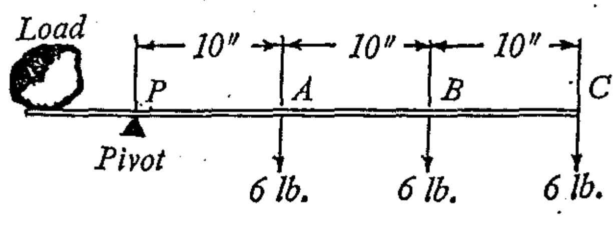
## Moment of a Force

The *moment* of a force about a point is the turning effect about the point produced by the force. It is calculated by multiplying the force by the perpendicular distance from the point to the line of action of the force.

*Example:*




In the diagram showing a lever, a 6-lb. force may be exerted at $A$, $B$ or $C$. The moment about $P$ of each force is shown below:

If force is applied at $A$, moment $= 6 \times 10 = 60$ (lb. in.).

If force is applied at $B$, moment $= 6 \times 20 = 120$ (lb. in.).

If force is applied at $C$, moment $= 6 \times 30 = 180$ (lb. in.).

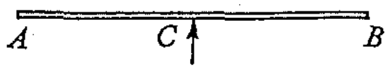
The moment of a force is used in many tools such as wrenches, pliers, scissors and levers. It is used to create a turning motion in bicycles and crank shafts.

Harris, R.F., Joyce, W.S. & Butterhill, R.D., (1961). Section VI – Statics (A.H. Heywood, Ed.). *In Senior Technical Math (pp. 362-372).* MacMillan of Canada.

Harris, R.F., Joyce, W.S. & Butterhill, R.D., (1961). Section VI – Statics (A.H. Heywood, Ed.). *In Senior Technical Math (pp. 362-372).* MacMillan of Canada.

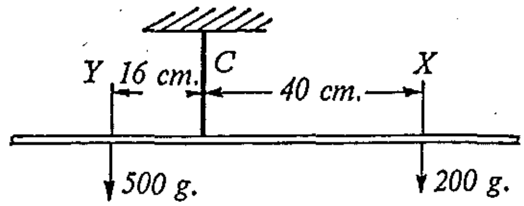


## Centre of Gravity of a Uniform Rod




Since a yard stick may be balanced at its mid-point $C$ the equilibrant which balances the entire weight is a single force upwards at $C$. This point is the *centre of gravity* of the rod. Hence the entire weight of a uniform rod may be treated as a single force acting downward at the mid-point of the rod.

## Experiment to Illustrate the Principal of Moments

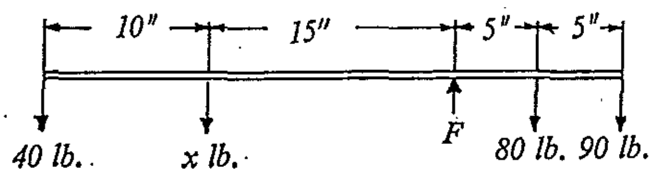



A metre stick is suspended by a string to balance at its centre of gravity $C$. A weight is suspended on one side at the point $X$, while another weight is adjusted on the other side at $Y$ until the system balances. When equilibrium is obtained, the clockwise moment of the force at $X$ is found to be equal to the counter-clockwise moment of the force at $Y$.

## Principal of Moments

If a body which is free to rotate about a point is in equilibrium when a number of forces act upon it, then the sum of the clockwise moments about the point is equal to the sum of the counter-clockwise moments.

*Example:* From the diagram, calculate the force $x$ lb. if the system is in equilibrium.




