

Unit 12

Solving Word Problems

Topic A: Value mixture problems

Solving value mixture problems

Topic B: Concentration mixture problems

Solving mixture problems

Topic C: Motion and business problems

- Distance, speed and time problems
- Business problems

Topic D: Mixed problems

Solving mixed problems

Unit 12: Summary

Unit 12: Self-test

Topic A: Value Mixture Problems

Solving Value Mixture Problems

Steps for solving word problems:

Steps for Solving Word Problems	
▪	Organize the facts given from the problem (make a table).
▪	Identify and label the unknown quantity (let x = unknown).
▪	Draw a diagram if it will make the problem clearer.
▪	Convert words into a mathematical equation .
▪	Solve the equation and find the solution(s).
▪	Check and state the answer .

Table for value mixture problems:

Item	Value of the item	Number of items	Total value
Item A	value of A	# of A	(value of A) \times (# of A) = amount of A
Item B	value of B	# of B	(value of B) \times (# of B) = amount of B
Item C	value of C	# of C	(value of C) \times (# of C) = amount of C
...
Total or mixture			total value

Let x = unknown

Value of item A + Value of item B + Value of item C + ... = Total value of the mixture

Example: Susan has **\$5.95** in nickels, dimes and quarters. If she has two **less** than **three times quarters of dimes**, and **three more nickels than quarters**. How many of each coin does she have?

- Let x = number of quarters
- Organize the facts:

Coin	Value of the coin	Number of coins	Total value (in cents)	
Quarter	25 ¢	x	$25x$	(value of 25¢) \times (# of 25¢)
Dime	10 ¢	$3x - 2$	$10(3x - 2)$	(value of 10¢) \times (# of 10¢)
Nickel	5 ¢	$x + 3$	$5(x + 3)$	(value of 5 ¢) \times (# of 5¢)
Total			$\$5.95 = 595 \text{ ¢}$	

Convert \$ to ¢

- Equation: $25x + 10(3x - 2) + 5(x + 3) = 595$ value of quarters + value of dimes + value of nickels = 595¢
- Solve for x : $25x + 30x - 20 + 5x + 15 = 595$ Remove parentheses.
 $60x - 5 = 595$ Combine like terms.
 $60x = 600$ Solve for x .
 $x = \boxed{10}$

- Check:

Number of quarters	$x = 10$
Number of dimes	$3x - 2 = 3(10) - 2 = 28$
Number of nickels	$x + 3 = 10 + 3 = 13$

$$25x + 10(3x - 2) + 5(x + 3) = 595$$

Equation

$$25 \cdot 10 + 10(3 \cdot 10 - 2) + 5(10 + 3) = 595$$

Substitute x for 10.

$$250 + 280 + 65 = 595$$

Check LS = RS

$$595 = 595 \quad \text{Correct!}$$

LS = RS

- State the answer:

Number of quarters	10
Number of dimes	28
Number of nickels	13

Example: Damon purchased \$1.00, \$1.19, and \$1.20 Canadian stamps with a **total value of \$23.72**. If the number of \$1.19 stamps is **7 more than the number of \$1.00** stamps, and the number of **\$1.20** stamps is **8 more than three times** of **\$1.00** stamps. How many of each did Damon receive?

- Let x = number of \$1.00 stamps
- Organize the facts:

Stamps	Value of the stamps	Number of stamps	Total value
\$1.00	\$1.00	x	$1.00x$
\$1.19	\$1.19	$7 + x$	$1.19(7 + x)$
\$1.20	\$1.20	$8 + 3x$	$1.20(8 + 3x)$
Total			\$23.72

(value of \$1.00) \times (# of \$1.00)

(value of \$1.19) \times (# of \$1.19)

(value of \$1.20) \times (# of \$1.20)

Value of \$1.00 + value of \$1.19 + value of \$1.20 = \$20.68

- Equation: $1.00x + 1.19(7 + x) + 1.20(8 + 3x) = 23.72$

- Solve for x : $1x + 8.33 + 1.19x + 9.6 + 3.6x = 23.72$

Remove parentheses.

$$5.79x + 17.93 = 23.72$$

Combine like terms.

$$579x + 1793 = 2372$$

Remove decimals ($\times 100$)

$$579x = 579$$

Divide both sides by 579.

$$x = 1$$

- State the answer:

Number of \$1.00	$x = 1$
Number of \$1.19	$7 + x = 7 + 1 = 8$
Number of \$1.20	$8 + 3x = 8 + 3 \cdot 1 = 11$

Topic B: Concentration Mixture Problems

Solving Mixture Problems

Table of concentration mixture:

Item	Concentration	Volume	Amount
Item A	concentration of A	volume of A	(concentration of A) \times (volume of A) = amount of A
Item B	concentration of B	volume of B	(concentration of B) \times (volume of B) = amount of B
...
Mixture	concentration of mixture	volume of mixture	(concentration of mixture) \times (volume of mixture) = amount of mixture

Let x = unknown

$$\text{Amount of item A} + \text{Amount of item B} + \dots = \text{Amount of the mixture}$$

Example: A shrimp meal is 35% protein and a fish meal is 25% protein. Susan wants a 750 grams mixture that is 30% protein. How many grams of protein each meal should she have?

