Topic B: Trigonometric Functions

Sides and Angles

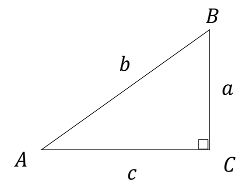
Trigonometry: the study of the relationships between sides and angles of right triangles and trigonometric functions.

Right triangle review: a triangle that has a 90° angle (right-angled triangle).

Sides and angles:

- $< C$ is a right angle (90^0).
- Sides are labeled with lower case letters (or two capital letters).

  Example: The side a or BC, the side b or AB, the side c or AC.



- Angles are labeled with uppercase letters.

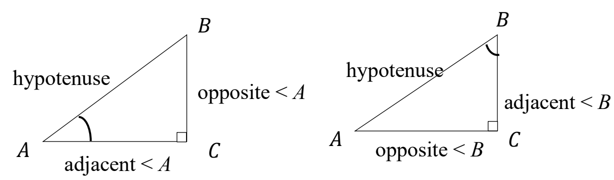
  Example: $< A$, $< B$, $< C$

- Side a will be the side opposite angle A; side b will be the side opposite angle B; and side c will be the side opposite angle C.

Hypotenuses, adjacent, and opposite:

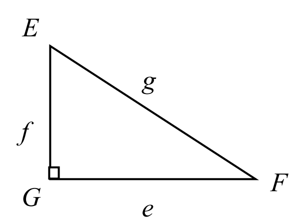
- The longest side of the triangle is the hypotenuses (the side opposite the 90° angle).
- "Opposite" and "adjacent" refer to sides that are opposite or adjacent to the two acute angles ($< A$ and $< B$) of the triangle.
- Adjacent side: the side next to the acute angle.
- Opposite side: the side opposite the acute angle.

  (An acute angle $< 90^0$.)



Example: Fill in the blanks in each of the following

1)	Side EG (or f) is ________angle F.	opposite
2)	Side FG (or e) is ________angle F.	adjacent
3)	Side EF (or g) is the ___________ .	hypotenuse
4)	Side EG (or f) is ________angle E.	adjacent
5)	Side FG (or e) is ________angle E.	opposite
6)	Side EG is opposite to angle _____ .	F



Trigonometric Functions

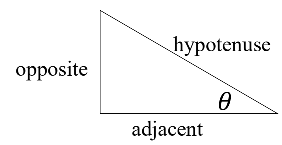
Trigonometric functions (of right triangles):

- There are six trigonometric functions (or ratios): sine (sin), cosine (cos), tangent (tan), secant (sec), cosecant (csc), and cotangent (cot).
- The lengths of the sides are used to define the trigonometric functions (or ratios).

Sine, cosine, and tangent (three main trigonometric functions):

- The sine of the angle $\theta = \dfrac{\text{the length of the opposite side}}{\text{the length of the hypotenuse}}$

  $\boxed{\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}}$



θ is a Greek letter that uses for an angle.

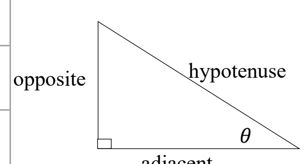
- The cosine of the angle $\theta = \dfrac{\text{the length of the adjacent side}}{\text{the length of the hypotenuse}}$
- $\boxed{\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}}$
- The tangent of the angle $\theta = \dfrac{\text{the length of the opposite side}}{\text{the length of the adjacent}}$

  $\boxed{\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent}}}$

Secant, cosecant, and cotangent: the inverse trigonometric functions.

