Unit 7

Equations

Topic A: Properties of equations

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- Solving one-step equations
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Unit 7 Summary

Unit 7 Self-test

Topic A: Properties of Equations

Introduction to Equations

Equation: a mathematical sentence that contains two expressions and separated by an equal sign (both sides of the equation have the same value).

Example: $4 + 3 = 7$, $9x - 4 = 5$, $2y - \frac{1}{3} = y$

To solve an equation is the process of finding a particular value for the variable in the equation that makes the equation true (left side = right side or LS = RS).

Example: For the equation $x + 4 = 5$

only $x = 1$ can make it true, since $1 + 4 = 5$ (LS = RS)

Solution of an equation: the value of the variable in the equation that makes the equation true.

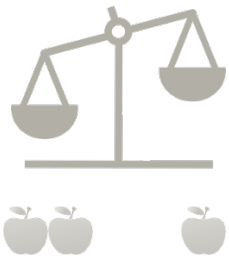
Example: For the equation $x + 4 = 5$, $x = 1$ is the solution.

Examples: Indicate whether each of the given number is a solution to the given equation.

1)	**2:**	$4x - 3 = 5$	$4 \cdot 2 - 3 \overset{?}{=} 5$	$5 \overset{\surd}{=} 5$	Yes	Replace x with 2.
2)	**15:**	$\frac{-3}{15}y = -3$	$\frac{-3}{15}(15) \overset{?}{=} -3$	$-3 \overset{\surd}{=} -3$	Yes	Replace y with 15.
3)	$\frac{1}{2}$**:**	$8t = 3$	$8(\frac{1}{2}) \overset{?}{=} 3$	$4 \neq 3$	No	Replace t with $\frac{1}{2}$.

An equation behaves like a pair of balanced scales. The scales remain balanced when the same weight is put on to or taken away from each side. Always do the same thing on both sides to keep an equation true.

Left side = Right side (LS = RS)



Left side ≠ Right side (LS ≠ RS)

Solving One-Step Equations

To solve a one-step addition equation: $x + a = b$

Isolate the variable "***x***" by **subtracting** the same number ***a*** from each side of the equation (to get rid of the constant ***a*** on the left side of the equal sign so that the letter x is on its own).

Example: Solve $x + 7 = 9$ $a = 7$

$x + 7 - 7 = 9 - 7$ Subtract 7 from both sides.

$x = 2$

or $x + 7 = 9$

$-7 \quad -7$

Solution: $x = 2$

Check: substitute the solution into the equation to verify that is true.

(Left side = Right side).

$x + 7 = 9$ Original equation

$2 + 7 \overset{?}{=} 9, \quad 9 \overset{\surd}{=} 9$ LS = RS (correct) Replace x with 2.

Example: Solve $u + \frac{2}{5} = \frac{3}{5}$

$u + \frac{2}{5} - \frac{2}{5} = \frac{3}{5} - \frac{2}{5}$ Subtract $\frac{2}{5}$ from both sides.

$u = \frac{1}{5}$

or $u + \frac{2}{5} = \frac{3}{5}$

$-\frac{2}{5} \quad -\frac{2}{5}$

Solution: $u = \frac{1}{5}$

Check: $u + \frac{2}{5} = \frac{3}{5}$ Replace u with $\frac{1}{5}$.

$\frac{1}{5} + \frac{2}{5} \overset{?}{=} \frac{3}{5}, \quad \frac{3}{5} \overset{\surd}{=} \frac{3}{5}$ LS = RS (correct)

To solve a one-step subtraction equation: $\boxed{x - a = b}$

Isolate the variable by **adding** the same number ***a*** to each side of the equation.

Example: Solve $x - 5 = 8$ $a = 5$

$x - 5 + \mathbf{5} = 8 + \mathbf{5}$ Add 5 to both sides.

Solution: $x = \boxed{13}$

To solve a one-step multiplication equation: $\boxed{a\,x = b}$

Isolate the variable "***x***" by **dividing** the same number ***a*** from each side of the equation.

Example: Solve $6x = 42$ $a = 6$

$\frac{6x}{\mathbf{6}} = \frac{42}{\mathbf{6}}$ Divide both sides by 6.

