Unit 15

Graphing Linear Equations

Topic A: Cartesian graphing

- The Coordinate plane
- Graphing linear equations

Topic B: The slope of a straight line

- Slope
- Vertical and horizontal lines

Topic C: Graphing a linear equation

- Slope-intercept equation of a line
- Graphing using the slope and the *y*-intercept
- Graphing linear equations
  - Intercept method

Topic D: Writing equations of lines

- Finding an equation of a line

Unit 15 Summary

Unit 15 Self-test

Topic A: Cartesian Graphing

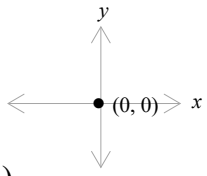
The Coordinate Plane

The coordinate plane (or Cartesian / rectangular coordinate system): a powerful tool to mark a point and solution of linear equations on a graph.

- Coordinate axes:

  x axis - the horizontal line.

  y axis - the vertical line.



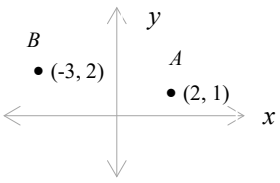
- The origin: the intersection of the *x* and *y* axes (both lines are 0 at the origin).

Ordered pair (*x*, *y*): a pair of numbers (each point on the plane corresponds to an ordered pair).

(x , y)

1st-coordinator (abscissa) 2nd-coordinator (ordinate)

Example: Point *A*: (2, 1)
Point *B*: (-3, 2)



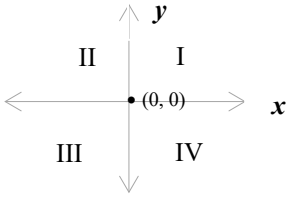
Example: (coke, \$0.90) , (juice, \$1.25)

Coordinate: the numbers in an ordered pair (the *x*-distance and the *y*-distance from a given origin).

Example: the coordinate of the point *A* is (2, 1) and the point *B* is (-3, 2).

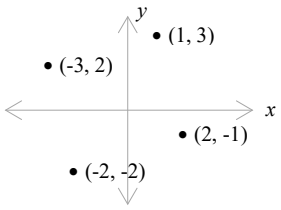
Four Quadrants:

Quadrant	(*x*, *y*)	Example
The 1st quadrant I	(+*x*, +*y*)	(+2, +3)
The 2nd quadrant II	(-*x*, +*y*)	(-2, +3)
The 3rd quadrant III	(-*x*, -*y*)	(-2, -3)
The 4th quadrant IV	(+*x*, -*y*)	(+2, -3)



Example: Plot the points and name the quadrants.

(1, 3) (-3, 2) (-2, -2) (2, -1)



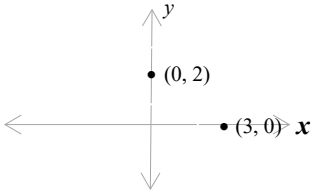
(1, 3): I , **(-3, 2):** II , **(-2, -2):** III, **(2, -1):** IV

***x* - intercept (*x*, 0):** the point at which the graph crosses the *x* - axis.

Example: (*x*, *y*) = (3, 0)

***y* - intercept (0, *y*):** the point at which the graph crosses the *y* - axis.

Example: (*x*, *y*) = (0, 2)



Points are on the axes.

Graphing Linear Equations

A linear (first-degree) equation: an equation whose graph is a straight line.

A linear equation in two variables: a linear equation that contains two variables, such as $2x + y = 3$.

The standard form of linear equation in two variables: $Ax + By = C$

Standard Form	Example
$Ax + By = C$	$5x - 7y = 4$

Solutions of equations: solutions for a linear equation in two variables are an ordered pair. They are the particular values of the variables in the equation that makes the equation true.

Example: Find the ordered pair solution of the given equation.

$2x - 3y = 7$, when $x = 2$.

