274.

Solution

Multiply 537 first by 4, then by 7 and then by 2 as shown. Add these products.

537	
×274	
2148	
3759	
1074	
147138	

When multiplying decimal fractions, great care must be taken in where to place the decimal point in the product.

Example 2

Multiply 1.35 by 0.409.

Solution

As above, multiply 1.35 first by 9, then by 0 and then by 4. Arrange these products as shown. Since the factors have a total of five decimal places, the product must also have five decimal places.

1.35	2 decimal places
×0.409	3 decimal places
1215	
000	
540	
0.55215	5 decimal places



Now complete Exercise 5 and check your answers.

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

1. Complete the times table chart.

- 2. Use the times table chart to help answer the following.
 - a. the product of 0 and any number is always ______.
 - b. the product of 1 and any number is always ______.
 - c. The product of 5 and any number always ends in a _____ or a _____.
 - d. Look at the products in the 9's row or column. Except for the 0 product, the sum of the digits in each product is always ______.
- 3. Multiply

a. 41	b. 96	c. 944	d. 5062
<u>×5</u>	<u>×13</u>	<u>×87</u>	×804
e. 83	f. 152	g. 602	h. 9065
<u>×1.1</u>	×0.07	<u>×0.504</u>	<u>×1.83</u>

i. 16.3	j. 19.84	k. 0.15	1. 0.004
<u>×0.07</u>	<u>×62</u>	<u>×3.7</u>	<u>×0.0205</u>

- 4. Find the products.
 - a. 17.7 × 0.6
 - b. 3.004×0.008
 - c. 0.006×0.02
 - d. 0.079×0.0001
 - e. 0.008×0.008
- 5. Tim earns \$1408.45 bi-weekly. There are 26 bi-weekly pay days in one year. How much will he earn in a year?

6. Holly drives an average speed of 84 km/hr in her car. How far will she travel in 4.5 hours?

Answers are on page 57.

Unit 6: Dividing whole numbers and decimal fractions

When one number is divided by another, the answer is called a quotient. The number being divided is called a dividend, while the dividing number is called a divisor. There is one number that can never be divided into another. It is impossible to divide zero into any number. For example, $5 \div 0$ is impossible. We say that division by zero is undefined.

There are three ways of indicating division:

$$\frac{\text{quotient}}{\text{divisor}) \text{dividend}}$$

dividend ÷ divisor = quotient
$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

For example, the problem of dividing 10 by 2 can be written as:

 $2\overline{)10}$ or $10 \div 2$ or $\frac{10}{2}$

We read $2\overline{)10}$ as "2 into 10", $10 \div 2$ as "10 divided by 2" and $\frac{10}{2}$ as "10 over 2".

Example 1

Find $860 \div 12$ by long division.

Solution

Write the question as 12 into 860. Now, 12 into 8? There are no 12's in 8. 12 into 86 goes 7 times. Place 7 above the 6 and $7 \times 12 = 84$. Place 84 below 86 and subtract. Bring down the 0. Now 12 goes into 20 once. Place the 1 above the 0 and $1 \times 12 = 12$. Place 12 below 20 and subtract. Since there is nothing to bring down, 8 is the remainder. We write the quotient as 71 R8.

71	
12)860)
<u>84</u>	
20)
12)
8	3

860 ÷ 12 = 71 R8

Whenever we divide whole numbers we often get a remainder. We use the letter R to indicate the remainder.

When dividing decimal fractions, we must take special care in dealing with the decimal points. We can make the task simpler by rearranging the question so that we always divide by a whole number.

Example 2

Find $9.407 \div 0.23$ by long division.

Solution

Write as 0.23 into 9.407. Multiply both the divisor and dividend by 100 and rewrite as 23 into 940.7. This allows us to divide by a whole number. Place the decimal point in the quotient directly above the decimal point in the dividend. Divide as in Example 1. Notice that 23 goes into 20 zero times.

0.23)9.407
<u>40.9</u> 23)940.7
<u>92</u>
20
0
207
<u>207</u>
0

 $9.407 \div 0.23 = 40.9$

The process of long division can sometimes produce endless non-zero remainders. We can deal with this by rounding the quotient to an appropriate place value.

Example 3

Find $13 \div 0.7$ by long division. Round the answer to the nearest hundredth.

Solution

Write as 0.7 into 13. Multiply both the divisor and dividend by 10. Rewrite as 7 into 130. Now we can divide by a whole number. Since we must round the quotient to two places after the decimal point, write three zeros after the 130. Divide as before.

Arithmetic and Estimation

0.7)13
7)130
$\frac{18.571}{7)130.000}$
<u>7</u> 60
<u> 56</u> 40
$\frac{35}{50}$
<u> </u>
10
3

Rounding the quotient, $13 \div 0.7 = 18.57$.



Now complete Exercise 6 and check your answers.

1.	Divide.		
	a. 56 ÷ 7	b. $\frac{54}{9}$	c. 72 ÷ 8
	d. 83 ÷ 1	e. $\frac{0}{6}$	f. 6 ÷ 0
	g. $\frac{63}{9}$	h. 43 ÷ 43	i. 850 ÷ 10
	j. 6.3 ÷ 10	k. 94.7 ÷ 100	1. 0.3 ÷ 0.03
2.	Divide.		
	a. 5)124	b. 7)7650	c. 12)6012
	d. 9)111.06	e. 41)125.46	f08)112
	g. 0.06)96	h. 1.5)47.55	i. 0.22)0.6666
	j. 0.048)2.4	k. 1.02)91.8	1. 3.4)346.902

Arithmetic and Estimation

- 3. Divide. Round the answers to the nearest hundredths.
 - a. $3\overline{)61}$ b. $7\overline{)20}$ c. $0.6\overline{)11}$

 d. $11\overline{)6}$ e. $1.3\overline{)5.4}$ f. $0.17\overline{)9.2}$

 g. $0.41\overline{)18.63}$ h. $23\overline{)842.96}$
- 4. Three children decide to share the cost of their mother's \$40 gift. How much should each pay? Round the answer to the nearest cent.

5. Kim drove 310 km on 28.5 litres of gas. How far does she travel on one litre of gas? Round the answer to the nearest tenth.

6. Tracy bought 9 apples for \$1.71. How much did each apple cost?

Answers are on page 58.

Unit 7: Order of operations

Consider the following calculation:

 $2 + 3 \times 4$

Is it $5 \times 4 = 20$ or 2 + 12 = 14? In other words, do we add first or multiply first? Mathematicians have decided upon the following order of operations:

1. Brackets, such as (), { } or []

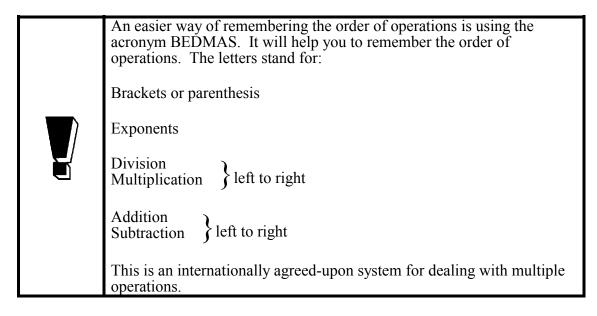
First, look for brackets and perform all operations within the brackets.