- Let x = the protein volume of the shrimp meal
- The protein volume of fish meal = $750 - x$

The protein volume of *mixture* – The protein volume of *shrimp* meal = The protein volume of *fish* meal
(If there is a total mixture protein volume of 750 g, then $750 - x$ must be the protein volume of fish meal.)

- Organize the facts:

Meal	Concentration	Protein volume	Amount
Shrimp meal	35% = 0.35	x	$0.35x$
Fish meal	25% = 0.25	$750 - x$	$0.25(750 - x)$
Mixture	30% = 0.30	750	$0.3(750)$

(concentration of shrimp meal) \times (volume of shrimp meal)

(concentration of fish meal) \times (volume of fish meal)

(concentration of mixture) \times (volume of mixture)

- Equation: $0.35x + 0.25(750 - x) = (0.3)(750)$

Remove parentheses.

Amount of shrimp meal + Amount of fish meal = Amount of mixture

- Solve for x : $0.35x + 187.5 - 0.25x = 225$

Combine like terms.

$$0.1x = 37.5$$

Divide both sides by 0.1.

- State the answer:
 - Shrimp meal: $x = 375 \text{ g}$
 - Fish meal: $750 - x = 750 - 375$
 $= 375 \text{ g}$

Example: How much **8% sugar solution** must be added to **15 liters of 27% solution** to make a **20% solution**?

- Let x = volume of 8% solution

- Volume of 20% = $x + 15$

$$\text{Volume of 20\%} = \text{Volume of 8\%} + \text{Volume of 27\%}$$

Mixture

- Organize the facts:

Solution	Concentration	Volume	Amount
8%	0.08	x	$0.08x$
27%	0.27	15	$(0.27)(15)$
20%	0.2	$x + 15$	$0.2(x + 15)$

(concentration of 8%) \times (volume of 8%)

(concentration of 27%) \times (volume of 27%)

(concentration of 20%) \times (volume of 20%)

- Equation: **$0.08x + (0.27)(15) = 0.2(x + 15)$**

Amount of 8% + Amount of 27% = Amount of 20%

- Solve for x : $0.08x + 4.05 = 0.2x + 3$

Combine like terms.

$$-0.12x = -1.05$$

Divide both sides by -0.12.

$$x = \boxed{8.75}$$

- State the answer: 8.75 liters of 8% sugar solution must be added to 15 liters of 27% solution.

Topic C: Motion and Business Problems

Distance, Speed and Time Problems

Formulas of motion:

- Distance = Speed · Time $d = r t$
- Speed = $\frac{\text{Distance}}{\text{Time}}$ $r = \frac{d}{t}$
- Time = $\frac{\text{Distance}}{\text{Speed}}$ $t = \frac{d}{r}$

Example: Adam walks for **4.4 hours** at a rate of **2 km per hour**. **How far** does he walk?

Equation: $d = r t$

$$= (2 \text{ km/h}) (4.4 \text{ h}) = 8.8 \text{ km}$$

$$t = 4.4 \text{ h}, \quad r = 2 \text{ km/h}, \quad d = ?$$

km/h: km per hour

Table of motions:

Condition	Speed (r)	Time (t)	Distance (d)
Condition A	r	t	$d = r t$
Condition B	r	t	$d = r t$
...
Total			

Example: Two cyclists are **60 km apart** and are travelling towards each other. Their **speeds differ** by **1.5 km** per hour. What is the **speed of each** cyclist if they meet after **2 hours**?