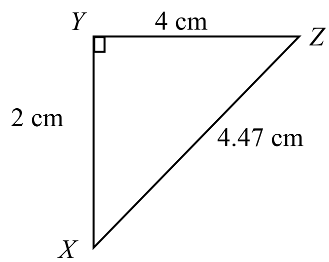
- Secant is the inverse of cosine: $\boxed{\sec\theta = \dfrac{1}{\cos\theta}}$
- Cosecant is the inverse of sine: $\boxed{\csc\theta = \dfrac{1}{\sin\theta}}$
- Cotangent is the inverse of tangent: $\boxed{\cot\theta = \dfrac{1}{\tan\theta}}$

Six trigonometric functions:

Trigonometric function	Read	Diagram	Memory aid
$\sin\boldsymbol{\theta} = \dfrac{\textbf{opposite side}}{\textbf{hypotenuse}}$	Sine of θ		Soh
$\cos\boldsymbol{\theta} = \dfrac{\textbf{adjacent side}}{\textbf{hypotenuse}}$	Cosine of θ		Cah
$\tan\boldsymbol{\theta} = \dfrac{\textbf{opposite side}}{\textbf{adjacent side}}$	Tangent of θ		Toa
$\sec\boldsymbol{\theta} = \dfrac{1}{\cos\boldsymbol{\theta}}$	Secant of θ		Inverse of cosine
$\csc\boldsymbol{\theta} = \dfrac{1}{\sin\boldsymbol{\theta}}$	Cosecant of θ		Inverse of sine
$\cot\boldsymbol{\theta} = \dfrac{1}{\tan\boldsymbol{\theta}}$	Cotangent of θ		Inverse of tangent

Sine, Cosine, and Tangent

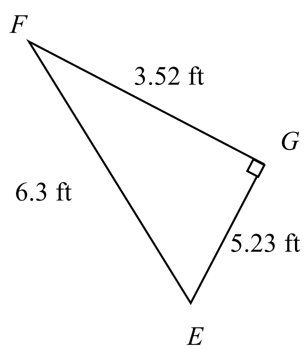
Example: Find the sine, cosine, and tangent for each of the following.



$\sin X = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{4 \text{ cm}}{4.47 \text{ cm}} \approx \boxed{0.8949}$	Soh
$\cos X = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{2 \text{ cm}}{4.47 \text{ cm}} \approx \boxed{0.4474}$	Cah
$\tan X = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{4 \text{ cm}}{2 \text{ cm}} = \boxed{2}$	Toa
$\sin Z = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{2 \text{ cm}}{4.47 \text{ cm}} \approx \boxed{0.4474}$	
$\cos Z = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{4 \text{ cm}}{4.47 \text{ cm}} \approx \boxed{0.8949}$	
$\tan Z = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{2 \text{ cm}}{4 \text{ cm}} = \frac{1}{2} = \boxed{0.5}$	

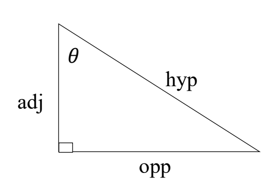
The sine of one angle in the right triangle is equal to the cosine of the other angle in that same right triangle.

Example: Find the sine, cosine, and tangent for each of the following.



$\sin F = \frac{\text{opp}}{\text{hyp}} = \frac{5.23 \text{ ft}}{6.3 \text{ ft}} \approx \boxed{0.8302}$	Soh
$\cos F = \frac{\text{adj}}{\text{hyp}} = \frac{3.52 \text{ ft}}{6.3 \text{ ft}} \approx \boxed{05587}$	Cah
$\tan F = \frac{\text{opp}}{\text{adj}} = \frac{5.23 \text{ ft}}{3.52 \text{ ft}} = \boxed{1.4858}$	Toa
$\sin E = \frac{\text{opp}}{\text{hyp}} = \frac{3.52 \text{ ft}}{6.3 \text{ ft}} \approx \boxed{0.5587}$	
$\cos E = \frac{\text{adj}}{\text{hyp}} = \frac{5.23 \text{ ft}}{6.3 \text{ ft}} \approx \boxed{0.8302}$	
$\tan E = \frac{\text{opp}}{\text{adj}} = \frac{3.52 \text{ ft}}{5.23 \text{ ft}} = \boxed{0.673}$	

Memory Aid:

Sine, cosine, and tangent	Trigonometric function	Memory aid	Diagram
Sine	$\sin \boldsymbol{\theta} = \frac{\textbf{opp}}{\textbf{hyp}}$	Soh	
Cosine	$\cos \boldsymbol{\theta} = \frac{\textbf{adj}}{\textbf{hyp}}$	Cah	
Tangent	$\tan \boldsymbol{\theta} = \frac{\textbf{opp}}{\textbf{adj}}$	Toa	

Topic C: Solving Right Triangles

Trigonometry Using a Calculator

Find the trigonometric functions of an angle:

Example: Find each of the following using a scientific calculator.