$2(2) - 3y = 7$	Replace x with 2.
$4 - 3y = 7$	Subtract 4 from both sides.
$-3y = 3$	Divide -3 both sides.
$y = -1$	

Check: $2 \cdot 2 - 3(-1) \overset{?}{=} 7$, $7 = 7$. The ordered pair solution is $(2, -1)$ (x, y).

The graph of an equation is the diagram obtained by plotting the set of points where the equation is true (or satisfies the equation).

Procedure to graph a linear equation

Steps

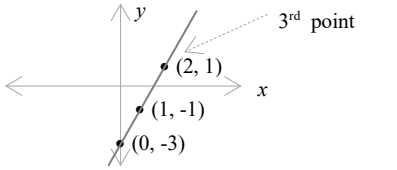
- Choose two values of x, calculate the corresponding y, and make a table.
- Plot these two points on the coordinate plane.
- Connect the points with a straight line.
  (Any two points determine a straight line.)
- Check with the third point.

  Is third point (2, 1) on the line? Yes. Correct!

Example: Graph $2x - y = 3$

x	$y = 2x - 3$	(x, y)	
0	$2 \cdot 0 - 3 = -3$	(0, -3)	Isolate y. y-intercept
1	$2 \cdot 1 - 3 = -1$	(1, -1)	

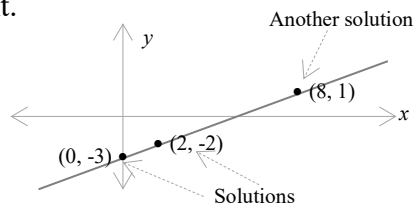
Select x — Calculate y — Ordered pair



x	$y = 2x - 3$	(x, y)
2	$2 \cdot 2 - 3 = 1$	(2, 1)

Example: Graph $y = \frac{1}{2}x - 3$ and determine another point.

x	y	(x, y)
0	- 3	(0, -3)
2	- 2	(2, -2)



Topic B: The Slope of a Straight Line

Slope

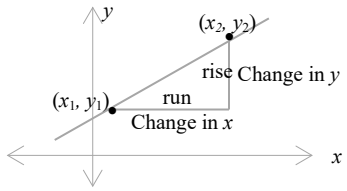
Recall: the graph of a linear equation is a straight line.

Slope (*m*) (grade or pitch): the slope of a straight line is the rate of change. It is a measure of the "steepness" or "incline" of the line and indicates whether the line rises or falls.

A line with a positive slope rises from left to right and a line with a negative slope falls.

The slope formula:

The slope formula	
$\text{Slope} = \frac{\text{the change in } y}{\text{the change in } x} = \frac{\text{rise}}{\text{run}}$	The slope of the straight line that passes through two points (x_1, y_1) and (x_2, y_2): $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{y_1 - y_2}{x_1 - x_2}$ $x_1 \neq x_2$



Example: Determine the slope containing points (3, -2) and (4, 1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{4 - 3} = \frac{3}{1} = 3$$ $(x_1, y_1) = (3, -2)$, $(x_2, y_2) = (4, 1)$

or $$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-2 - 1}{3 - 4} = \frac{-3}{-1} = 3$$

Example: Determine the slope of **$6x - y - 5 = 0$.**

x	$y = 6x - 5$	(x, y)
0	$6 \cdot 0 - 5 = -5$	$(x_1, y_1) = (0, -5)$
1	$6 \cdot 1 - 5 = 1$	$(x_2, y_2) = (1, 1)$

Choose Calculate

Solve for y from $6x - y - 5 = 0$

$6x - 5 = y$ (add y both sides.)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-5)}{1 - 0} = \frac{6}{1} = 6 \quad \text{or} \quad m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-5 - 1}{0 - 1} = \frac{-6}{-1} = 6$$

Other points on the line will obtain the same slope m.

x	$y = 6x - 5$	(x, y)
2	7	(2, 7)
-1	-11	(-1, -11)

Choose Calculate

$$m = \frac{-11 - 7}{-1 - 2} = \frac{-18}{-3} = 6$$ $(x_1, y_1) = (2, 7)$, $(x_2, y_2) = (-1, 11)$

Vertical and Horizontal Lines

Horizontal line: a line that is parallel to the x-axis.