2. Multiplying and dividing

Second, perform all multiplying and dividing going from left to right.

3. Adding and subtracting

The last operations to be performed are adding and subtracting, from left to right.



We can now answer the above question:

 $2 + 3 \times 4 = 2 + 12 = 14$

Example 1

Find 50 - 2(3 + 8)

Solution

First, add 3 + 8 inside the brackets.

50 - 2(3 + 8) = 50 - 2(11)

Next, multiply 2(11). This expression means 2×11 . The operation of multiplication is always indicated when a number is placed in front of brackets.

50 - 2(11) = 50 - 22

Last, subtract 50 - 22.

50 - 22 = 28

When working towards a solution, it is safest to perform just one operation at a time.

Example 2

Calculate $7 + 3 \div 0.5 \times 10$

Solution

First the division, then the multiplication and finally the adding.

$$7 + 3 \div 0.5 \times 10$$

= 7 + 6 × 10
= 7 + 60
= 67

Example 3

Calculate $2[1.4 \div 0.7 \times 0.5 \div (0.05 + 0.2)]$

Solution

First, the innermost brackets. Then, within brackets do multiplying or dividing going from left to right. Finally, multiply by 2.

 $2[1.4 \div 0.7 \times 0.5 \div (0.05 + 0.2)]$ = 2[1.4 ÷ 0.7 × 0.5 ÷ (0.25)] = 2[2 × 0.5 ÷ (0.25)] = 2[1.0 ÷ (0.25)] = 2[4] = 8



Now complete Exercise 7 and check your answers.

1.	Calculate		
	a. $12 + 6 \div 3$	b. $36 \div 9 \times 2$	c. $36 \times 2 \div 9$
	d. $10 \div 5 + 5$	e. 2(8.1 - 0.6)	f. $99 \div (10 - 0.1)$
	g. $99 + 10 \div 0.1$	h. $6.5 - 3.2 \div 0.8$	i. 16 + 4(1.0 - 0.5)
2.	Calculate		
	a. $16 - (2 + 3 - 1) \div 2$	b. $18 \div 3 + 3 \times 5 - 2$	
	c. $3[0.3+3(3+0.3)]$	d. $14 - 7 \div (0.3 + 0.4)$	4) × 0.2
	(0, 7)(0, 1, 7)		- 0 1)
	e. (9 - 7)(9 + 7)	f. 0.5(1.0 - 0.1)(1.0	+ 0.1)
	g. 4+6(8.1 - 1.1) - 25	h. 20 - $12 \div 2 \times 6(0)$	8 - 0 3)
	D . 1 0 (0.1 1 .1, 2 0	n. 20 12 2×0(0.	

3. Donna has saved \$180. She plans to save \$55 every week for the next 8 weeks. How much will she have saved altogether after 8 weeks?

4. Four people went to dinner. Three people had the \$12.95 dinner special while the fourth person had a \$14.95 dinner. The wine totaled \$22.50. If they split the bill four ways, how much should each pay?

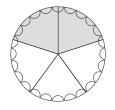
Answers are on page 58.

Unit 8: Operations with fractions

Common fractions are numbers that express parts of a whole. Suppose a pie is cut into five equal pieces and we eat two of them. Then we can say that we ate two-fifths or $\frac{2}{5}$ of the pie.

A common fraction is always made of two numbers. The top number is called the numerator and the bottom number is called the denominator.

2 numerator 5 denominator



When working with common fractions it is important to remember that the whole is the same as the number one. If we ate the whole pie, it is the same as eating $\frac{5}{5}$ of the pie. Fractions can be used to represent numbers less than one, more than one or equal to one.

Proper fractions are less than one. Their numerators are less than their denominators. For example, $\frac{1}{2} < 1$, $\frac{3}{4} < 1$, $\frac{9}{10} < 1$ and $\frac{1}{40} < 1$.

Improper fractions are equal to or greater than one. For example, $\frac{4}{3} > 1$, $\frac{10}{10} = 1$, $\frac{9}{4} > 1$ and

$$\frac{13}{12} > 1$$

Mixed numbers are composed of a whole number and a proper fraction. Mixed numbers are always greater than one. For example, $1\frac{1}{4} > 1$, $5\frac{3}{8} > 1$, $10\frac{1}{2} > 1$ and $2\frac{4}{7} > 1$.

The rectangles shown below have been divided into four equal parts. The shaded part can be described by the improper fraction as $\frac{9}{4}$ or by the mixed number $2\frac{1}{4}$.



Example 1

Write $\frac{13}{5}$ as a mixed number.

Solution

The idea is to find the number of $\frac{5}{5}$ in $\frac{13}{5}$ or

$$\frac{13}{5} = \frac{5}{5} + \frac{5}{5} + \frac{3}{5} = 1 + 1 + \frac{3}{5} = 2\frac{3}{5}$$

The quickest way to write an improper fraction as a mixed fraction is to divide the numerator by the denominator.

$$\frac{13}{5} \Rightarrow \frac{2}{13} \Rightarrow 2\frac{3}{5}$$
$$\frac{10}{3} \Rightarrow 2\frac{3}{5}$$

Example 2

Write $4\frac{3}{8}$ as an improper fraction.

Solution

The idea is to find the number of eighths in $4^3/_8$.

$$4\frac{3}{8} = 1 + 1 + 1 + 1 + \frac{3}{8} = \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{3}{8} = \frac{35}{8}$$

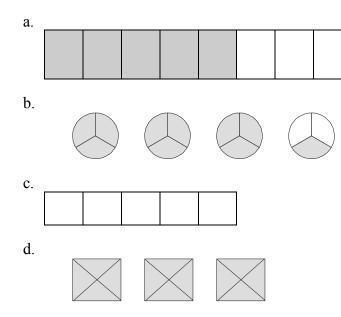
The quickest way to change a mixed number to an improper fraction is to multiply the whole number by the denominator and add this product to the numerator.

$$4\frac{3}{8} = \frac{4 \times 8 + 3}{8} = \frac{32 + 3}{8} = \frac{35}{8}$$



Now complete Exercise 8 and check your answers.

1. Write the fraction that describes the shaded regions.



2. Write as mixed numbers.

3.

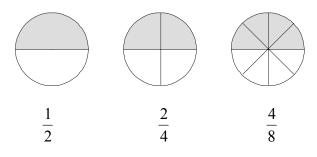
a. $\frac{7}{5}$	b. $\frac{19}{2}$	c. $\frac{41}{9}$	d. $\frac{12}{4}$		
e. $\frac{8}{8}$	f. $\frac{64}{11}$	g. $\frac{43}{22}$	h. $\frac{10}{9}$		
i. $\frac{84}{7}$					
Write as improper fractions.					

a. $2\frac{1}{2}$ b. $10\frac{3}{4}$ c. $4\frac{3}{5}$ d. 8 e. $33\frac{1}{3}$ f. $9\frac{5}{8}$ g. $14\frac{9}{10}$ h. $1\frac{73}{100}$ i. $15\frac{7}{12}$

4. Bill has 31 quarters. How much money, in dollars and quarters is this? Answers are on page 58.

Unit 9: Equivalent fractions

In each of the following, one half of the pie is shaded.