Condition	Speed (r)	Time (t)	Distance ($d = r t$)
Bike A	r	2	$2 r$
Bike B	$r - 1.5$	2	$2 (r - 1.5)$
Total			60 km

- Equation: $2r + 2(r - 1.5) = 60$ Distance of A + Distance of B = 60km
 $2r + 2r - 3 = 60$ Remove parentheses.
 $4r = 63$ Combine like terms.
- Bike A: $r = 15.75 \text{ km/h}$ Divide both sides by 4.
- Bike B: $r - 1.5 = 15.75 - 1.5 = 14.25 \text{ km/h}$

Example: Mike **boats** at a **speed** of **28 km** per hour in still water. The **river** flows at a **speed** of **5 km** per hour. **How long** will it take Mike to boat **3 km downstream?** **3 km upstream?**

Condition	Speed (r)	Distance (d)	Time ($t = \frac{d}{r}$)
Downstream	$r = 28 + 5 = 33 \text{ km/h}$	$d = 3 \text{ km}$	$t = \frac{d}{r} = \frac{3 \text{ km}}{33 \text{ km/h}}$
Upstream	$r = 28 - 5 = 23 \text{ km/h}$	$d = 3 \text{ km}$	$t = \frac{d}{r} = \frac{3 \text{ km}}{23 \text{ km/h}}$

Downstream (fast): speed of boat + speed of river

Upstream (slower): speed of boat - speed of river

- Downstream: $t = \frac{d}{r} = \frac{3 \text{ km}}{33 \text{ km/h}} \approx 0.091 \text{ h}$
- Upstream: $t = \frac{d}{r} = \frac{3 \text{ km}}{23 \text{ km/h}} \approx 0.13 \text{ h}$

Business Problems

Business math formulas:

Business problems	Formulas
Percent increase	Percent increase = $\frac{\text{New value} - \text{Original value}}{\text{Original value}}$, $x = \frac{N - O}{O}$
Percent decrease	Percent decrease = $\frac{\text{Original value} - \text{New value}}{\text{Original value}}$, $x = \frac{O - N}{O}$
Sales tax	Sales tax = Sales \times Tax rate
Commission	Commission = Sales \times Commission rate
Discount	Discount = Original price \times Discount rate Sale price = Original price – Discount
Markup	Markup = Selling price \times Markup rate Original price = Selling price – Markup
Simple interest	Interest = Principle \cdot Interest rate \cdot Time, $I = P r t$ Balance = Principle + Interest
Compound interest	Balance = Principle $(100\% + \text{Interest rate})^t$ Balance = $P (100\% + r)^t$

Example: A product increased production from **230 last month** to **250 this month**. Find the **percent increase**.

- New value (N): 250 This month.
- Original value (O): 230 Last month.
- Percent increase: $x = \frac{N - O}{O} = \frac{250 - 230}{230} \approx 0.087 = \boxed{8.7\%}$ About 8.7% increase.

Example: A product was **reduced** from **\$59** to **\$39**. What was the percent **reduction**?

Percent decrease: $x = \frac{O - N}{O} = \frac{59 - 39}{59} \approx 0.339 = \boxed{33.9\%}$ 33.9 % decrease.

Example: Find the **sales tax** for a **\$999** laptop with a **tax rate** of **7%**.

$$\begin{aligned} \text{Sales tax} &= \text{Seles} \times \text{Tax rate} \\ &= (\$999) (7\%) = (\$999) (0.07) = \boxed{\$69.93} \end{aligned}$$

Example: Find the **commission** for a **\$950,000** house with a **commission rate** of **5%**.

$$\begin{aligned} \text{Commission} &= \text{Sales} \times \text{Commission rate} \\ &= (\$950,000) (5\%) = (\$950,000) (0.05) = \boxed{\$47,500} \end{aligned}$$

Example: A men's coat was *originally* priced at **\$159**, and is on sale at a **25% discount**. Find the *discount* and *sale price*.

- **Discount** = Original price \times Discount rate
 $= (\$159) (25\%)$
 $= (\$159) (0.25)$
 $= \$39.75$
- **Sale price** = Original price – Discount
 $= \$159 - \39.75
 $= \$119.25$

Example: A condo was sold at **\$399,000**, with a *markup rate* of **8%**. What was the *markup* and *original price*?

- **Markup** = Selling price \times Markup rate
 $= (\$399,000) (8\%)$
 $= (\$399,000) (0.08)$
 $= \$31,920$
- **Original price** = Selling price – Markup
 $= \$399,000 - \$31,920$
 $= \$367,080$

Example: Jo borrowed **\$150,000** mortgage from a bank. Find the interest at **3%** per year for **3.5 years**, and also find the *total* amount that Jo paid the bank.