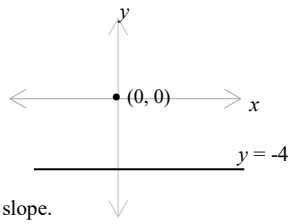
- It has a zero slope ($m = 0$).
- With a y-intercept $y = b$ or $(0, b)$. Where b is any constant.

Example: $y = -4$

x	y	(x, y)
1	-4	(1, -4)
2	-4	(2, -4)

Choose Given

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(-4)}{2-1} = \frac{0}{1} = 0$$



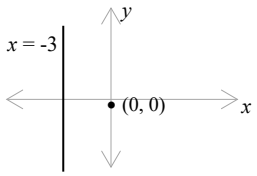
The horizontal line $y = -4$ has a zero slope.

Vertical line: a line that is parallel to the y-axis.

- It has an infinite slope ($m = \infty$).
- With a x-intercept $x = a$ or $(a, 0)$.

Example: $x = -3$

x	y	(x, y)
-3	3	(-3, 3)
-3	-1	(-3, -1)



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{-3-(-3)} = \frac{-4}{0} = \infty$$

The vertical line $x = -3$ has an infinite slope.

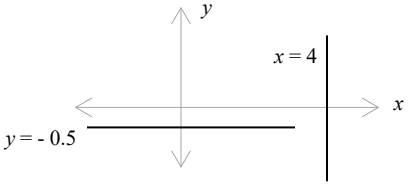
Summary of horizontal and vertical lines:

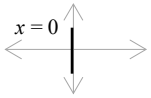
Line	Equation	Slope (m)	Example	Graph
Horizontal line	$y = b$	$m = 0$	$y = 2$	
Vertical line	$x = a$	$m = \infty$	$x = 1$	

Example: **1)** Graph $y = -0.5$

2) Graph $x = 4$

3) Graph $x = 0$



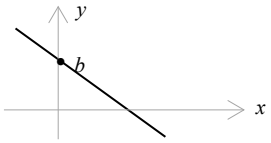


Topic C: Graphing a Linear Equation

Slope-Intercept Equation of a Line

Slope - intercept form of a linear equation

Slope - intercept equation of a line	
$y = \boldsymbol{m}\, x + \boldsymbol{b}$	m: the slope of the line b: y-intercept



Recall: *y* - intercept: the point at which the line crosses the y axis. $b = (0, y)$

Example: Identify the slope and y-intercept of the following equations.

1) $y = -3x - 5$

$y = \boldsymbol{m}\, x + \boldsymbol{b}$

$y = -3x - 5$

The slope: $\boldsymbol{m} = -3$

y-intercept: $\boldsymbol{b} = -5$ or $(0, -5)$

2) $3y - 2x = 1$

$3y = 2x + 1$ Add $2x$ on both sides.

$y = \frac{2}{3}x + \frac{1}{3}$ Divide both sides by 3.

The slope: $\boldsymbol{m} = \frac{2}{3}$ $y = \boldsymbol{m}\, x + \boldsymbol{b}$

y-intercept: $\boldsymbol{b} = \frac{1}{3}$ or $(0, \frac{1}{3})$

3) $4x + \frac{1}{3}y = 5$

$4x \cdot 3 + \frac{1}{3}y \cdot 3 = 5 \cdot 3$ Multiply 3 by each term.

$12x + y = 15$ Subtract $12x$ from both sides.