The common fractions $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ are called equivalent fractions because they represent the same value. Here, $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

There are many, actually infinitely many, ways of expressing the value 1/2.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \dots$$

For the ease of understanding, it is always best to express fractions in the lowest terms. A fraction is said to be in the lowest terms when neither the numerator or the denominator is divisible by the same prime number. For example, $\frac{10}{15}$ is not in the lowest terms since both 10 and 15 are divisible by 5.

Prime numbers are divisible only by one or themselves:

2, 3, 5, 7, 11, 13, 17, 19, 23, ...

Example 1

Write 140 as a product of primes.

Solution

The process is to keep dividing by the prime numbers 2, 3, 5, 7, 11, ...

Divide 140 by 2.

 $140 = 2 \times 70$

Divide 70 by 2.

 $140 = 2 \times 2 \times 35$

35 is not divisible by 2 or 3, so divide 35 by 5.

 $140 = 2 \times 2 \times 5 \times 7$

The quotient 7 is a prime so we are done.

The method used in Example 1 is not the only way to write 140 as a product of primes. We could easily have obtained the same answer as follows:

 $140 = 14 \times 10 = 2 \times 7 \times 2 \times 5 = 2 \times 2 \times 5 \times 7$

Factoring the numerator and denominator of a fraction is one way of helping to reduce the fraction to lowest terms. Prime numbers that occur in both the numerator and denominator can then be cancelled and replaced with a one. For example:

$$\frac{15}{20} = \frac{3 \times 5}{2 \times 2 \times 5} = \frac{3 \times 5}{2 \times 2 \times 5} = \frac{3}{2 \times 2} = \frac{3}{4}$$

Example 2

Reduce:

a.
$$\frac{30}{63}$$
 b. $\frac{66}{24}$ c. $5\frac{6}{42}$

Solution

Use the method in Example 1 to factor the numerator and denominator. Replace crossed out factors with ones. Then multiply the remaining factors.

a.
$$\frac{30}{63} = \frac{2 \times 3 \times 5}{3 \times 3 \times 7} = \frac{2 \times 3^{1} \times 5}{3_{1} \times 3 \times 7} = \frac{10}{21}$$

b.
$$\frac{66}{24} = \frac{2 \times 3 \times 11}{2 \times 2 \times 2 \times 3} = \frac{2^{1} \times 3^{1} \times 11}{2_{1} \times 2 \times 2 \times 3_{1}} = \frac{11}{4}$$

c.
$$5\frac{6}{42} = 5\frac{2 \times 3}{2 \times 3 \times 7} = 5\frac{2^{1} \times 3^{1}}{2_{1} \times 3_{1} \times 7} = 5\frac{1}{7}$$

There are many short cuts to reducing fractions and you may already be aware of some. If you have a method that works, keep using it. Check with your instructor on the validity of your method.



Now complete Exercise 9 and check your answers.

Exercise 9

1.	Factor (write as a product of primes).							
	Example: $60 = 2 \times 30 = 2 \times 2 \times 15 = 2 \times 2 \times 3 \times 5$							
	a. 80	b. 18	c. 35	d. 49				
	e. 77	f. 100	g. 144	h. 20				
	i. 200	j. 150						

- 2. List the prime numbers that are greater than 20 but less than 40.
- 3. Factor the numerator. Factor the denominator. Then reduce to lowest terms by cancelling.

a.
$$\frac{9}{24}$$
 b. $\frac{3}{6}$
 c. $\frac{8}{12}$
 d. $\frac{10}{15}$

 e. $\frac{21}{21}$
 f. $\frac{25}{20}$
 g. $6\frac{10}{35}$
 h. $\frac{32}{64}$

 i. $\frac{60}{70}$
 j. $\frac{26}{20}$
 k. $\frac{32}{24}$
 l. $12\frac{18}{40}$

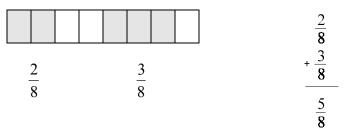
m.
$$\frac{8}{80}$$
 n. $\frac{51}{17}$ o. $\frac{26}{78}$ p. $9\frac{88}{110}$

4. Is 2001 a prime number?

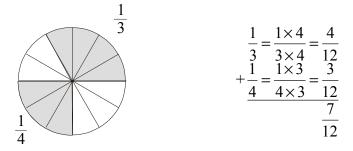
Answers are on page 59.

Unit 10: Adding and subtracting common fractions

Notice how we can add the shaded parts of the whole.



The fractions $\frac{2}{8}$ and $\frac{3}{8}$ have a common denominator. Before adding or subtracting fractions, the fractions must be written with a common denominator. The idea behind adding fractions that do not have a common denominator, like $\frac{1}{3}$ and $\frac{1}{4}$ is shown below.



Notice that the method for writing $\frac{1}{3}$ as $\frac{4}{12}$ is the reverse method of reducing fractions to lowest terms.

Example 1

Add
$$\frac{5}{6} + \frac{3}{8}$$

Solution

We need a common denominator. Think as follows. The larger denominator is 8. Will 6 divide evenly into 8? No. Now double the 8. $8 \times 2 = 16$. Will 6 divide evenly into 16? No. Triple 8. $8 \times 3 = 24$. Will 6 divide evenly into 24? Yes. The common denominator is 24.

Multiply $\frac{5}{6} \times \frac{4}{4}$ and $\frac{3}{8} \times \frac{3}{3}$ to get equivalent fractions. Rewrite the answer as a mixed number.

$$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24} + \frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24} - \frac{29}{24} = \frac{29}{24} = 1\frac{5}{24}$$

Example 2

Subtract $9\frac{4}{5} - 6\frac{3}{10}$

Solution

The common denominator is 10. Subtract the fractions and the whole numbers. Reduce the answer to the lowest terms.

$$9\frac{4}{5} = 9\frac{4 \times 2}{5 \times 2} = 9\frac{8}{10}$$
$$-\frac{6\frac{3}{10} = 6\frac{3}{10} = 6\frac{3}{10}}{3\frac{5}{10}}$$

 $3\frac{5}{10} = 3\frac{1}{2}$

Example 3

Subtract $4\frac{1}{3} - 1\frac{1}{2}$

Solution

The common denominator is 6. Notice that we cannot subtract $\frac{3}{6}$ from $\frac{2}{6}$. We must borrow a whole $\frac{6}{6}$ from the 4. Then add this $\frac{6}{6}$ to $\frac{2}{6}$. Now we can subtract.