Solution: $x = \boxed{7}$

Example: Solve $\frac{4y}{5} = \frac{4}{15}$ $a = \frac{4}{5}$, $\frac{4y}{5} = \frac{4}{5}y$

$\frac{4y}{5} \div \frac{\mathbf{4}}{\mathbf{5}} = \frac{4}{15} \div \frac{\mathbf{4}}{\mathbf{5}}$ Divide both sides by $\frac{4}{5}$.

$\frac{4y}{5} \cdot \frac{5}{4} = \frac{\overset{1}{4}}{\underset{3}{15}} \cdot \frac{\overset{1}{5}}{\underset{1}{4}}$

Solution: $y = \boxed{\frac{1}{3}}$

To solve a one-step division equation: $\boxed{\frac{x}{a} = b}$

Isolate the variable by **multiplying** the same number ***a*** to each side of the equation.

Example: Solve $\frac{x}{7} = 6$ $a = 7$

$\frac{x}{7} \cdot 7 = 6 \cdot 7$ Multiply both sides by 7.

Solution: $x = \boxed{42}$

Example: Solve $-\frac{1}{5}y = 8$ $a = -5$

$-\frac{1}{5}\,\mathbf{(-5)}\,y = 8\,\mathbf{(-5)}$ Multiply both sides by -5.

Solution: $y = \boxed{-40}$

Properties of Equality

Basic rules for solving one-step equations:

- Add, subtract, multiply or divide the same quantity to both sides of an equation can result in a valid equation.
- Remember to always do the same thing to both sides of the equation (balance).

Properties for solving equations:

Properties	Equality	Example
Addition property of equality	$A = B \quad A + \boldsymbol{C} = B + \boldsymbol{C}$	Solve $x - 6 = 3$ $x - 6 + \boldsymbol{6} = 3 + \boldsymbol{6}$ $x = \mathbf{9}$
Subtraction property of equality	$A = B \quad A - \boldsymbol{C} = B - \boldsymbol{C}$	Solve $y + 5 = -8$ $y + 5 - \mathbf{5} = -8 - \mathbf{5}$ $y = \mathbf{-13}$
Multiplication property of equality	$A = B \quad A \cdot \boldsymbol{C} = B \cdot \boldsymbol{C}$	Solve $\frac{m}{9} = 2$ $\mathbf{9} \cdot \frac{m}{9} = 2 \cdot \mathbf{9}$ $m = \mathbf{18}$
Division property of equality	$A = B \quad \frac{A}{\boldsymbol{C}} = \frac{B}{\boldsymbol{C}} \quad (C \neq 0)$	Solve $3n = -15$ $\frac{3n}{\mathbf{3}} = \frac{-15}{\mathbf{3}}$ $n = \mathbf{-5}$

Example: Solve the following equations.

1) $-9 + x = 5$

$-9 + x + \boldsymbol{9} = 5 + \boldsymbol{9}$ Property of addition.

$x = \boxed{14}$

Check: $-9 + \boldsymbol{14} \overset{?}{=} 5$ $\quad 5 \overset{\surd}{=} 5$ Replace x with 14.

2) $t + \frac{2}{5} = -\frac{1}{5}$

$y + \frac{2}{5} - \mathbf{\frac{2}{5}} = -\frac{1}{5} - \mathbf{\frac{2}{5}}$ Property of subtraction.

$y = \boxed{-\frac{3}{5}}$

3) $\frac{-1}{6}x = 7$

$\mathbf{-6} \cdot \frac{-1}{6}x = 7(\mathbf{-6})$ Property of multiplication.

$x = \boxed{-42}$

4) $-5x = 30$

$\frac{-5x}{\mathbf{-5}} = \frac{30}{\mathbf{-5}}$ Property of division.

$x = \boxed{-9}$

5) $0.7y = -0.63$

$\frac{0.7y}{\mathbf{0.7}} = \frac{-0.63}{\mathbf{0.7}}$ Property of division.

$y = \boxed{-0.9}$

6) $y - 3\frac{2}{5} = 2\frac{3}{10}$

$y - 3\frac{2}{5} + \mathbf{3\frac{2}{5}} = 2\frac{3}{10} + \mathbf{3\frac{2}{5}}$ Property of addition.