*Solution:*

Taking moments about the fulcrum $F$,

$$\text{Clockwise moments} = \text{Counter-clockwise moments.}$$

$$(80 \times 5) + (90 \times 10) = (40 \times 25) + (x \times 15),$$

$$400 + 900 = 1000 + x \times 15,$$

$$15x = 300,$$

$$x = 20 \text{ lb.}$$

CHAPTER XXV

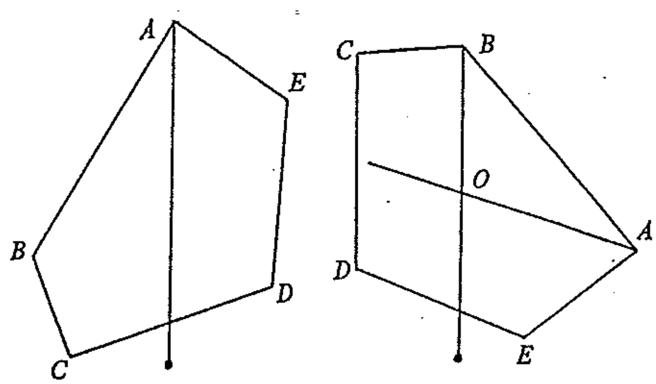
# CENTRE OF GRAVITY AND FRICTION

## Centre of Gravity

The *centre of gravity* of a body is the point at which the entire weight of the body may be considered to act. The body is made up of many small particles, each of which has its own weight. All these small weights will balance about some central point called the centre of balance or *Centre of Gravity*. (C.G.)

## Experiment to Locate the C.G.

A piece of cardboard of any shape $ABCDE$ is hung from a pin at $A$, and a weighted string also hangs from the pin. A line drawn along the string will then separate the cardboard into two equal areas.




The cardboard is then set up to hang from a point $B$ as shown and again a line is drawn along the string. The two lines cross at $O$, which is the C. G. of the cardboard.

### Exercise

B

1. Using a ruler and compasses only, draw a circle with radius $1\frac{1}{2}$ in. Draw two chords not parallel to each other. Construct the right bisectors of these two chords, and mark the point where they intersect at $O$. Point $O$ is the centre of the area of the circle and its C.G.

Harris, R.F., Joyce, W.S. & Butterhill, R.D., (1961). Section VI – Statics (A.H. Heywood, Ed.). *In Senior Technical Math (pp. 362-372)*. MacMillan of Canada.

2. Using a ruler and compasses only, construct a triangle $ABC$ having sides $1\frac{1}{2}$ in., 2 in. and 3 in. Draw a line from $A$ to the middle point of the opposite side. This line is called a median and divides the area of the triangle into halves. Draw another median from $B$ and have it meet the first median at point $O$. Point $O$ is called the centroid of the triangle and is its centre of area and C.G.

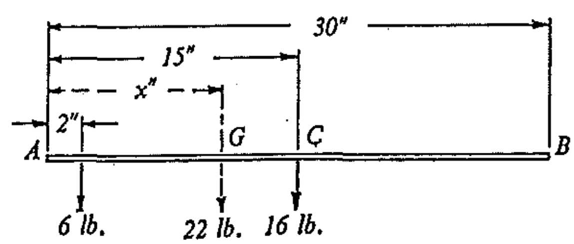
3. Show, using diagrams, how the C.G. of a rectangle, square, rhombus, and parallelogram, may be obtained by drawing two straight lines in each case. Show why the C.G. of a trapezium or U-shaped figure cannot be obtained in the same way.
Describe an experimental way of finding the C.G. of such areas.

## Mathematical Calculation of Centre of Gravity

*Example:* A uniform metal rod $AB$, 30 in. long and 16 lb. in weight has a mass of 6 lb. attached 2 in. from $A$. Find the C.G.

*Solution:*

The weight of the rod acts at the mid-point C. Let the C.G. of the two weights be at $G$ so that $AG = x$ in.




Then the entire 22 lb. may be considered to act at $G$. If moments are taken about $A$, the moment of the total weight acting at $G$ must be equal to the sum of the moments of the original two weights.

$$(22 \times x) = (6 \times 2) + (16 \times 15)$$

$$x = 11\tfrac{5}{11}.$$

The C.G. is $11\frac{5}{11}$ in. from $A$.

Harris, R.F., Joyce, W.S. & Butterhill, R.D., (1961). Section VI – Statics (A.H. Heywood, Ed.). *In Senior Technical Math (pp. 362-372).* MacMillan of Canada.

## Effect of removing or discharging mass

Consider a rectangular plank of homogeneous wood. Its centre of gravity will be at its geometrical center: – i.e., half-way along its length, half-way across its breadth, and at half depth. Let the mass of the plank be W kg and let it be supported by means of a wedge placed under the centre of gravity as shown in Figure 2.2. The plank will balance.