Arithmetic and Estimation

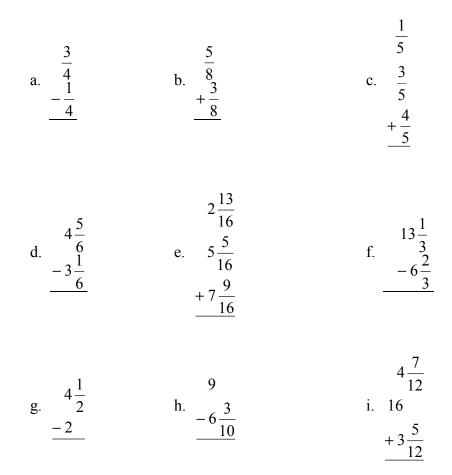
$$4\frac{1}{3} = 4\frac{1\times 2}{3\times 2} = 4\frac{2}{6}$$
$$-\frac{1}{2} = 1\frac{1\times 3}{2\times 3} = 1\frac{3}{6}$$
$$4\frac{2}{6} = 3\frac{6}{6} + \frac{2}{6} = 3\frac{8}{6}$$
$$-\frac{1\frac{3}{6}}{2\frac{6}{6}} = 1\frac{3}{6}$$
$$2\frac{5}{6}$$



Now complete Exercise 10 and check your answers.

Exercise 10

1. Add or subtract as indicated. Reduce your answers to the lowest terms.



2. Add, then reduce your answers to the lowest terms.

a.
$$\frac{\frac{1}{4}}{\frac{+\frac{1}{2}}{2}}$$
 b. $\frac{\frac{2}{3}}{\frac{+\frac{1}{4}}{4}}$ c. $\frac{\frac{7}{8}}{\frac{+\frac{3}{4}}{\frac{+\frac{3}{4}}{4}}}$

d.
$$\frac{\frac{2}{15}}{\frac{+\frac{7}{10}}{\frac{10}{2}}}$$
 e. $\frac{16\frac{3}{4}}{\frac{+24\frac{5}{6}}{\frac{10}{2}}}$ f. $\frac{2\frac{2}{3}}{\frac{+\frac{11}{12}}{\frac{+\frac{11}{12}}{\frac{11}{2}}}}$

Arithmetic and Estimation

$$\begin{array}{cccc}
\frac{1}{2} & 4\frac{2}{5} \\
g. \frac{1}{3} & h. 17\frac{1}{3} \\
+\frac{1}{4} & +6\frac{5}{6}
\end{array}$$

3. Subtract, then reduce your answers to lowest terms.

a.
$$\frac{5}{6}$$

 $-\frac{1}{3}$
b. $\frac{59}{100}$
c. $7\frac{5}{8}$
 $-\frac{3}{10}$
c. $-2\frac{1}{4}$

d.
$$-4\frac{3}{4}$$
 e. $\frac{19\frac{1}{2}}{-11\frac{2}{3}}$ f. $\frac{5\frac{3}{8}}{-\frac{3}{4}}$

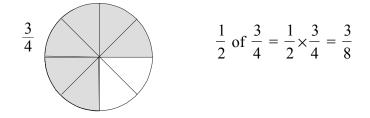
g.
$$\frac{10\frac{3}{16}}{-5\frac{2}{3}}$$
 h. $\frac{22\frac{7}{10}}{-21\frac{3}{4}}$

- 4. If Jack ate half a pie and Jill ate a third of the same pie, how much was left over?
- 5. Roy worked $3\frac{1}{4}$ hours on Monday, $5\frac{1}{2}$ hours on Wednesday and $4\frac{1}{3}$ hours on Friday. How many hours altogether did he work?
- 6. Joan cut $7\frac{3}{4}$ inches off a 12 inch board. How much of the board was left?

Answers are on page 59.

Unit 11: Multiplying common fractions

The pie below is cut into 8 parts. $\frac{3}{4}$ of the pie is shaded. Suppose we want to find $\frac{1}{2}$ of $\frac{3}{4}$ or $\frac{1}{2} \times \frac{3}{4}$. Notice that the word "of" means "times" in arithmetic.



When multiplying fractions, multiply the numerator times the numerator and the denominator times the denominator.

Example 1

Multiply $\frac{2}{3} \times \frac{4}{5}$

Solution

Simply multiply numerators and denominators.

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

Example 2

Multiply $2\frac{3}{4} \times 5$

Solution

Write both numbers as improper fractions, then multiply numerators and denominators. Rewrite the answer as a mixed number.

$$2\frac{3}{4} \times 5 = \frac{11}{4} \times \frac{5}{1} = \frac{11 \times 5}{4 \times 1} = \frac{55}{4} = 13\frac{3}{4}$$

It is sometimes possible to cancel common factors from the question before multiplying.

Example 3

Multiply $\frac{4}{21} \times 3\frac{1}{2}$

Solution

Write $3\frac{1}{2}$ as $\frac{7}{2}$. Factor all numerators and denominators. Cancel common factors. If we did not cancel first, then $\frac{4}{21} \times \frac{7}{2} = \frac{28}{42}$ and we would have to reduce $\frac{28}{42}$ to lowest terms regardless.

$$\frac{4}{21} \times 3\frac{1}{2} = \frac{4}{21} \times \frac{7}{2} = \frac{2 \times 2 \times 7}{3 \times 7 \times 2} = \frac{2^1 \times 2 \times 7^1}{3 \times 7_1 \times 2_1} = \frac{2}{3}$$

The steps to multiplying fractions are:

Step 1: Rewrite any mixed numbers as improper fractions.

Step 2: Factor, if possible, all numerators and denominators.

Step 3: Cancel factors common to both the numerator and denominator.

Step 4: Multiply the remaining factors.



Now complete Exercise 11 and check your answers.

Exercise 11

1. Multiply, then write the products in the lowest terms.

a.
$$\frac{2}{3} \times \frac{7}{11}$$
 b. $\frac{5}{12} \times \frac{3}{5}$ c. $\frac{5}{8} \times \frac{8}{5}$ d. $\frac{7}{8} \times 1\frac{1}{2}$

e.
$$3\frac{1}{4} \times 5$$
 f. $2\frac{7}{10} \times 5\frac{1}{3}$ g. $12 \times \frac{3}{4}$ h. $1\frac{1}{2} \times 1\frac{3}{5}$

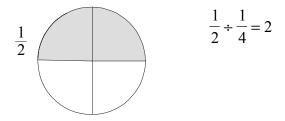
i.
$$2\frac{7}{8} \times 3\frac{1}{5}$$
 j. $\frac{3}{4} \times 7\frac{2}{9}$

- 2. Shelly exercised $\frac{3}{4}$ of an hour every day. How many hours in a week did she exercise?
- 3. Bill did $\frac{2}{3}$ of the 69 math questions. How many questions did he not do?
- 4. Carol spent three quarters of the $1\frac{1}{2}$ hour long English class staring out the window. How much time did she spend doing this?

Answers are on page 60.