$y = 2\frac{3}{10} + 3\frac{4}{10}$ The LCD = 10

$y = \boxed{5\frac{7}{10}}$

Topic B: Solving Equations

Solving Multi-Step Equations

Multi-step equation: an equation that requires more than one step to solve it.

Steps for solving multi-step equations:

- Simplify the equation and remove parentheses if necessary.
- Combine like terms on each side of the equation.
- Collect the variable (letter) terms on one side of the equation and the numerical terms (numbers) on the other side.
- Isolate the variable and find the solution: make the coefficient of the variable (number in front of the variable) equal to one.
- Check: substitute the solution back into the equation to verify that it is true (LS = RS).

Example: Solve $\mathbf{9x + 6 = 12}$

- Simplify: $3x + 2 = 4$ — Divide each term by 3.
- Combine like terms: $3x + 2 - 2 = 4 - 2$ — Subtract 2 from both sides.

$3x = 2$

Variable term Constant term

- Isolate the variable $\frac{3x}{3} = \frac{2}{3}$ — Divide both sides by 3.

Solution: $\boxed{x = \frac{2}{3}}$

- Check: $9x + 6 = 12$ — Original equation.

$9 \cdot \frac{2}{3} + 6 \overset{?}{=} 12 \qquad 12 = 12$ √ — Replace x with $\frac{2}{3}$.

Example: Solve $\mathbf{13t - 10 = 3}$

$13t - 10 + 10 = 3 + 10$ — Add 10 to both sides.

$13t = 13$

$\frac{13t}{13} = \frac{13}{13}$ — Divide both sides by 13.

$\boxed{t = 1}$ — Solution.

Example: Solve $\mathbf{2(x - 4) + 5x + 3 = 3(2 - 3x)}$. — Remove parentheses.

$2x - 8 + 5x + 3 = 6 - 9x$ — Combine like terms.

$7x - 5 = 6 - 9x$

$7x - 5 + 5 = 6 - 9x + 5$ — Add 5 to both sides.

$7x = 11 - 9x$

$7x + 9x = 11 - 9x + 9x$ — Add $9x$ to both sides.

$16x = 11$ — Divide both sides by 16.

$\boxed{x = \frac{11}{16}}$

Equations Solving Strategy

Procedure for solving equations

Equation solving strategy
▪ Clear the fractions or decimals if necessary. ▪ Simplify and remove parentheses if necessary. ▪ Combine like terms on each side of the equation. ▪ Collect the variable terms on one side of the equation and the constants on the other side. ▪ Isolate the variable (to get the variable alone on one side of the equation). ▪ Check the solution with the original equation.

Steps for solving equations:

Steps	Example
	Solve $\frac{1}{5}(y+10)=3y-\frac{9}{5}y$
▪ Eliminate the denominators if the equation has fractions.	$5\cdot\frac{1}{5}(y+10)=5(3y)-5(\frac{9}{5}y)$ Multiply each term by 5.
▪ Remove parentheses.	$y+10=15y-9y$
▪ Combine like terms.	$y+10=6y$
▪ Collect variable terms on one side and the constants on the other side.	$y+10-10=6y-10$ $y=6y-10$ Subtract 10 from both sides. $y-6y=6y-10-6y$ Subtract $6y$ from both sides.
▪ Isolate the variable.	$-5y=-10$ Divide both sides by -5. $y=\frac{-10}{-5}$ $y=2$
▪ Check with the original equation.	$\frac{1}{5}(2+10)\overset{?}{=}3\cdot2-\frac{9}{5}\cdot2$ Replace y with 2. $5\cdot\frac{1}{5}(2+10)\overset{?}{=}5\cdot3\cdot2-5\cdot\frac{9}{5}\cdot2$ Multiply each term by 5. $(2+10)\overset{?}{=}30-18$ $12\overset{\surd}{=}12$ LS = RS (correct)

Equations Involving Decimals / Fractions

Equations involving decimals

Tip: Multiply every term of both sides of the equation by a multiple of 10 (10, 100, 1000, etc.) to clear the decimals (based on the number with the largest number of decimal places in the equation).

Steps

- Multiply each term by 100 to clear the decimal.
- Collect the variable terms on one side of the equation and the constants on the other side.
- Isolate the variable.