- **Interest** = Principle \cdot Interest rate \cdot Time
 $I = P r t = (\$150,000) (3\%) (3.5)$
 $= (\$150,000) (0.03) (3.5)$
 $= \$15,750$
- **Balance** = Principle + Interest
 $= \$150,000 + \$15,750$
 $= \$165,750$

Example: David deposited **\$3,000** in an account at **4.5% interest compounded** per year for **5 years**. How much was in the account at the end of **5 years**?

$$\begin{aligned}
 \text{Balance} &= \text{Principle} (100\% + \text{Interest rate})^t && \text{Compound interest} \\
 &= P (100\% + r)^t \\
 &= \$3,000 (100\% + 4.5\%)^5 \\
 &= \$3,000 (1 + 0.045)^5 \\
 &\approx \$3,738.55
 \end{aligned}$$

Topic D: Mixed Problems

Solving Mixed Problems

Example: After a *ten percent reduction*, a toy is on sale for *twenty-nine dollars*. What was the *original price*?

- Let x = original price
- Equation: $x - 10\% x = 29$
 $1x - 0.1x = 29$
 $0.9x = 29$

Original price – Reduction = Sale price
 $x = 1 \cdot x$
- Answer: $x = \frac{29}{0.9} \approx \boxed{\$32.22}$

The original price was \$32.22.

Example: William receives a *1.5% raises* bring his salary *to \$39,000*. What was his salary *before the raise*?

- Let x = Tom's salary before the raise
- Equation: $x + 1.5\% x = 39,000$
 $1x + 0.015x = 39,000$
 $1.015x = 39,000$

Raise = $(1.5\%)(\text{Previous salary}) = 1.5\% x$
Previous salary + Raise = Current salary
- Answer: $x = \frac{39000}{1.015} \approx \boxed{\$38423.65}$

Tom's salary before the raise was \$38423.65.

Example: Bob deposits a certain amount of money in a *chequing account* that earns *2.5%* in annual interest, and deposits *\$2000 less than* that in a *saving account* that pays *1.5%* in annual interest. If the total interest from *both* accounts at the end of the year is *\$95*, how much is deposited in each account?

- Let x = money deposited in the saving account

Account	Deposit	Interest rate	Interest
Chequing account	x	2.5%	$0.025x$
Saving account	$x - 2000$	1.5%	$0.015(x - 2000)$

Total interest = \$95

- Equation: $0.025x + 0.015(x - 2000) = 95$

2.5% of saving + 1.5% of checking = \$95

 $0.025x + 0.015x - 30 = 95$

Combine like terms.

 $0.04x = 125$
- Answer: Chequing account: $x = \frac{125}{0.04} = \boxed{\$3125}$

\$3125 in the chequing account.

Saving account: $x - 2000 = 3125 - 2000 = \boxed{\$1125}$

\$1125 in the saving account.

Example: A string *103 meters* long is cut into *four pieces*. The *second* is *four times* as long *as* the *first*. The *third* piece is *five meters longer* than the *first*. *The fourth* piece is *twice* as long *as* the *third*. How long is each piece of string?

- Let x = the length of the first piece.

1 st piece	x
2 nd piece	$4x$
3 rd piece	$x + 5$
4 th piece	$2(x + 5)$

- Equation: $x + 4x + (x + 5) + 2(x + 5) = 103$

$$1^{\text{st}} + 2^{\text{nd}} + 3^{\text{rd}} + 4^{\text{th}} = 103$$

$$x + 4x + x + 5 + 2x + 10 = 103$$

Combine like terms.

$$8x + 15 = 103$$

$$8x = 88$$

$$x = \boxed{11 \text{ m}}$$

- Answer:

1 st piece	$x = \boxed{11 \text{ m}}$
2 nd piece	$4x = 4(11) = \boxed{44 \text{ m}}$
3 rd piece	$x + 5 = 11 + 5 = \boxed{16 \text{ m}}$
4 th piece	$2(x + 5) = 2(11 + 5) = \boxed{32 \text{ m}}$

Example: A fruit punch that contains *25% fruit juice*. *How much water* would you have to add to *1 liter* of punch to get a new drink that contains *10% fruit juice*?

- Let x = water to add to 1 L of punch to get a 10% fruit juice.

	Concentration	Volume	Amount
Fruit punch	25 %	1 (L)	$0.25(1)$
New drink	10 %	$x + 1$	$0.1(x + 1)$

- Equation: $0.25(1) = 0.1(x + 1)$

Amount of 25% = Amount of 10%

$$0.25 = 0.1x + 0.1$$

Multiply 100 for each term.

$$25 = 10x + 10$$

Combine like terms.

$$15 = 10x$$

Divide both sides by 10.

- Answer: $x = \boxed{1.5 \text{ L}}$

It needs to add 1.5 L of water to get a new drink that contains 10% fruit juice.