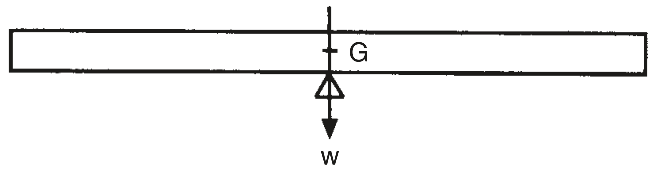



Fig. 2.2

Now let a short length of the plank, of mass w kg, be cut from one end such that its centre of gravity is d metres from the centre of gravity of the plank. The other end, now being of greater mass, will tilt downwards. Figure 2.3(a) shows that by removing the short length of plank a resultant moment of w × d kg m has been created in an anti-clockwise direction about G.

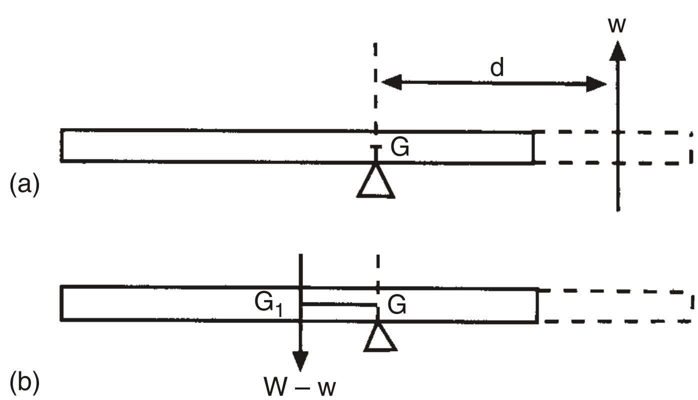



Fig. 2.3

Now consider the new length of plank as shown in Figure 2.3(b). The centre of gravity will have moved to the new half-length indicated by the distance G to $G_1$. The new mass, (W – w) kg, now produces a tilting moment of $(W - w) \times GG_1$ kg m about G.

Since these are simply two different ways of showing the same effect, the moments must be the same, i.e.

$$(W - w) \times GG_1 = w \times d$$

or

$$GG_1 = \frac{w \times d}{W - w} \text{ metres}$$

Derrett, D.R., Barrass, C.B., *Ship Stability for Masters and Mates*, Butterworth-Heinemann, 2000.

From this it may be concluded that when mass is removed from a body, the centre of gravity of the body will move directly away from the centre of gravity of the mass removed, and the distance it moves will be given by the formula:

$$GG_1 = \frac{w \times d}{\text{Final mass}} \text{ metres}$$

where $GG_1$ is the shift of the centre of gravity of the body, w is the mass removed, and d is the distance between the centre of gravity of the mass removed and the centre of gravity of the body.

## Application to ships

In each of the above figures, G represents the centre of gravity of the ship with a mass of w tonnes on board at a distance of d metres from G. G to $G_1$ represents the shift of the ship's centre of gravity due to discharging the mass.

In Figure 2.4(a), it will be noticed that the mass is vertically below G, and that when discharged G will move vertically upwards to $G_1$.

In Figure 2.4(b), the mass is vertically above G and the ship's centre of gravity will move directly downwards to $G_1$.

In Figure 2.4(c), the mass is directly to starboard of G and the ship's centre of gravity will move directly to port from G to $G_1$.