Example

Solve $\mathbf{0.34}x - \mathbf{0.12} = \mathbf{-4.26}x$.

$\mathbf{100}(0.34x) - \mathbf{100}(0.12) = \mathbf{100}(-4.26x)$

The largest number of decimal place is two.

$34x - 12 = -426x$ Add 12 to both sides.

$34x + 426x = 12$ Add $426x$ to both sides.

$460x = 12$

$x \approx 0.026$

Example: Solve $\mathbf{0.4}y + \mathbf{0.08} = \mathbf{0.016}$

The largest number of decimal place is three.

$\mathbf{1000}(0.4y) + \mathbf{1000}(0.08) = \mathbf{1000}(0.016)$ Multiply each term by 1000.

$400y + 80 = 16$ Combine like terms.

$400y = -64$ Divide both sides by 400.

$y = -0.16$

Equations involving fractions

Steps

- Multiply each term by the LCD.
- Collect the variable terms on one side of the equation and the constants on the other side.
- Isolate the variable.

Example

Solve $\frac{t}{3} + \frac{3}{4} = -\frac{t}{2} - \frac{1}{3}$.

$\mathbf{12} \cdot \frac{t}{3} + \mathbf{12} \cdot \frac{3}{4} = \mathbf{12}(-\frac{t}{2}) - \mathbf{12} \cdot \frac{1}{3}$

$2 \mid 3\ 4\ 2\ 3$

$3 \mid 3\ 2\ 1\ 3$

$1\ 2\ 1\ 1$

LCD = 2 × 3 × 2 = 12

$4t + 9 = -6t - 4$ Add $6t$ to both sides.

$10t = -13$ Subtract 9 from both sides.

$t = \frac{-13}{10} = -1\frac{3}{10}$

Divide both sides by 10.

Topic C: One Solution, No Solutions, Infinite Solutions

Types of Equations

Types of equations: a mathematical equation can be a contradiction, an identity, or a conditional equation.

Contradiction equation: an equation which is never true, regardless of the value of the variable, and thus has no solution.

Example: $3(x + 1) - 3x = -7$ Distribute property.

$3x + 3 - 3x = -7$ Combine like terms.

$3 = -7$ False, $3 \neq -7$

No solution There are no real numbers that can make this equation true.

Note: If the resulting equation is a **false** statement with **no variables**, it is a contradiction equation.

Identity equation: an equation which is always true for every value of the variable and thus has an infinite number of solutions (the solution is all real numbers).

Example: $12x - 3(2 + 4x) = -6$ Distribute property.

$12x - 6 - 12x = -6$ Combine like terms.

$-6 = -6$

The solution is all real numbers.

The equation is always true no matter what value is substituted for the variable.

Note: If the resulting equation is a **true** statement and with **no variables**, it is an identity equation.

Conditional equation: an equation is true only for the certain value of the variable (one solution).

Example: $2x - 3 = -7x$ Add $7x$ to both sides.

$9x - 3 = 0$ Add 3 to both sides.

$9x = 3$

$x = \frac{1}{3}$ Divide both sides by 9.

If $x = \frac{1}{3}$, the equation is true, otherwise, the equation is false.

Summary: types of equations

Types of equations	Characteristic	Solution
Contradiction equation	Always false	No solution
Identity equation	Always true	All real numbers
Conditional equation	It is true only for the certain value.	One solution

Example: Determine each equation as a contradiction, an identity, or a conditional equation.

1) $4x - (3 - x) = 5(x - 1)$ Remove parentheses.

$4x - 3 + x = 5x - 5$ Combine like terms.

$5x - 3 = 5x - 5$

$5x - 3 - 5x = 5x - 5 - 5x$ Subtract $5x$ from both sides.

$-3 = -5$

No solution – contradiction equation

The resulting equation is a false statement with no variables.

2) $\frac{y}{2} + 2(y - 3) = 2 - 3y$ Multiply each term by 2.

$2 \cdot \frac{y}{2} + 2 \cdot 2(y - 3) = 2 \cdot 2 - 2(3y)$ Remove parentheses.

$y + 4y - 12 = 4 - 6y$ Combine like terms.

$5y - 12 + \mathbf{12} = 4 - 6y + \mathbf{12}$ Add 12 to both sides.