1) $\sin 132^0 = ?$

Type in: $\boxed{\sin}$ 132 Display: **0.7431**… $\sin 132^0 \approx 0.7431$

Or 132 $\boxed{\sin}$ with some calculators.

2) $\cos 25^0 = ?$

Type in: $\boxed{\cos}$ 25 $\boxed{=}$ Display: **0.9063 …** $\cos 25^0 \approx 0.9063$

Or 25 $\boxed{\cos}$ with some calculators.

3) $\tan 48^0 = ?$

Type in: $\boxed{\tan}$ 48 Display: **1.11061**… $\tan 48^0 \approx 1.1106$

Or 48 $\boxed{\tan}$ with some calculators.

Find an angle when given the trigonometric function (ratio):

Example: Find each of the following using a scientific calculator.

1) $\sin A = 0.5446$, $\quad < A = ?$

Type in: $\boxed{2ndF}$ $\boxed{\sin^{-1}}$ 0.5446 $\boxed{=}$ Display: **32.997333**… $< A \approx 33^0$

Or $\boxed{INV}$ with some calculators.

2) $\tan B = 0.57736$, $\quad < B = ?$

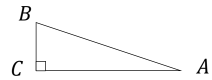
Type in: $\boxed{2ndF}$ $\boxed{\tan^{-1}}$ 0.57736 $\boxed{=}$ Display: **30.000418**… $< B \approx 30^0$

Or $\boxed{INV}$ with some calculators.

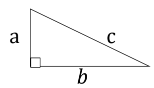
Solving Triangles

Angles in a triangle: the sum of the three internal angles in a triangle is always 180^0.

$< A + < B + < C = 180^0$



Pythagorean theorem review: a relationship between the three sides of a right triangle.



$c = \sqrt{a^2 + b^2}$

There are six elements (or parts) in a triangle, that is, three sides and three internal angles.

Solving a triangle: to solve a triangle means to know all three sides and all three angles.

Example: 1) Solve for the variable.

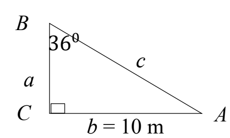
$\tan 32^0 = \frac{7}{x}$, $x = ?$

$x \cdot \tan 32^0 = \frac{7}{\cancel{x}} \cdot \cancel{x}$ Multiply both sides by x.

$\frac{x \cdot \cancel{\tan 32^0}}{\cancel{\tan 32^0}} = \frac{7}{\tan 32^0}$ Divide both sides by tan 32^0.

$\boldsymbol{x} = \frac{7}{\tan 32^0} \approx \boxed{11.2}$ Use a calculator.

2) Find side c if $b = 10\text{m}$ and $< B = 36^0$.



$\sin 36^0 = \frac{10\text{m}}{c}$ $\sin = \frac{\text{opp}}{\text{hyp}}$

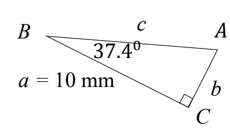
$\sin 36^0 \cdot c = \frac{10\text{m}}{\cancel{c}} \cdot \cancel{c}$ Multiply both sides by c.

$\frac{\cancel{\sin 36^0}}{\cancel{\sin 36^0}} \cdot c = \frac{10\text{m}}{\sin 36^0}$ Divide both sides by $\sin 36^0$.

$c = \frac{10\text{m}}{\sin 36^0} \approx \boxed{17.01 \text{ m}}$

Example: Solve the triangle ($< A = ?$ $b = ?$ $c = ?$). Find all unknown sides and angles.