$y = -12x + 15$ $y = \boldsymbol{m}\, x + \boldsymbol{b}$

The slope: $\boldsymbol{m} = -12$

y-intercept: $\boldsymbol{b} = 15$ or $(0, 15)$

Graphing Using the Slope and the y - Intercept

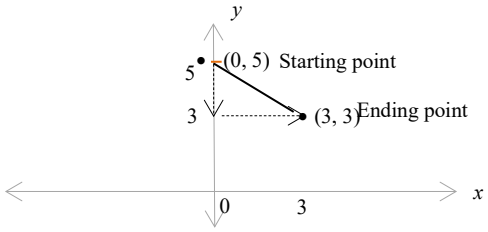
Slope-intercept equation: $y = mx + b$

$$\begin{cases} m = \text{slope} \\ b = y - \text{intercept} \end{cases}$$

The slope and a point can determine a straight line.

Example: Graph the equation using the slope and the y-intercept. $\boldsymbol{y = \frac{-2}{3}x + 5}$

- Plot the y-intercept (0, 5).
- Determine the rise and run: $m = \frac{-2}{3}$ — The change in y: the rise (move 2 units down, $\because y$ is negative). The change in x: the run (move 3 units to the right, $\because x$ is positive).
- Plot another point by moving 2 units down and 3 units to the right (3, 3).
- Connect the two points with a straight line.

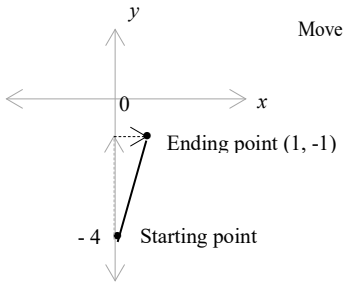


Starting point: y-intercept

Example: Graph the equation using the slope and the y-intercept. $\boldsymbol{-9x + 12 = -3y}$

- Convert to the slope - intercept form. $3y = 9x - 12$ — Divide each term by (-1).

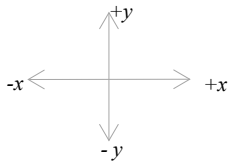
  $y = 3x - 4$ — Divide both sides by 3.
- y-intercept: $(0, -4)$ — $y = mx + b$
- Slope: $m = 3 = \frac{3}{1}$



Move 3 units up and 1 unit to the right (both x & y are positive).

Tip: $m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$

$$\begin{cases} +y: \text{ move up} \\ -y: \text{ move down} \\ +x: \text{ move to the right} \\ -x: \text{ move to the left} \end{cases}$$



Graphing Linear Equations
- Intercept Method

Recall: The *x* intercept is the point at which the line crosses the *x* axis. $(a, 0)$

The *y* intercept is the point at which the line crosses the *y* axis. $(0, b)$

Find the intercepts:

Example: Determine the intercepts of the line $5x - y = 6$.

- The *x*-intercept: let $y = 0$, and solve for *x*. $5x - 0 = 6, \quad 5x = 6$

  $x = \frac{6}{5} = 1.2$ Divide both sides by 5.

  $(1.2,\ 0)$

- The *y*-intercept: let $x = 0$, and solve for *y*. $5 \cdot 0 - y = 6, \quad -y = 6$

  $y = -6$ Divide both sides by -1.

  $(0, -6)$

Procedure to graph a linear equation using the intercept method

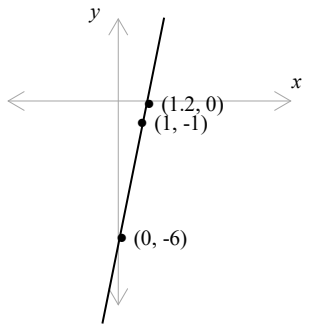
Steps

- Choose $x = 0$ and calculate the corresponding *y*.
- Choose $y = 0$ and calculate the corresponding *x*.
- Plot these two points on the coordinate plane.
- Connect the points with a straight line.
- Check with the third point.

  Is third point (1, -1) on the line? Yes. Correct!