$5y = 16 - 6y$

$5y + \mathbf{6y} = 16 - 6y + \mathbf{6y}$ Add 6y to both sides.

$11y = 16$ Divide both sides by 11.

$\boldsymbol{y \approx 1.455}$

One solution – conditional equation

3) $4t - 3(t + 4) = t - 12$ Distribute property.

$4t - 3t - 12 = t - 12$ Combine like terms.

$t - 12 = t - 12$ Add 12 to both sides.

$+12 \quad +12$

$t = t$ Subtract t from both sides.

$-t \quad -t$

$0 = 0$

All real numbers – identity equation

The resulting equation is a true statement with no variables.

Topic D: Writing and Solving Equations

Number Problems

Number problems - examples

English phrase	Algebraic expression / equation
Seven more than the difference of a number and four.	$(x - 4) + 7$
The quotient of five and the product of six and a number.	$\frac{5}{6x}$
The product of nine and a number, decreased by five.	$9x - 5$
Ten less than three times two numbers is seven more than their sum.	$3xy - 10 = x + y + 7$
The sum of the squares of two numbers is nine less than their product.	$x^2 + y^2 = xy - 9$
Two more than the quotient of $11x$ by 5 is seven times that number.	$2 + \frac{11x}{5} = 7x$

Let x = a number, y = a number

Steps for solving word problems:

Procedure for solving word problems
▪ **Organize** the **facts** given from the problem.
▪ Identify and **label** the **unknown** quantity (**let x = unknown**).
▪ Draw a **diagram** if it will make the problem clearer.
▪ Convert words into a mathematical **equation**.
▪ **Solve** the equation and find the **solution**(s).
▪ **Check** the solution with the original equation (check it back into the problem – is it logical? if necessary).

Example: The product of nine and a number is twenty-seven. Determine the value of this number.

- Organize the facts and assign the unknown quantity:

Facts	The **product** of **9** and x is **27**
Unknown	Let x = number

- Write an equation: $9 \cdot x = 27$ or $9x = 27$
- Solve the equation: $\frac{9x}{9} = \frac{27}{9}$ Divide both sides by 9.

  $x = 3$
- Check: $9 \cdot 3 \overset{?}{=} 27$ Replace x with 3.

  $27 \overset{\surd}{=} 27$ LS = RS (correct)

Answer: The value of the number is 3.

Example: Eight less than two times a number is five less than the number divided by two.

Find the number. Let x = number

- Organize the facts: -8 $2x$ $=$ -5 $\frac{x}{2}$
- Equation: $\mathbf{2x - 8 = \frac{x}{2} - 5}$ Multiply each term by 2.

$2(2x) - 2 \cdot 8 = 2(\frac{x}{2}) - 2 \cdot 5$ Remove parentheses.

$4x - 16 = x - 10$ Combine like terms.

$3x = 6$ Divide both sides by 3.

- Solution: $\boxed{x = 2}$
- Check: $2(\mathbf{2}) - 8 \overset{?}{=} \frac{\mathbf{2}}{2} - 5$ Replace x with 2.

$4 - 8 \overset{?}{=} 1 - 5$

$-4 \overset{\surd}{=} -4$ LS = RS (correct)

Answer: The number is 2.

Example: There are **three** numbers, the **first** is **four less** than **three times** the **second**, and the **third** is **two more** than the **first**. The **sum** of these **three** numbers **is fifteen**.

Find each number.

- Organize the facts:

Number	Words	Algebraic expression
2nd number	Let 2nd number = x	x
1st number	4 less than 3 times the 2nd number	$3x - 4$
3rd number	2 more than the 1st number	$(3x - 4) + 2$
Sum	The sum of three numbers is 15	1st # + 2nd # + 3rd # = 15

- Equation: $\mathbf{(3x - 4) + x + [(3x - 4) + 2)] = 15}$ Remove parentheses.

$3x - 4 + x + 3x - 2 = 15$ Combine like terms.

$7x - 6 = 15$ Add 6 to both sides.

$7x = 21$ Divide both sides by 7.

- Solution: $\boxed{x = 3}$

1st Number	$3x - 4 = 3 \cdot 3 - 4 = 5$
2nd Number	$x = 3$
3rd Number	$(3x - 4) + 2 = (3 \cdot 3 - 4) + 2 = 7$

- Check: $5 + 3 + 7 \overset{?}{=} 15$ Yes!