$< A + < B + < C = 180^0$



- $< \boldsymbol{A} = 180^0 - (< C + < B)$

  $= 180^0 - (90^0 + 37.4^0)$

  $= \boxed{52.6^0}$

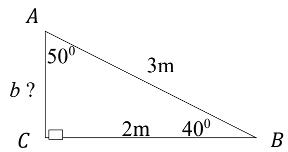
- $\tan B = \frac{b}{a}$ $\tan = \frac{\text{opp}}{\text{adj}}$

  $b = a\,(\tan B) = 10\,(\tan 37.4) \approx \boxed{7.65 \text{ mm}}$ Multiply both sides by a and reverse the sides.

- $\boldsymbol{c} = \sqrt{a^2 + b^2} = \sqrt{10^2 + 7.65^2} \approx \boxed{12.59 \text{ mm}}$ Pythagorean theorem.

Example: Find the missing part of each triangle.

1)



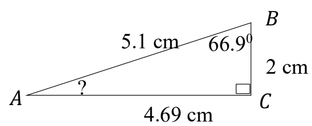
$\cos 50^0 = \frac{b}{3}$ $\cos = \frac{\text{adj}}{\text{hyp}}$

$3 \cdot \cos 50^0 = \frac{b}{\cancel{3}} \cdot \cancel{3}$ Multiply both sides by 3.

$3\,(\cos 50^0) = b$

$\boldsymbol{b} = 3\,(\cos 50^0) \approx \boxed{1.928 \text{ m}}$ Reverse the sides of the equation.

2)



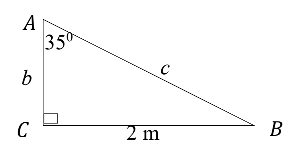
$\sin A = \frac{2}{5.1}$ $\sin = \frac{\text{opp}}{\text{hyp}}$

$< \boldsymbol{A} = \sin^{-1}\left(\frac{2}{5.1}\right) \approx \boxed{23.1^0}$ $\boxed{\text{2nd F}}$ $\boxed{\sin^{-1}}$

Example: Solve the right triangle. Find all unknown sides and angles.

1) **< B:** $< \boldsymbol{B} = 180^0 - (< C + < A)$ $(< B = ? \quad b = ? \quad c = ?)$

$= 180^0 - (90^0 + 35^0) = \boxed{55^0}$



b: $\tan 35^0 = \frac{2}{b}$ $\tan = \frac{\text{opp}}{\text{adj}}$

$b \cdot \tan 35^0 = \frac{2}{\cancel{b}} \cdot \cancel{b}$ Multiply both sides by *b*.

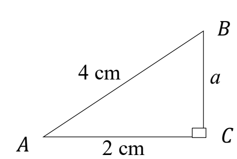
$\frac{b\,\cancel{\tan 35^0}}{\cancel{\tan 35^0}} = \frac{2}{\tan 35^0}$ Divide both sides by $\tan 35^0$.

$\boldsymbol{b} = \frac{2}{\tan 35^0} \approx \boxed{2.856 \text{ m}}$

***c*:** $\boldsymbol{c} = \sqrt{a^2 + b^2} = \sqrt{2^2 + 2.856^2} \approx \boxed{3.487 \text{ m}}$ Pythagorean theorem.

2) ***a*:** $\boldsymbol{a} = \sqrt{c^2 - b^2} = \sqrt{4^2 - 2^2} \approx \boxed{3.464 \text{ m}}$ $(a = ? \quad < B = ? \quad < A = ?)$

***<A*:** $\cos A = \frac{2}{4} = 0.5$ $\cos = \frac{\text{adj}}{\text{hyp}}$



$< \boldsymbol{A} = \cos^{-1} A = \cos^{-1} 0.5 = \boxed{60^0}$ $\boxed{\text{2nd F}}$ $\boxed{\cos^{-1}}$

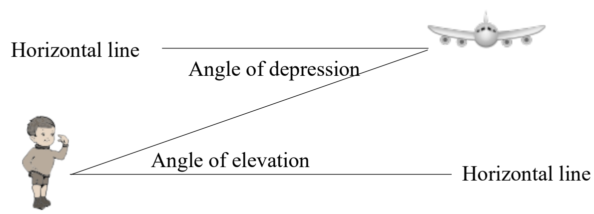
***<B*:** $< \boldsymbol{B} = 180^0 - (90^0 + 60^0) = \boxed{30^0}$

Check: $< A + < B + < C \overset{?}{=} 180^0$, $\quad 60^0 + 30^0 + 90^0 \overset{\surd}{=} \boxed{180^0}$ Correct!