Example: $5x - y = 6$

x	$y = 5x - 6$	(x, y)	Intercept
0	-6	(0, -6)	*y*-intercept
1.2	0	(1.2, 0)	*x*-intercept



x	$y = 5x - 6$	(x, y)
1	-1	(1, -1)

$(5 \cdot 1 - y = 6, \quad -y = 6 - 5, \quad y = -1)$

Topic D: Writing Equations of Lines

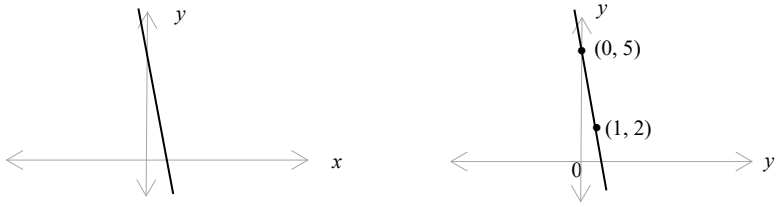
Finding an Equation of a Line

Equation of a straight line:

Straight-line equation	Equation	Example
Point-slope form	$y - y_1 = m\,(x - x_1)$	$y - 3 = -4\,(x + 2)$ $\quad m = -4 \quad y_1 = 3,\ x_1 = -2$
Slope-intercept form	$y = mx + b$	$y = 3x - \frac{4}{5}$ $\quad m = 3\ ,\ b = -\frac{4}{5}$

Finding an equation of a line from the graph:

Example: Write the slope intercept equation of the given line. $y = m\,x + b$



- Choose two points on the given line, such as (0, 5) and (1, 2).
- The slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{1-0} = \frac{-3}{1} = -3$ $\quad (x_1, y_1) = (0, 5),\ (x_2, y_2) = (1, 2)$
- y-intercept: $b = 5$ $\quad$ The line crosses the y-axis at (0, 5).
- Equation of the line: $\boxed{y = -3x + 5}$ $\quad y = m\,x + b$: $\quad m = -3, \quad b = 5$

Finding an equation of a line when the slope and a point are given:

Example: Write an equation for a line passing the point (5, 3) with slope $m = -4$.

- Start with: $y = mx + b$ $\quad$ Replace (x, y) by (5, 3) & m by -4.
- Solve for b: $3 = -4 \cdot 5 + b$ $\quad$ Add 20 on both sides.
- y-intercept: $b = 23$
- Equation of the line: $\boxed{y = -4x + 23}$ $\quad y = m\,x + b$: $\quad m = -4, \quad b = 23$

Finding an equation of a line when two points are given:

Example: Write an equation for a line that passes through the points (2, 1) and (3, -5).

- The slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-1}{3-2} = \frac{-6}{1} = -6$ $\quad (x_1, y_1) = (2, 1)\ ,\ (x_2, y_2) = (3, -5).$

  Substitute values into point-slope equation: $y - y_1 = m\,(x - x_1)$
- Point-slope equation: $y - 1 = -6\,(x - 2)$ $\quad$ Replace (x_1, y_1) with (2, 1) & m with – 6.
- Slope-intercept form: $y - 1 = -6x + 12$ $\quad$ Remove parentheses.

  $\boxed{y = -6x + 13}$ $\quad$ Add 1 on both sides, $y = m\,x + b$.

Unit 15: Summary

Graphing Linear Equations

The coordinate plane: a powerful tool to mark a point and solution of linear equation on a graph.

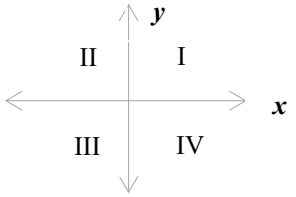
- Coordinate axes: x axis and y axis.
- The origin: the intersection of the x and y axes (both lines are 0 at the origin).

Ordered pair (x, y): a pair of numbers (each point on the plane corresponds to an ordered pair).

Coordinate: the numbers in an ordered pair (the x-distance and the y-distance from a given origin).

Four quadrants:

Quadrant	(x, y)
The 1st quadrant I	($+x$, $+y$)
The 2nd quadrant II	($-x$, $+y$)
The 3rd quadrant III	($-x$, $-y$)
The 4th quadrant IV	($+x$, $-y$)



x – intercept (x, 0): the point at which the graph crosses the x - axis.

y – intercept (0, y): the point at which the graph crosses the y - axis.