Consecutive Integers

Consecutive integers:

English phrase	Algebraic expression	Example
Two consecutive integers	x , $x+1$	If $x = \mathbf{1}$, $x+1 = \mathbf{2}$
Three consecutive integers	x , $x+1$, $x+2$	If $x = \mathbf{1}$, $x+1 = \mathbf{2}$, $x+2 = \mathbf{3}$
Two consecutive **odd** integers	x , $x+2$	If $x = \mathbf{1}$, $x+2 = \mathbf{3}$
Three consecutive **odd** integers	x , $x+2$, $x+4$	If $x = \mathbf{1}$, $x+2 = \mathbf{3}$, $x+4 = \mathbf{5}$
Two consecutive **even** integers	x , $x+2$	If $x = \mathbf{2}$, $x+2 = \mathbf{4}$
Three consecutive **even** integers	x , $x+2$, $x+4$	If $x = \mathbf{2}$, $x+2 = \mathbf{4}$, $x+4 = \mathbf{6}$

Examples:

English phrase	Equation
The difference of two consecutive integers is one.	$(x+1) - x = 1$
The sum of three consecutive odd integers is nine.	$x + (x+2) + (x+4) = 9$
The product of two consecutive even integers is eight.	$x(x+2) = 8$
Three consecutive even integers whose sum is twelve.	$x + (x+2) + (x+4) = 12$

Example: The **sum** of **three consecutive odd** integers is **twenty-one**, find each number.

- Organize the facts:

1st consecutive odd number	x
2nd consecutive odd number	$x+2$
3rd consecutive odd number	$x+4$

- Write an equation: $x + (x+2) + (x+4) = 21$ Combine like terms.
- Solve the unknown: $3x + 6 = 21$ Subtract 6 from both sides.

$3x = 15$ Divide both sides by 3.

$\boxed{x = 5}$

1st consecutive even number	$x = 5$
2nd consecutive even number	$x + 2 = 5 + 2 = 7$
3rd consecutive even number	$x + 4 = 5 + 4 = 9$

- Check: $5,\ 7,\ 9 \overset{?}{=}$ consecutive odd integers Yes!

$5 + (5+2) + (5+4) \overset{?}{=} 21$ Replace x with 5.

or $5 + 7 + 9 \overset{?}{=} 21$

$21 \overset{\surd}{=} 12$ LS = RS (correct)

- State the answer: $\boxed{x = 5,\quad x + 2 = 7,\quad x + 4 = 9}$

Mixed Problems

Example: The second angle of a triangle is twelve times as large as the first. The third angle is five degrees more than the second angle. Find the measure of each angle.

1st angle	x
2nd angle	$12x$
3rd angle	$12x + 5^0$

- Equation $x + 12x + (12x + 5^0) = 180^0$ — The sum of three angles of a triangle is 180^0.

  $25x + 5^0 = 180^0$ — Remove parentheses and combine like terms.

  $25x = 175^0$ — Subtract 5^0 from both sides.

- Solve: $x = \frac{175}{25} = \boxed{7^0}$ — Divide both sides by 25.
- The answer:

1st angle	$x = 7^0$
2nd angle	$12x = 12\ (7) = 84^0$
3rd angle	$12x + 5^0 = 12\ (7^0) + 5^0 = 89^0$

- Check: $7^0 + 84^0 + 89^0 \overset{?}{=} 180^0$

  $180^0 \overset{\surd}{=} 180^0$ — Yes!

Example: The perimeter of a rectangle is 164 meters. The width is 13 meters shorter than the length. Find the dimensions (width and length).

- List the facts and sign the unknown quantity:

Facts	Perimeter $P = 164$ m
Unknown	Let l = length, width = $l - 13$

- Equation: $2\,l + 2(l - 13) = 164$ — The perimeter of a rectangle: $P = 2l + 2w$

  $4\,l - 26 = 164$ — Remove parentheses and combine like terms.

  $4\,l = 190$ — Divide both sides by 4.

  Length: $\boxed{l = 47.5 \text{ m}}$
- Find the width: $w = l - 13$

  $w = 47.5 - 13$ — Substitute 47.5m for l in the equation.