Angles of Depression and Elevation

Angle of depression: the angle between a horizontal line and the line of sight for an object below the horizontal. The word "depression" means "fall" or "drop".

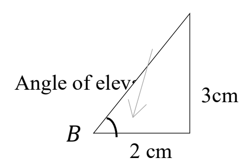
Angle of elevation: the angle between a horizontal line and the line of sight for an object above the horizontal. The word "elevation" means "rise" or "move up".



Example: 1) Find the angle of elevation.

$\tan B = \frac{3}{2} = 1.5$ $\quad$ $\tan = \frac{\text{opp}}{\text{adj}}$

$< \boldsymbol{B} = \tan^{-1} B = \tan^{-1} \frac{3}{2} \approx \boxed{56.3^0}$ $\quad$ $\boxed{\text{2nd F}}$ $\boxed{\tan^{-1}}$

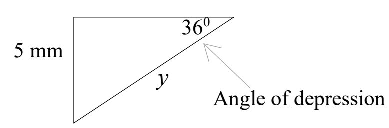


2) Find y if the angle of depression is 36^0.

$\sin 36^0 = \frac{5}{y}$ $\quad$ $\sin = \frac{\text{opp}}{\text{hyp}}$

$\boldsymbol{y} = \frac{5}{\sin 36^0} \approx \boxed{8.507 \text{ mm}}$

(Divide both sides by $\sin 36^0$ and multiply both sides by y.)



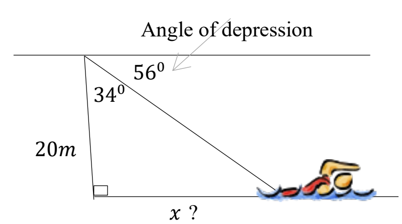
Example: From the top of a rock wall, the angle of depression to a swimmer is 56^0. If the wall is 20m high, how far from the base of the wall is the swimmer?

$90^0 - 56^0 = 34^0$

$\tan 34^0 = \frac{x}{20}$ $\quad$ $\tan = \frac{\text{opp}}{\text{adj}}$

$\boldsymbol{x} = 20\,(\tan 34^0) \approx \boxed{13.49 \text{ m}}$

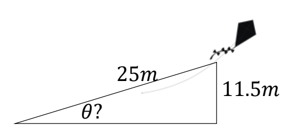
(Multiply both sides by 20 and reverse the sides of the equation.)



Example: Mike has let 25 m of string out on his kite. He is flying it 11.5 m above his eye level. Find the angle of elevation of the kite.

$\sin \theta = \frac{11.5}{25} \approx 0.46$ $\quad$ $\sin = \frac{\text{opp}}{\text{hyp}}$

$\theta = \sin^{-1} 0.46 \approx \boxed{27.4.^0}$



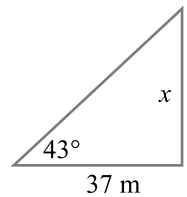
Applications of Trigonometry

Example: When Brandon stands 37 m from the base of a building and sights the top of the building, he is looking up at an angle of 43°. How high is the building?

$\tan 43^0 = \frac{x}{37}$ tan = $\frac{\text{opp}}{\text{adj}}$

$(37)\tan 43^0 = \frac{x}{\cancel{37}} \cdot \cancel{37}$ Multiply both sides by 37.

$x = (37)\tan 43^0 \approx \boxed{34.5 \text{ m}}$



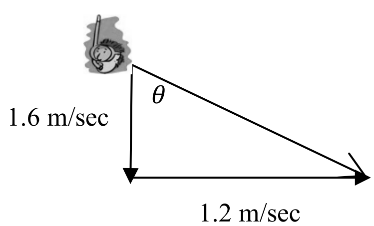
The building is approximately **34.5 m** high.