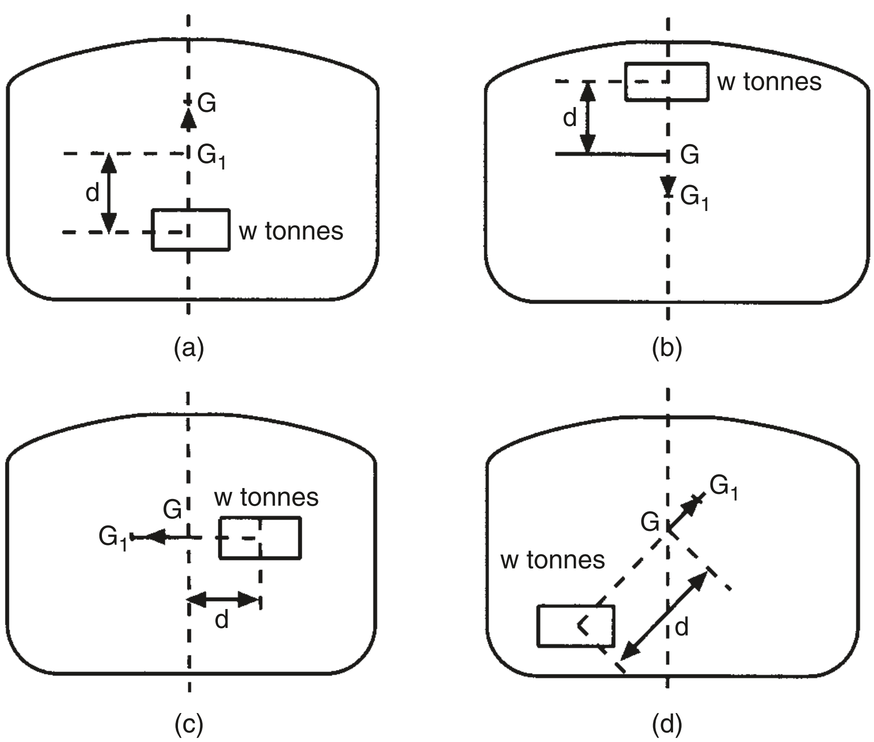



Fig. 2.4 Discharging a mass w.

In Figure 2.4(d), the mass is below and to port of G, and the ship's centre of gravity will move upwards and to starboard.

In each case:

$$GG_1 = \frac{w \times d}{\text{Final displacement}} \text{ metres}$$

## Effect of adding or loading mass

Once again consider the plank of homogeneous wood shown in Figure 2.2. Now add a piece of plank of mass w kg at a distance of d metres from G as shown in Figure 2.5(a).

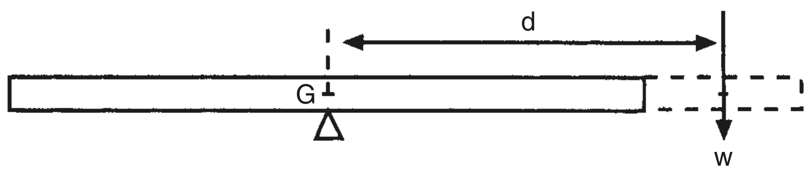



Fig. 2.5(a)

The heavier end of the plank will again tilt downwards. By adding a mass of w kg at a distance of d metres from G a tilting moment of w × d kg m about G has been created.

Now consider the new plank as shown in Figure 2.5(b). Its centre of gravity will be at its new half-length ($G_1$), and the new mass, (W + w) kg, will produce a tilting moment of (W + w) × $GG_1$ kg m about G.

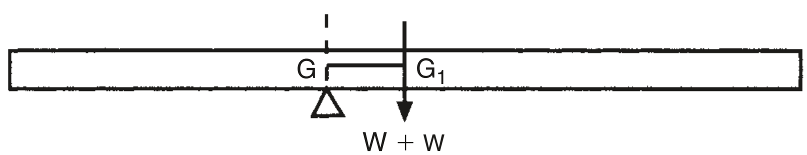



Fig. 2.5(b)

These tilting moments must again be equal, i.e.

$$(W + w) \times GG_1 = w \times d$$

or

$$GG_1 = \frac{w \times d}{W + w} \text{ metres}$$

From the above it may be concluded that when mass is added to a body, the centre of gravity of the body will move directly towards the centre of

Derrett, D.R., Barrass, C.B., *Ship Stability for Masters and Mates*, Butterworth-Heinemann, 2000.

gravity of the mass added, and the distance which it moves will be given by the formula:

$$GG_1 = \frac{w \times d}{\text{Final mass}} \text{ metres}$$

where $GG_1$ is the shift of the centre of gravity of the body, w is the mass added, and d is the distance between the centres of gravity.