  $= 34.5$ m

  Width: $\boxed{w = 34.5 \text{ m}}$

	Formulas
Original price	Original price = Sale price + Discount
Discount	Discount = Discount rate $\times$ Original price
Sale price	Sale price = Original price – Discount

Example: After a 35% reduction, a women's jacket is on sale for \$30.55. What is the discount? What was the original price?

- Organize the facts:

Sale price	\$30.55
Discount rate	35 %
Unknown	Let x = original price

- Discount: Discount = Discount rate $\times$ Original price

  $= (35\%)\, x$

- Equation: Original price = Sale price + Discount

  $x = 30.55 + 35\%\, x$

  or $x = 30.55 + 0.35\, x$ Convert percent to decimal.

- Solve: $x - 0.35\, x = 30.55$ Subtract $0.35x$ from both sides.

  $0.65\, x = 30.55$ Combine like terms. $x = 1 \cdot x$

  $x = \frac{30.55}{0.65} = 47$ Divide both sides by 0.65.

  $\boxed{x = \$47}$

- State the answer: The original price was \$47.

Example: A \$159.99 instant pot is labeled "30% off". What is the sale price?

Original price	\$159.99
Discount rate	30 %
Unknown	Let $\boldsymbol{x}$ = sale price

- Equation: Sale price = Original price – Discount

  $x = 159.99 - (30\%)\,(159.99)$ Discount = Discount rate $\times$ Original price

  $x = 159.99 - (0.3)\,(159.99)$ Convert percent to decimal.

  $x \approx 111.99$

- State the answer: The sale price is \$111.99.

Unit 7: Summary

Equations

Equation: a mathematical sentence that contains two expressions and separated by an equal sign.

To solve an equation is the process of finding a particular value for the variable in the equation that makes the equation true.

Solution of an equation: the value of the variable in the equation that makes the equation true.

An equation behaves like a pair of balanced scales. The scales remain balanced when the same weight is put on to or taken away from each side. Always do the same thing on both sides to keep an equation true (LS = RS).

Basic rules for solving one-step equations:

- Add, subtract, multiply or divide the same quantity to both sides of an equation can result in a valid equation.
- Remember to always do the same thing to both sides of the equation (balance).

Properties for solving equations:

Properties	Equality		Example
Addition property of equality	$A = B$	$A + \boldsymbol{C} = B + \boldsymbol{C}$	Solve $x - 6 = 3$ $x - 6 + \boldsymbol{6} = 3 + \boldsymbol{6}$ $x = \mathbf{9}$
Subtraction property of equality	$A = B$	$A - \boldsymbol{C} = B - \boldsymbol{C}$	Solve $y + 5 = -8$ $y + 5 - \mathbf{5} = -8 - \mathbf{5}$ $y = \mathbf{-13}$
Multiplication property of equality	$A = B$	$A \cdot \boldsymbol{C} = B \cdot \boldsymbol{C}$	Solve $\frac{m}{9} = 2$ $\mathbf{9} \cdot \frac{m}{9} = 2 \cdot \mathbf{9}$, $m = \mathbf{18}$
Division property of equality	$A = B$	$\frac{A}{\boldsymbol{C}} = \frac{B}{\boldsymbol{C}}$ $(C \neq 0)$	Solve $3n = -15$ $\frac{3n}{\mathbf{3}} = \frac{-15}{\mathbf{3}}$ $n = \mathbf{-5}$

Steps for solving equations:

Equation solving strategy
▪ Clear the fractions or decimals if necessary. ▪ Simplify and remove parentheses if necessary. ▪ Combine like terms on each side of the equation. ▪ Collect the variable terms on one side of the equation and the constants on the other side. ▪ Isolate the variable. ▪ Check the solution with the original equation.

Types of equations:

Types of equations	Characteristic	Solution
Contradiction equation	Always false	No solution
Identity equation	Always true	All real numbers
Conditional equation	It is true only for the certain value.	One solution

- If the resulting equation is a false statement with no variables, it is a contradiction equation.
- If the resulting equation is a true statement and with no variables, it is an identity equation.