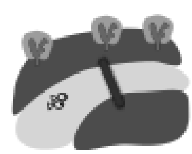
Example: Tom tries to swim straight across a river. He can swim at 1.6 m/sec, but the river is flowing at 1.2 m/sec. At what angle to his intended direction will Tom actually travel?

$\tan \theta = \frac{1.2}{1.6} = 0.75$ tan = $\frac{\text{opp}}{\text{adj}}$

$< \theta = \tan^{-1} 0.75 \approx \boxed{36.87^0}$ $\boxed{\text{2nd F}}$ $\boxed{\tan^{-1}}$

Tom will travel about **36.87°** off course.



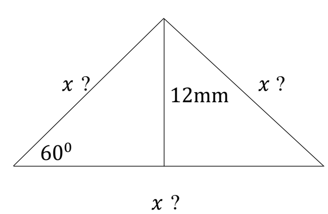


Example: An equilateral triangle has a height of 12 mm. Find the length of each side.

$\sin 60^0 = \frac{12}{x}$ Each angle = 60^0 (an equilateral triangle.)

$\boldsymbol{x} = \frac{12}{\sin 60^0} \approx \boxed{13.86 \text{ mm}}$

(Multiply both sides by x and divide both sides by $\sin 60^0$.)



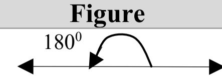
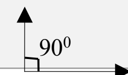
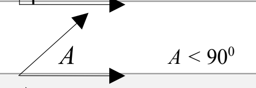
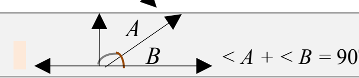
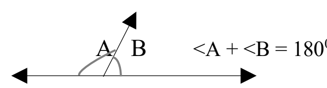
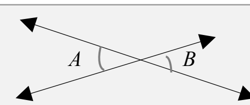
The length of each side is about **13.86 mm**.

Unit 10:   Summary

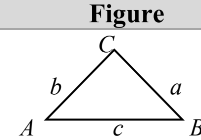
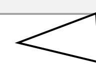
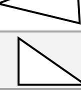
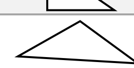
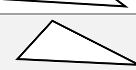
Trigonometry

An angle can vary from 0 to 360 degrees (360^0).

Classifying angles:

Angle	Definition	Figure
Straight angle	An angle of exactly 180^0.	
Right angle	An angle of exactly 90^0.	
Acute angle	An angle between 0 and 90^0.	
Obtuse angle	An angle between 90 and 180^0.	
Reflex angle	An angle between 180 and 360^0.	
Complementary angles	Two angles whose sum is exactly 90^0.	
Supplementary angles	Two angles whose sum is exactly 180^0.	
Vertical angles	Two angles formed by the intersection of two straight lines. $<A$ and $< B$ are vertical angels.	

Classify triangles:

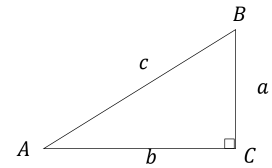
Name of triangle	Definition	Figure
Equilateral triangle	A triangle that has three equal sides and three equal angles. $a = b = c$, $< A = < B = < C = 60^0$	
Isosceles triangle	A triangle that has two equal sides and two equal angles. $a = b$, $< A = < B$	
Acute triangle	A triangle that has three acute angles ($< 90^0$).	
Right triangle	A triangle that has a right angle ($= 90^0$).	
Obtuse triangle	A triangle that has an obtuse angle ($> 90^0$).	
Scalene triangle	A triangle that has three unequal sides.	

Angles in a triangle: the sum of the three angles in a triangle is always 180^0.

$$\boxed{< X + < Y + < Z = 180^0}$$

How to use a protractor:

- Place the protractor so that the center hole is over the angle's vertex.
- Line up the base line of the protractor with one of the sides of the angle.
- Read the angle over the second side of the angle.