Unit 12: Dividing common fractions

How many quarters are there in one half? The drawing suggests that there are two quarters in the shaded half of the pie. Mathematically, we are asking, what is one half divided by one quarter, or



Notice also that $\frac{1}{2} \times \frac{4}{1} = 2$. The number 4 is the reciprocal of $\frac{1}{4}$. The reciprocal of a fraction is found by interchanging the numerator and the denominator. Note that your calculator has a reciprocal key $\frac{1}{x}$

Division questions can be rewritten as multiplication questions. Dividing by a number is the same as multiplying by the reciprocal of that number.

Example 1

Divide $\frac{1}{2} \div \frac{2}{3}$

Solution

Rewrite as a multiplication question by changing the "÷" to a "×" and the $\frac{2}{3}$ to its reciprocal $\frac{3}{2}$, then multiply. $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{1 \times 3}{2 \times 2} = \frac{3}{4}$

Example 2

Divide $6\frac{3}{8} \div 5$

Solution

Rewrite as improper fractions. Then rewrite as a multiplication question. Then multiply and rewrite the answer as a mixed number.

$$6\frac{3}{8} \div 5 = \frac{51}{8} \div \frac{5}{1} = \frac{51}{8} \times \frac{1}{5} = \frac{51 \times 1}{8 \times 5} = \frac{51}{40} = 1\frac{11}{40}$$



Now complete Exercise 12 and check your answers.

Exercise 12

- 1. Divide a. $\frac{1}{2} \div \frac{3}{5}$ b. $\frac{1}{2} \div \frac{1}{2}$ c. $\frac{3}{4} \div 2$ d. $\frac{7}{12} \div \frac{4}{5}$ e. $1\frac{1}{2} \div \frac{3}{4}$ f. $\frac{1}{8} \div 2\frac{1}{4}$ g. $12 \div 2\frac{2}{5}$ h. $3\frac{2}{3} \div 5\frac{1}{4}$
 - i. $3 \div \frac{1}{2}$ j. $4\frac{1}{2} \div 3\frac{7}{8}$
- 2. Kim made nine plates in $2\frac{1}{4}$ hours. How much time did it take her to make one plate?

3. Greg cut two thirds of the cake into five equal pieces and then ate one piece. How much of the whole cake did Greg eat?

Answers are on page 60.

Unit 13: Conversions between common fractions and decimal fractions

It is often convenient to write a common fraction as a decimal fraction. For example, we think of three quarters as 75 cents or \$0.75. In other words

$$\frac{3}{4} = 0.75$$

To write a common fraction as a decimal fraction, divide the numerator by the denominator.

$$\frac{3}{4} = 3 \div 4 \text{ or } 4)\overline{3}$$

Example 1

Write $\frac{3}{8}$ as a decimal fraction.

Solution

Write $\frac{3}{8}$ as a long division question. Write 3 as 3.00 and add more decimal place zeros as needed.

 $\begin{array}{r}
 0.375 \\
 8)3.000 \\
 \underline{24} \\
 60 \\
 \underline{56} \\
 40 \\
 \underline{40} \\
 0 \\
 \underline{3} \\
 8 = 0.375
 \end{array}$

Arithmetic and Estimation

Example 2

Write $5\frac{1}{3}$ as a decimal fraction.

Solution

Use long division to divide the fraction $\frac{1}{3}$. Note that the quotient has repeating threes. Use a bar over the repeating digit. Place the whole number 5 before the decimal, 5.3 $\overline{3}$.

$$3\overline{\underline{)1.00}}^{\underline{.3\overline{3}}}$$

$$5\frac{1}{3} = 5.\overline{3}$$



Now complete Exercise 13 and check your answers.

Exercise 13

1. Rewrite as decimal fractions and then memorize the results.

a.
$$\frac{1}{2}$$
 b. $\frac{1}{3}$
 c. $\frac{2}{3}$
 d. $\frac{1}{4}$
 e. $\frac{2}{4}$

 f. $\frac{3}{4}$
 g. $\frac{1}{5}$
 h. $\frac{2}{5}$
 i. $\frac{3}{5}$
 j. $\frac{4}{5}$

2. Rewrite as decimal fractions.

a.
$$\frac{1}{6}$$
b. $\frac{5}{6}$ c. $\frac{1}{8}$ d. $\frac{5}{8}$ e. $2\frac{1}{4}$ f. $\frac{1}{12}$ g. $\frac{3}{10}$ h. $\frac{3}{100}$ i. $1\frac{5}{12}$ j. $6\frac{7}{8}$

3. Rewrite as decimal fractions and note the pattern.

a. $\frac{1}{9}$	b. $\frac{2}{9}$	c. $\frac{3}{9}$
d. $\frac{4}{9}$	e. $\frac{5}{9}$	f. $\frac{6}{9}$
g. $\frac{7}{9}$	h. $\frac{8}{9}$	i. $\frac{9}{9}$

4. Write as decimal fractions and note the pattern.

a.
$$\frac{20}{99}$$
 b. $\frac{52}{99}$ c. $\frac{1}{99}$
d. $\frac{13}{99}$ e. $\frac{5}{99}$ f. $\frac{98}{99}$

Answers are on page 60.

Unit 14: Conversions between decimal fractions and common fractions

There are three types of decimal fractions:

- 1. Terminating decimals have a fixed number of digits. For example, 0.5, 3.625 and 10.128 496.
- 2. Repeating decimals have one digit or a group of digits which repeat forever. A bar is placed over the repeating digit or group of repeating digits. For example:

 $\begin{array}{l} 0.333 \ 333 \ 333 \ldots = \ 0.\overline{3} \\ 6.934 \ 934 \ 934 \ldots = \ 6.\overline{934} \\ 8.056 \ 262 \ 626 = \ 8.05\overline{62} \end{array}$

3. Non-repeating decimals have an infinite number of digits but no repeating digit or group of digits. For example:

0.123 456 789 101 112... 6.505 505 550 555 505... 3.141 592 748 605 213...

Both terminating and repeating decimals can be written as common fractions. Non-repeating decimals cannot be written as common fractions. Study the following:

$$0.7 = \frac{7}{10} \qquad \qquad 0.83 = \frac{83}{100}$$

$$0.029 = \frac{29}{1000} \qquad \qquad 0.2011 = \frac{2011}{10000}$$

$$5.60891 = 5\frac{60891}{100000} \qquad \qquad 0.029 = \frac{29}{1000}$$

When a decimal is written as a fraction, the decimal digits appear in the numerator of the fraction and the denominator has the same number of zeros as there are decimal digits.

Example 1

Write 8.0306 as a common fraction. Reduce to the lowest term.

Solution

Write as a mixed number. The decimal part of 8.0306 has four digits, so the denominator is 10 000.

$$8.0306 = 8\frac{306}{10000} = 8\frac{153}{5000}$$



Now complete Exercise 14 and check your answers.

Exercise 14

1.	Write as com	mon fractions b	out do not reduc	ce to the lowes	t terms.
	a. 0.5	b. 0.61	c. 0.907	d. 1.1	e. 6.02
	f. 0.70	g. 0.001	h. 27.95	i. 0.0105	j. 1.502
2.	Write as com	mon fractions a	and reduce to th	e lowest terms	
	a. 0.5	b. 0.25	c. 0.125	d. 0.2	
	e. 0.04	f. 0.75	g. 0.6	h. 0.4	
3.	Write as deci	mal fractions.			
	a. $\frac{7}{9}$	b. $\frac{14}{99}$	c. $\frac{2}{9}$	d. $\frac{82}{99}$	

Answers are on page 62.