Steps for solving word problems:

Procedure for solving word problems
▪ **Organize** the **facts** given from the problem. ▪ Identify and **label** the **unknown** quantity (**let x = unknown**). ▪ Draw a **diagram** if it will make the problem clearer. ▪ Convert words into a mathematical **equation**. ▪ **Solve** the equation and find the **solution**(s). ▪ **Check** the solution with the original equation (check it back into the problem – is it logical? if necessary).

Consecutive integers:

English phrase	Algebraic expression	Example
Three consecutive integers	x , $x+1$, $x+2$	If $x=\mathbf{1}$, $x+1=\mathbf{2}$, $x+2=\mathbf{3}$
Three consecutive **odd** integers	x , $x+2$, $x+4$	If $x=\mathbf{1}$, $x+2=\mathbf{3}$, $x+4=\mathbf{5}$
Three consecutive **even** integers	x , $x+2$, $x+4$	If $x=\mathbf{2}$, $x+2=\mathbf{4}$, $x+4=\mathbf{6}$

	Formulas
Original price	Original price = Sale price + Discount
Discount	Discount = Discount rate $\times$ Original price
Sale price	Sale price = Original price – Discount

Unit 7: Self-Test

Equations

Topic A

1. Indicate whether each of the given number is a solution to the given equation.

a) 2: $9x - 7 = 11$

b) 17: $\frac{-5}{17}y = -9$

c) $\frac{2}{3}$**:** $9m = 6$

2. Solve the following equations.

a) $x - 7 = 12$

b) $y + \frac{3}{8} = \frac{5}{8}$

c) $m - 6 = 17$

d) $9t = 72$

e) $\frac{3x}{2} = \frac{9}{16}$

f) $\frac{y}{13} = -4$

g) $-21 + x = 7$

h) $y + \frac{4}{9} = -\frac{3}{9}$

i) $\frac{-4}{14}x = -2$

j) $-19t = 38$

k) $0.8y = -0.64$

l) $x - 4\frac{2}{3} = 3\frac{2}{9}$

Topic B

3. Solve the following equations.

a) $14t + 5 = 8$

b) $7m - 23 = 40$

c) $7(x - 3) + 3x - 5 = 2(5 - 4x)$

d) $\frac{1}{7}(y + 12) = 4y - \frac{3}{7}y$

e) $0.63x - 0.29 = -3.56x$

f) $0.5t + 0.05 = 0.025$

g) $\frac{x}{4} + \frac{2}{5} = -\frac{x}{2} - \frac{1}{5}$

Topic C

4. Determine each equation as a contradiction, an identity, or a conditional equation.

a) $5(y + 2) - 5y = -8$

b) $8x - 4(3 + 2x) = -12$

c) $7t - 9 = -3t$

d) $5y - (4 - y) = 6(y - 2)$

e) $\frac{x}{3} + 3(x - 4) = 5 - 8x$

f) $7m - 5(m + 3) = 2m - 15$

Topic D

5. Write each of the following as an algebraic expression.

a) Nine more than the difference of a number and seven.

b) The quotient seven and the product of nine and a number.

c) The product of eleven and a number, decreased by eight.

6. Write each of the following as an algebraic expression or equation.

a) Thirteen less than four times two numbers is six more than their sum.

b) The sum of the squares of two numbers is twenty-six less than their product.

c) Five more than the quotient of $5x$ by 23 is eleven times that number.

d) The difference of two consecutive integers is one.

e) The sum of three consecutive odd integers is fifteen.

f) The product of two consecutive even integers is forty-eight.

g) Three consecutive odd integers whose sum is twenty-one.

7. Solve each problem by writing and solving an equation.

a) The product of seven and a number is forty-two. Determine the value of this number.

b) Three less than four times a number is nine less than the number divided by four. Find the number.

c) There are three numbers, the first is three less than five times the second, and the third is four more than the first. The sum of these three numbers is twenty. Find each number.

d) The sum of three consecutive odd integers is twenty-seven, find each number.

e) The second angle of a triangle is seven times as large as the first. The third angle is thirty degrees more than the second angle. Find the measure of each angle.

f) The perimeter of a rectangle is 128 meters. The width is 8 meters shorter than the length. Find the dimensions (width and length).

g) After a 20% reduction, a TV is on sale for $199.99. What is the discount? What was the original price?

h) A $379.99 laptop is labeled "10% off". What is the sale price?