Unit 12: Summary

Solving Word Problems

Steps for solving word problems:

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- Organize the **facts** given from the problem (make a table).
- Identify and label the unknown quantity (**let x = unknown**).
- Draw a **diagram** if it will make the problem clearer.
- Convert words into a mathematical **equation**.
- Solve** the equation and find the solution(s).
- Check** and state the **answer**.

Table for value mixture problems:

Let x = unknown

Item	Value of the item	Number of items	Total value
Item A	value of A	# of A	(value of A) \times (# of A) = amount of A
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Item C	value of C	# of C	(value of C) \times (# of C) = amount of C
...
Total or mixture			total value

$$\text{Value of item A} + \text{Value of item B} + \text{Value of item C} + \dots = \text{Total value of the mixture}$$

Formulas of motion:

Distance = Speed \cdot Time

$$d = r t$$

$$t = \frac{d}{r}$$

$$r = \frac{d}{t}$$

Table of motions:

Condition	Speed (r)	Time (t)	Distance (d)
Condition A	r	t	$d = r t$
Condition B	r	t	$d = r t$
...
Total			

- Downstream (fast): speed of boat + speed of river
- Upstream (slower): speed of boat - speed of river

Table of concentration mixture:

Let x = unknown

Item	Concentration	Volume	Amount
Item A	concentration of A	volume of A	(concentration of A) \times (volume of A) = amount of A
Item B	concentration of B	volume of B	(concentration of B) \times (volume of B) = amount of B
...
Mixture	concentration of mixture	volume of mixture	(concentration of mixture) \times (volume of mixture) = amount of mixture

$$\text{Amount of item A} + \text{Amount of item B} + \dots = \text{Amount of the mixture}$$

Business math formulas:

Business problems	Formulas
Percent increase	Percent increase = $\frac{\text{New value} - \text{Original value}}{\text{Original value}}$, $x = \frac{N - O}{O}$
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Unit 12: Self-Test

Solving Word Problems

Topic A

1. Robert has \$2.50 in nickels, dimes and quarters. If he has two more than five times quarters of dimes, and two less nickels than quarters. How many of each coin does he have?
2. William purchased \$1.00, \$1.19, and \$1.20 Canadian stamps with a total value of \$27.13. If the number of \$1.19 stamps is 5 more than the number of \$1.00 stamps, and the number of \$1.20 stamps is 6 more than four times of \$1.00 stamps. How many of each did Damon receive?

Topic B

3. A lamb meal is 36% protein and a pork meal is 25% protein. Peter wants an 860 grams mixture that is 28% protein. How many grams of protein each meal should he have?
4. How much 5% salt solution must be added to 18 liters of 32% solution to make a 25% solution?

Topic C

5. Two cyclists are 72 km apart and are travelling towards each other. Their speeds differ by 2 km per hour. What is the speed of each cyclist if they meet after 3 hours?
6. Linda boats at a speed of 17 km per hour in still water. The river flows at a speed of 3 km per hour. How long will it take Linda to boat 4 km downstream? 4 km upstream?
7. A product increased production from 400 last month to 420 this month. Find the percent increase.
8. A product was reduced from \$80 to \$62. What was the percent reduction?
9. Find the sales tax for a \$ 679 laptop with a tax rate of 9%.
10. Find the commission for a \$699,000 townhouse with a commission rate of 4%.

11. A women's dress was originally priced at \$199, and is on sale at a 15% discount. Find the discount and sale price.
12. A condo was sold at \$469,000, with a markup rate of 5%. What was the markup and original price?
13. Smith borrowed \$100,000 mortgage from a bank. Find the interest at 4% per year for 5 years, and also find the total amount that Smith paid the bank.
14. Susan deposited \$2,500 in an account at 3.2% interest compounded per year for 2 years. How much was in the account at the end of 2 years?

Topic D

15. After a five percent reduction, a toy is on sale for thirty-nine dollars. What was the original price?
16. Ruth receives a 2.5% raises bring her salary to \$34,000. What was her salary before the raise?
17. Amy deposits a certain amount of money in a chequing account that earns 1.5% in annual interest, and deposits \$1500 less than that in a saving account that pays 1.2% in annual interest. If the total interest from both accounts at the end of the year is \$76.50, how much is deposited in each account?
18. A string that is 52 meters long is cut into four pieces. The second is three times as long as the first. The third piece is seven meters longer than the first. The fourth piece is three times as long as the third. How long is each piece of string?
19. A fruit punch is 45% fruit juice. How much water would you have to add to 1.5 liter of punch to get a new drink that is 25% fruit juice?