Unit 15: Estimation

Estimation is a useful tool in mathematical problem solving. Estimating gives you an approximate or "ballpark" answer. People often estimate differently. There is no one correct way to estimate. It takes practice to get good at choosing numbers that make estimating easier. Here are some estimating strategies:

- 1. Round to numbers that are easy for you to compute. For example, about how much is 29×14 ? Some possible estimates are:
 - $25 \times 16 = 400$ $30 \times 10 = 300$ $30 \times 15 = 450$
- 2. Use numbers that make sense to you. When you round, sometimes the result will be an underestimate (less than the exact answer) and sometimes it will be an overestimate (more than the exact answer).

problem	estimate	the exact answer is
31×4	$30 \times 4 = 120$	a little more than 120
48×5	$50 \times 5 = 250$	a little less than 250
17×21	$20 \times 20 = 400$	about 400

3. Choose numbers that are compatible and easy to work with for the problem. For example, how much is 2775 divided by 6? Some possible estimates are:

 $3000 \div 6 = 500$ $2800 \div 7 = 400$

4. Use whole numbers to estimate fractions and decimals. For example:

problem	estimate
19.32 - 7.29	19 - 7 = 12
3.7×8.2	$4 \times 8 = 32$
$16\frac{2}{3} + 12\frac{1}{8}$	17 + 12 = 29
$20\frac{3}{4} \div 3\frac{1}{5}$	$21 \div 3 = 7$

5. Group numbers to make estimation easier. For example, 48 + 27 + 55 + 75 + 98 is approximately = 300. This is because 48 + 55 is about 100, 27 + 75 is about 100 and 98 is about 100.



Now complete Exercise 15 and check your answers.

Arithmetic and Estimation

Exercise 15

- 1. Decide whether these will total more or less than \$20.
 - a. \$7.99 + \$8.99 b. \$4.79 + \$8.39 + \$7.19
 - c. \$12.87 + \$6.99 d. \$4.99 + \$3.29 + \$6.89
 - e. \$5.89 + \$13.99 f. \$7.69 + \$5.49 + \$7.99
- 2. Estimate the following
 - a. \$4.99 + \$3.29 + \$6.89 b. 7.4 × 9.8

c.
$$2\frac{1}{3} + 4\frac{1}{8} + 3\frac{1}{4}$$
 d. 4397 - 1970

e.
$$125 + 98 + 73 + 142 + 59$$
 f. $174\frac{3}{4} - 49\frac{3}{5}$

- g. 17.4 + 9.1 + 2.8 + 10.7 h. 795 ÷ 4
- i. $9\frac{2}{3} \times 4\frac{7}{8}$ j. $8125 \div 42$

Answers are on page 61.



Now complete the Practice Test and check your answers.

Practice Test

- 1. Write the place value for each underlined digit.
 - a. 8<u>4</u> 926 b. 6.0<u>2</u>04
- 2. Underline the tenths digit in 43.09.
- 3. Use >, < or = to compare the numbers below.
 - a. 93.090.3b. 84____84.00c. 0.0020.02d. 99.910.000
- 4. Round each of the following as indicated.
 - a. 3 452 to the nearest hundred
 - b. 16.97 to the nearest tenth
 - c. 804.99 to the nearest ten
- 5. Add 30.7 + 12 + 960.9 + 84.63
- 6. Subtract 689.49 from 1 207.3
- 7. Multiply the following.
 - a. 24.06 <u>×0.58</u>
 b. 768 <u>×0.097</u>
- 8. Divide the following.
 - a. $8)\overline{3205}$ b. $1.3)\overline{0.052}$
- 9. Divide 7 by 0.11 and round the answer to the nearest hundredth.
- 10. Calculate the following.

a. $28 - 12 \div 3 \times 2$ b. 0.4[10 - 0.2(4.8 - 0.8)]

Arithmetic and Estimation

- 11. Donna won \$5 000 and decided to share half of it among her three children. How much should each child receive?
- 12. Write as mixed numbers.

a.
$$\frac{25}{3}$$
 b. $\frac{39}{6}$

13. Write as improper fractions.

a.
$$4\frac{3}{5}$$
 b. $16\frac{2}{3}$

14. Reduce to lowest terms.

a.
$$\frac{25}{30}$$
 b. $8\frac{12}{90}$

15. Add or subtract as indicated, then reduce the answers to lowest terms.

a.
$$\frac{2}{5}$$
 b. $\frac{7}{8}$ c. $18\frac{5}{12}$
 $+\frac{1}{3}$ $-\frac{1}{2}$ $+\frac{1}{3}$

d.
$$\frac{12}{-3\frac{7}{10}}$$
 e. $4\frac{3}{16}$
 $-\frac{3}{4}$

16. Multiply or divide, then reduce the answers to the lowest terms.

a.
$$6 \times \frac{3}{4}$$

b. $4\frac{1}{3} \div \frac{1}{2}$
c. $\frac{9}{10} \times \frac{5}{12}$
d. $3\frac{3}{8} \div 2\frac{2}{5}$

17. Write as decimal fractions.

a.
$$\frac{5}{8}$$
 b. $3\frac{4}{11}$

- 18. Write as common fractions.
 - a. 0.501 b. 12.3
- 19. Audrey claims to have read two-thirds of a 396 page report. How many pages did she read?
- 20. Bill did $\frac{1}{4}$ of his assignment the first night, then $\frac{1}{3}$ the second night. How much would he have left to do on the third night?
- 21. The recipe calls for $\frac{3}{4}$ cup of sugar if you want to serve 6 people. How much sugar should you use to serve 2 people?
- 22. Estimate the following.

a. 14.7 + 12.36 + 18.25 b. $5\frac{3}{4} \times 3\frac{1}{8} \times 9\frac{5}{6}$

23. In driving a distance of 489 km, Paul's car consumes 51.2 litres of gasoline. Estimate how many kilometres he can travel on one litre of gasoline.

Answers are on pages 61-62.