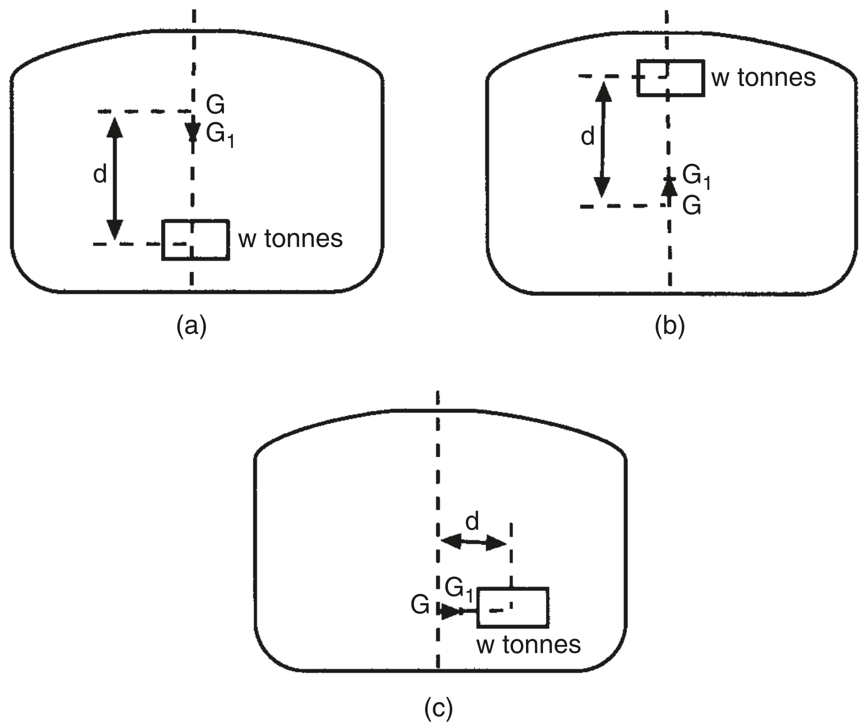



Fig. 2.6 Adding a mass w.

### Application to ships

In each of the above figures, G represents the position of the centre of gravity of the ship before the mass of w tonnes has been loaded. After the mass has been loaded, G will move directly towards the centre of gravity of the added mass (i.e. from G to $G_1$).

Also, in each case:

$$GG_1 = \frac{w \times d}{\text{Final displacement}} \text{ metres}$$

## Effect of shifting weights

In Figure 2.7, G represents the original position of the centre of gravity of a ship with a weight of 'w' tonnes in the starboard side of the lower hold having its centre of gravity in position $g_1$. If this weight is now discharged the ship's centre of gravity will move from G to $G_1$ directly away from $g_1$. When

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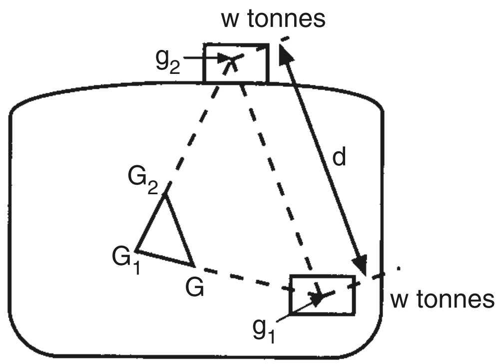


Fig. 2.7 Discharging, adding and moving a mass w.

the same weight is reloaded on deck with its centre of gravity at $g_2$ the ship's centre of gravity will move from $G_1$ to $G_2$.

From this it can be seen that if the weight had been shifted from $g_1$ to $g_2$ the ship's centre of gravity would have moved from G to $G_2$.

It can also be shown that $GG_2$ is parallel to $g_1g_2$ and that

$$GG_2 = \frac{w \times d}{W} \text{ metres}$$

where w is the mass of the weight shifted, d is the distance through which it is shifted and W is the ship's displacement.

The centre of gravity of the body will always move parallel to the shift of the centre of gravity of any weight moved within the body.

## Effect of suspended weights

The centre of gravity of a body is the point through which the force of gravity may be considered to act vertically downwards. Consider the centre of gravity of a weight suspended from the head of a derrick as shown in Figure 2.8.

It can be seen from Figure 2.8 that whether the ship is upright or inclined in either direction, the point in the ship through which the force of gravity may be considered to act vertically downwards is $g_1$, the point of suspension. Thus the centre of gravity of a suspended weight is considered to be at the point of suspension.

## Conclusions

1. The centre of gravity of a body will move directly *towards* the centre of gravity of any *weight added*.
2. The centre of gravity of a body will move directly *away* from the centre of gravity of any *weight removed*.

Derrett, D.R., Barrass, C.B., *Ship Stability for Masters and Mates*, Butterworth-Heinemann, 2000.