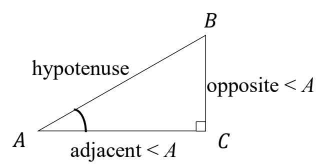


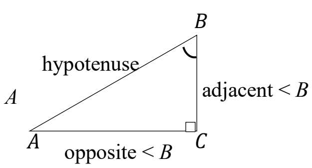
Sides and angles:

- Sides are labeled with lower case letters (or two capital letters).
- Angles are labeled with uppercase letters.
- Side a will be the side opposite angle A; side b will be the side opposite angle B; and side c will be the side opposite angle C.

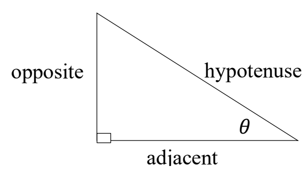
Hypotenuses, adjacent, and opposite:

- The longest side of the triangle is the hypotenuses (the side opposite the 90° angle).
- "Opposite" and "adjacent" refer to sides that are opposite or adjacent to the two acute angles (< A and < B) of the triangle.
- Adjacent side: the side next to the acute angle.
- Opposite side: the side opposite the acute angle.

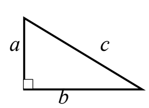




Six trigonometric functions:

Trigonometric function	Diagram	Memory aid
$\sin \boldsymbol{\theta} = \frac{\textbf{opposite}}{\textbf{hypotenuse}}$		Soh
$\cos \boldsymbol{\theta} = \frac{\textbf{adjacent}}{\textbf{hypotenuse}}$		Cah
$\tan \boldsymbol{\theta} = \frac{\textbf{opposite}}{\textbf{adjacent side}}$		Toa
$\csc \boldsymbol{\theta} = \frac{\mathbf{1}}{\sin A}$		Inverse of sine
$\sec \boldsymbol{\theta} = \frac{\mathbf{1}}{\cos A}$		Inverse of cosine
$\cot \boldsymbol{\theta} = \frac{\mathbf{1}}{\tan A}$		Inverse of tangent

Pythagorean theorem review: a relationship between the three sides of a right triangle.

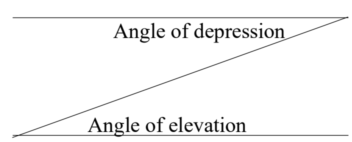


$$c = \sqrt{a^2 + b^2}$$

Solving a triangle: to solve a triangle means to know all three sides and all three angles.

Angle of depression: the angle between a horizontal line and the line of sight for an object below the horizontal.

Angle of elevation: the angle between a horizontal line and the line of sight for an object above the horizontal.



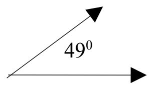
Unit 10: Self-Test

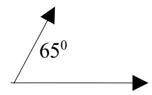
Trigonometry

Topic A

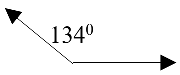
1. Label each of the following angles.

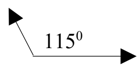
a)



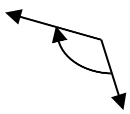


b)

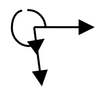




c)



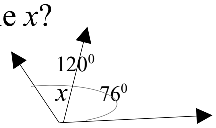
d)



2. What is the complementary angle to 42 degrees?

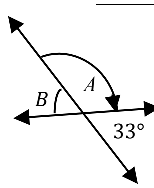
3. What is the supplementary angle to 146 degrees?

4. What is the size of the angle x?

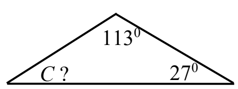


5. **a)** Two angles A and 33^0 that add together to measure 180^0 are said to be______?

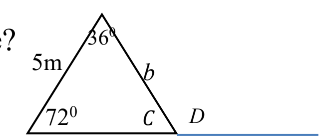
b) What is the size of angle A and B?