Answers

Exercise 1 - page 7

 a. ones d. ten thousand g. tenths j. thousands 	b. thousandthse. hundred thousandthsh. ones	c. hundredths f. millions i. hundredths
2. a. 8 <u>4</u> 2.56 b. 842. <u>5</u> 6 e. 63 521.009 842	c. 68 923.0 <u>5</u>	d. 84 <u>3</u> 907 000
3. a. $823 = 800 + 20 + 3$	b. $8.23 = 8 + \frac{2}{10} + \frac{3}{100}$	
c. $9\ 605 = 9\ 000 + 600 + 5$	d. $0.087 = \frac{0}{10} + \frac{8}{100} + \frac{3}{100}$	7 00
e. $3\ 842.65 = 3\ 000 + 800 + 40$	$+2+\frac{6}{10}+\frac{5}{100}$	
4. \$638		
5. \$50.32		
6. \$11.04		
7. 2 hundred dollar bills, 0 ten doll8. 18 300 000 dollars	lar bills, 3 loonies, 4 dimes an	nd 1 penny

Exercise 2 - page 10

Exercise 3 - page 13

1. a. 8400	b. 200	c. 100	d. 0	e. 1 343 00	0
f. 400					
2. a. 6	b. 19	c. 50	d. 56	e. 1	f. 3
g. 500	h. 2	i. 110	j. 0		
3. a. 43 830 000	b. 0.0	c. 0.50d.	0.4382 e	. 1 f. 6	50
4. a. 0.91	b. 16.58	c. 0.12d.	968.16 e	. 0.00f. 10.70	
g. 45.02	h. 5.31	i. 0.10	j. 11.68		
	¢04 10 1	Ф 4 4			

5. Mike \$20, Mary \$84 and Sandra \$44

Exercise 4 - pages 15-16

1. a. 941	b. 1 094	c. 45 134		
2. a. 1044	b. 90 315	c. 174 419		
3. a. 96.35	b. 155.79	c. 1 327.457		
4. a. 193.16	b. 9678.173	c. 0.958 5		
5. a. 143	b. 19	c. 78 989	d. 21.19	e. 0.020 57
f. 95.26	g. 3.823	h. 60 475.19	i. 0.005 899	
6. a. 82.17	b. 22.48	c. 0.497	d. 65.369 8	
7. \$3.17				
8. \$27.07				
9. a. \$280.51	b. \$1 171.96	c. \$1 138.27	d. \$991.92	e. \$541.92
10. a. 460	b. 8300			
100	<u>-320</u>			
0	7980			
80				
640				

Exercise 5 - pages 18-19

1.

1.												
	×	0	1	2	3	4	5	6	7	8	9	
	0	0	0	0	0	0	0	0	0	0	0	
	1	0	1	2	3	4	5	6	7	8	9	
	2	0	2	4	6	8	10	12	14	16	18	
	3	0	3	6	9	12	15	18	21	24	27	
	4	0	4	8	12	16	20	24	28	32	36	
	5	0	5	10	15	20	25	30	35	40	45	
	6	0	6	12	18	24	30	36	42	48	54	
	7	0	7	14	21	28	35	42	49	56	63	
	8	0	8	16	24	32	40	48	56	64	72	
	9	0	9	18	27	36	45	54	63	72	81	
2. a. 0				b. th	at nu	mber		c. 0 or 5				d. 9
3. a. 20)5			b. 12	248			c. 82 128				d. 4 069 848
e. 91	1.3			f. 10	.64			g. 303.408				h. 165 88.95
i. 1.	i. 1.141 j. 1 230.08				k. 0.555				1. 0.000 082			
4. a. 10).62				024 (c. (0.000	12		d. 0.000 007 9
e. 0.	000	064										
e. 0.000 064 5. \$36 619.70												

6. 378 km

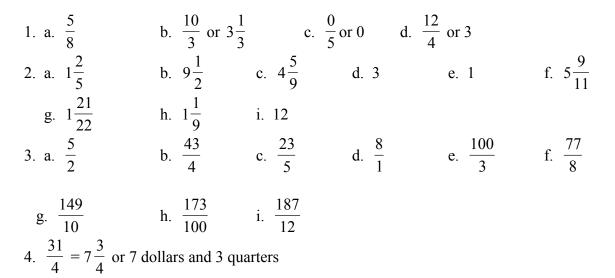
Exercise 6 - pages 23-24

1. a. 8 b. 6 c. 9 d. 83 e. 0 f. Undefined. It is impossible to divide any number by 0. j. 0.63 k. 0.947 l. 10 g. 7 h. 1 i. 85 d. 12.34e. 3.06f. 140j. 50k. 90d. 0.55e. 4.15f. 54.12 2. a. 24 R4 b. 1 092 R6 c. 501 i. 3.03 g. 1600 h. 31.7 k. 90 l. 102.03 3. a. 20.33 b. 2.86 c. 18.33 g. 45.44 h. 36.65 4. \$13.33 5. 10.9 km 6. \$0.19

Exercise 7 - page 27

1. a. 14 b. 8 c. 8 d. 7 e. 15 f. 10 g. 199 h. 2.5 i. 18 2. a. 14 b. 19 c. 30.6d. 12 e. 32 f. 0.495 g. 21 h. 2 3. \$180 + 8(\$55) = \$6204. $[12.95(3) + 14.95 + 22.50] \div 4 = \19.08

Exercise 8 - page 30



Exercise 9 - page 33

1. a. $80 = 2 \times 40 = 2 \times 2 \times 20 = 2 \times 2 \times 2 \times 10 = 2 \times 2 \times 2 \times 2 \times 5$ b. $18 = 2 \times 9 = 2 \times 3 \times 3$ c. $35 = 5 \times 7$ d. $49 = 7 \times 7$ e. $77 = 7 \times 11$ f. $100 = 2 \times 50 = 2 \times 2 \times 25 = 2 \times 2 \times 5 \times 5$ g. $144 = 2 \times 72 = 2 \times 2 \times 36 = 2 \times 2 \times 2 \times 18 = 2 \times 2 \times 2 \times 2 \times 9 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$ h. $20 = 2 \times 100 = 2 \times 2 \times 5$ i. $200 = 2 \times 100 = 2 \times 2 \times 2 \times 5 \times 5$ j. $150 = 2 \times 75 = 2 \times 3 \times 25 = 2 \times 3 \times 5 \times 5$ 2. 23, 29, 31, 373. a. $\frac{3}{8}$ b. $\frac{1}{2}$ c. $\frac{2}{3}$ d. $\frac{2}{3}$ e. 1 f. $\frac{5}{4}$ g. $6\frac{2}{7}$ h. $\frac{1}{2}$ i. $\frac{6}{7}$ j. $\frac{13}{10}$ k. $\frac{4}{3}$ l. $12\frac{9}{20}$ m. $\frac{1}{10}$ n. 3 o. $\frac{1}{3}$ p. $9\frac{4}{5}$ 4. No. $2001 = 3 \times 23 \times 29$

Exercise 10 - pages 37-38

1. a.
$$\frac{2}{4} = \frac{1}{2}$$
 b. $\frac{8}{8} = 1$ c. $\frac{8}{5}$ or $1\frac{3}{5}$ d. $1\frac{4}{6} = 1\frac{2}{3}$ e. $14\frac{27}{16} = 15\frac{11}{16}$
f. $6\frac{2}{3}$ g. $2\frac{1}{2}$ h. $2\frac{7}{10}$ i. 24
2. a. $\frac{3}{4}$ b. $\frac{11}{12}$ c. $\frac{13}{8}$ or $1\frac{5}{8}$ d. $\frac{25}{30} = \frac{5}{6}$ e. $40\frac{19}{12} = 41\frac{7}{12}$
f. $2\frac{19}{12} = 3\frac{7}{12}$ g. $\frac{13}{12} = 1\frac{1}{12}$ h. $27\frac{47}{30} = 28\frac{17}{30}$
3. a. $\frac{3}{6} = \frac{1}{2}$ b. $\frac{29}{100}$ c. $5\frac{3}{8}$ d. $8\frac{1}{4}$ e. $7\frac{5}{6}$
f. $4\frac{5}{8}$ g. $4\frac{25}{48}$ h. $\frac{19}{20}$
4. $1 - (\frac{1}{2} + \frac{1}{3}) = 1 - \frac{5}{6} = \frac{1}{6}$ of the pie was left over
5. $3\frac{1}{4} + 5\frac{1}{2} + 4\frac{1}{3} = 12\frac{13}{12} = 13\frac{1}{12}$ hours
6. $12 - 7\frac{3}{4} = 4\frac{1}{4}$ inches