6. What is the size of angle C in the following figure?



7. What is the size of angle C, D and the side b in the following figure?

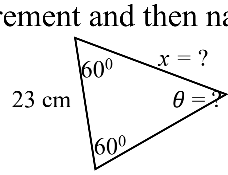


8. Match the following triangles to the letter with the best definition.

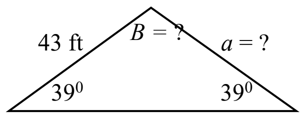
a) Equilateral triangle	i. has two equal sides
b) Isosceles triangle	ii. has three unequal sides
c) Supplementary angles	iii. Two angles whose sum is exactly 180^0.
d) Scalene triangle	iv. has three equal sides

9. Find the missing measurement and then name the kind of triangle.

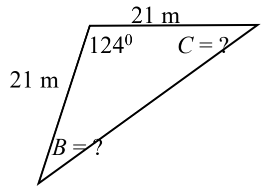
a)



b)



c)



d)

Topic B

10. Fill in the blanks in each of the following

a) Side ZY (or x) is ________angle X.

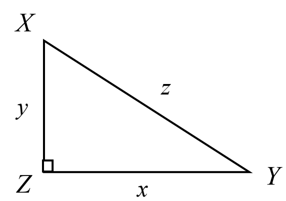
b) Side XZ (or y) is ________angle X.

c) Side XY (or z) is the ___________ .

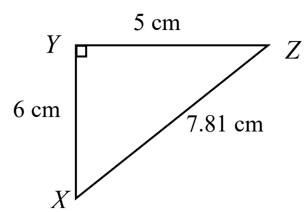
d) Side ZY (or x) is ________angle Y.

e) Side XZ (or y) is ________angle Y.

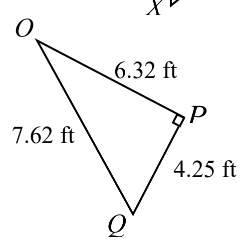
f) Side XZ (or y) is opposite to angle _____ .



11. Find the sine, cosine, and tangent of each acute angle.



12. Find the sine, cosine, and tangent of each acute angle.



Topic C

13. Use a calculator to find the trigonometric value of each angle.

a) sin 57° = ?

b) cos 36^0 = ?

c) tan 87^0 = ?

d) sin (   ) = 0.2165

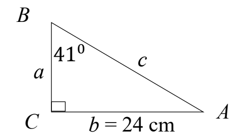
e) cos (   ) = 0.4567

f) tan (   ) = 1.2356

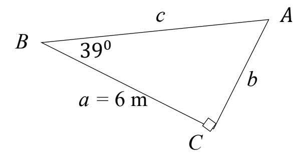
14. Solve for the variable.

$\tan 57^0 = \frac{12}{x}$, $x = ?$

15. Find side c if $b = 24$ cm and $< B = 41^0$.

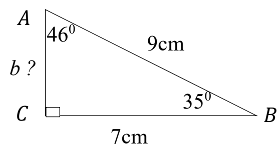


16. Solve the triangle ($< A = ?$ $b = ?$ $c = ?$).

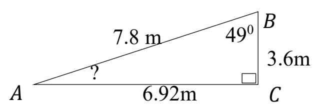


17. Find the missing part of each triangle.

a)

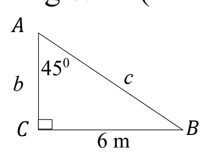


b)

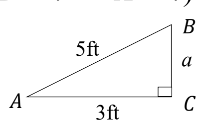


18. Solve the right triangle. ($< B = ?$ $b = ?$ $c = ?$)

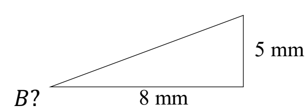
a)



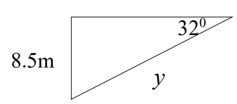
b) ($a = ?$ $< B = ?$ $< A = ?$)



19. **a)** Find the angle of elevation.



b) Find y if the angle of depression is 32^0.



20. From the top of a wall, the angle of depression to a boy is 43^0. If the wall is 24 m high, how far from the base of the wall is the boy?

21. Todd has let 34 m of string out on his kite. He is flying it 22.4 m above his eye level. Find the angle of elevation of the kite.

22. When Ann stands 28 m from the base of a building and sights the top of the building, she is looking up at an angle of 39°. How high is the building?

23. Damon tries to swim straight across a river. He can paddle at 1.3 m/sec, but the river is flowing at 1.5 m/sec. At what angle to his intended direction will Damon actually travel?

24. An equilateral triangle has a height of 41 cm. Find the length of each side.