Exercise 11 - page 41

1. a. $\frac{14}{33}$ b. $\frac{1}{4}$ c. 1 d. $\frac{21}{16} = 1\frac{5}{16}$ e. $\frac{65}{4} = 16\frac{1}{4}$ f. $14\frac{2}{5}$ g. 9 h. $2\frac{2}{5}$ i. $9\frac{1}{5}$ j. $5\frac{5}{12}$ 2. $\frac{3}{4} \times 7 = \frac{21}{4} = 5\frac{1}{4}$ hours in a week 3. $69 - (\frac{2}{3} \times 69) = 69 - 46 = 23$ questions or $(1 - \frac{2}{3}) \times 69 = \frac{1}{3} \times 69 = 23$ questions 4. $\frac{3}{4} \times 1\frac{1}{2} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$ or $1\frac{1}{8}$ hours

Exercise 12 - page 44

1. a. $\frac{5}{6}$ b. 1 c. $\frac{3}{8}$ d. $\frac{35}{48}$ e. 2 f. $\frac{1}{18}$ g. 5 h. $\frac{44}{63}$ i. 6 j. $1\frac{5}{31}$ 2. $2\frac{1}{4} \div 9 = \frac{9}{4} \times \frac{1}{9} = \frac{1}{4}$ hour per plate 3. $\frac{2}{3} \div 5 = \frac{2}{15}$ of the cake

Exercise 13 - page 47

1. a.
$$\frac{1}{2} = 0.5$$
 b. $\frac{1}{3} = 0.\overline{3}$ c. $\frac{2}{3} = 0.\overline{6}$ d. $\frac{1}{4} = 0.25$ e. $\frac{2}{4} = 0.5$
f. $\frac{3}{4} = 0.75$ g. $\frac{1}{5} = 0.2$ h. $\frac{2}{5} = 0.4$ i. $\frac{3}{5} = 0.6$ j. $\frac{4}{5} = 0.8$
2. a. $0.1\overline{6}$ b. $0.8\overline{3}$ c. 0.125 d. 0.625 e. 2.25
f. $0.08\overline{3}$ g. 0.3 h. 0.03 i. $1.41\overline{6}$ j. 6.875
3. a. $\frac{1}{9} = 0.\overline{1}$ b. $\frac{2}{9} = 0.\overline{2}$ c. $\frac{3}{9} = 0.\overline{3}$ d. $\frac{4}{9} = 0.\overline{4}$ e. $\frac{5}{9} = 0.\overline{5}$
f. $\frac{6}{9} = 0.\overline{6}$ g. $\frac{7}{9} = 0.\overline{7}$ h. $\frac{8}{9} = 0.\overline{8}$ i. $\frac{9}{9} = 1$
4. a. $\frac{20}{99} = 0.\overline{20}$ b. $\frac{52}{99} = 0.\overline{52}$ c. $\frac{1}{99} = 0.\overline{01}$
d. $\frac{13}{99} = 0.\overline{13}$ e. $\frac{5}{99} = 0.\overline{05}$ f. $\frac{98}{99} = 0.\overline{98}$

Exercise 14 - page 50

1. a. $\frac{5}{10}$	b. $\frac{61}{100}$	c. $\frac{907}{1000}$	d. $1\frac{1}{10}$	e. $6\frac{2}{100}$	f. $\frac{70}{100}$
	h. $27\frac{95}{100}$	i. $\frac{105}{10000}$	j. $1\frac{502}{1000}$		
2. a. $\frac{1}{2}$	b. $\frac{1}{4}$	c. $\frac{1}{8}$	d. $\frac{1}{5}$	e. $\frac{1}{25}$	f. $\frac{3}{4}$
g. $\frac{3}{5}$	h. $\frac{2}{5}$				
3. a. $0.\overline{7}$	b. $0.\overline{14}$	c. $0.\overline{2}$	d. $0.\overline{82}$		

Exercise 15 - page 52

1. a. less	b. more	c. less	d. less	e. less	f. more
2. (answers	may vary slig	ghtly)			
a. \$15	b. 70	c. 9	d. 2400	e. 500	f. 125
g. 40	h. 200	i. 50	j. 200		

Practice Test - pages 53-55

1. a. thousands b. hundredths 2. 43.09 3. a. > b. = c. < d. > 4. a. 3500 b. 17.0 c. 800 5. 1 088.23 6. 517.81 7. a. 13.9548 b. 74.496 8. a. 400 R5 b. 0.04 9. 63.64 10. a. 20 b. 3.68 11. \$833.33 12. a. $8\frac{1}{3}$ b. $6\frac{1}{2}$ 13. a. $\frac{23}{5}$ b. $\frac{50}{3}$ 14. a. $\frac{5}{6}$ b. $8\frac{2}{15}$ 15. a. $\frac{11}{15}$ b. $\frac{3}{8}$ c. $25\frac{3}{20}$ d. $8\frac{3}{10}$ e. $3\frac{7}{16}$ 16. a. $\frac{9}{2}$ or $4\frac{1}{2}$ b. $\frac{26}{3}$ or $8\frac{2}{3}$ c. $\frac{3}{8}$ d. $\frac{45}{32}$ or $1\frac{13}{32}$ 17. a. 0.625 b. 3.36

18. a. $\frac{501}{1000}$ b. $12\frac{1}{3}$ 19. $\frac{2}{3} \times 396 = 264$ pages 20. $1 - \left(\frac{1}{4} + \frac{1}{3}\right) = 1 - \frac{7}{12} = \frac{5}{12}$ of the assignment 21. $\frac{2}{6} \times \frac{3}{4} = \frac{1}{4}$ cup 22. (answers may vary slightly) a. 45 b. 180 23. 10 kilometres

Intermediate Mathematics

- Module 1 Arithmetic and Estimation
- Module 2 Measurement
- Module 3 Perimeter, Area and Volume
- Module 4 Ratio and Proportion
- Module 5 Percent
- Module 6 Geometry: Introduction
- Module 7 Geometry: Construction
- Module 8 Statistics
- Module 9 Signed (Rational) Numbers
- Module 10 Algebra: Equations
- Module 11 Algebra: Polynomials
- Module 12 Powers, Roots and Scientific Notation
- Module 13 Graphing
- Module 14 Trigonometry