A linear (first-degree) equation: an equation whose graph is a straight line.

A linear equation in two variables: a linear equation that contains two variables, such as $5x + 2y = 7$.

The standard form of linear equation in two variables: $Ax + By = C$

Standard Form	Example
$Ax + By = C$	$4x - 9y = 11$

Solutions of equations: solutions for a linear equation in two variables are an ordered pair. They are the particular values of the variables in the equation that makes the equation true.

Procedure to graph a linear equation:

- Choose two values of x, calculate the corresponding y, and make a table.
- Plot these two points on the coordinate plane.
- Connect the points with a straight line.
- Check with the third point – is third point on the line?

Slope (*m*) (grade or pitch): the slope of a straight line is the rate of change. It is a measure of the "steepness" or "incline" of the line and indicates whether the line rises or falls.

- A line with a positive slope rises from left to right and a line with a negative slope falls.
- **The slope formula:**

The slope formula	
$\text{Slope} = \frac{\text{the change in } y}{\text{the change in } x} = \frac{\text{rise}}{\text{run}}$	The slope of the straight line that passes through two points (x_1, y_1) and (x_2, y_2): $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{y_1 - y_2}{x_1 - x_2}$ $x_1 \neq x_2$

Horizontal and vertical lines:

Line	Equation	Slope (*m*)
Horizontal line	$y = b$	$m = 0$
Vertical line	$x = a$	$m = \infty$

The slope and a point can determine a straight line.

Procedure to graph a linear equation using the intercept method:

- Choose $x = 0$ and calculate the corresponding y.
- Choose $y = 0$ and calculate the corresponding x.
- Plot these two points on the coordinate plane.
- Connect the points with a straight line.
- Check with the third point - is third point on the line?

Equation of a straight line:

Straight-line equation	Equation
Point-slope form	$y - y_1 = m\,(x - x_1)$
Slope-intercept form	$y = mx + b$

{ m = slope
{ b = y − intercept

Finding an equation of a line from the graph:

- Choose two points on the given line.
- Calculate the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Determine the y-intercept on the line: b or $(0, y)$ — The line crosses the y-axis.
- Equation of the line: $y = m\,x + b$ — Replace m and b with values.

Finding an equation of a line when the slope and a point are given:

- Start with: $y = mx + b$ — Replace (x, y) & m with given values.
- Solve for b.
- Equation of the line: $y = mx + b$ — Replace m and b with values.

Finding an equation of a line when two points are given:

- Calculate the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Point-slope equation: $y - y_1 = m\,(x - x_1)$ — Replace (x_1, y_1) & m with values.
- Slope-intercept equation: $y = mx + b$ — Solve for y.

Unit 15: Self-Test

Graphing Linear Equations

Topic A

1. Plot the points and name the quadrants.

(2, -1) (-4, 3) (-1, -3) (3, 2)

2. Graph the following.

a) $y = 3x$

b) $7x - y = 3$

c) $x + 3y = 6$

3. Graph $y = \frac{1}{3}x - 4$ and determine another point.

4. Find the ordered pair solution of the given equation.

a) $3x - 5y = 11$, when $x = 2$.

b) $x - 0.6y = -3$, when $x = -6$.

c) $\frac{3}{4}x - 4y = 5$, when $x = -4$.

Topic B

5. Determine the slope containing points (4, -1) and (3, 5).

6. Determine the slope of $8x - y - 3 = 0$.

7. Graph the following.

a) $y = -0.9$

b) $x = 3$

c) $y = 0$

Topic C

8. Identify the slope and y-intercept of the following equations.

a) $y = -7x - 11$

b) $5y - 3x = 2$

c) $7x + \frac{1}{5}y = 2$

9. Graph the equation using the slope and the *y*-intercept.

a) $y = \frac{-3}{4}x + 5$

b) $-6x + 9 = -3y$

10. Determine the intercepts of the line $3x - y = 9$.

11. Graph the equation using the intercept method.

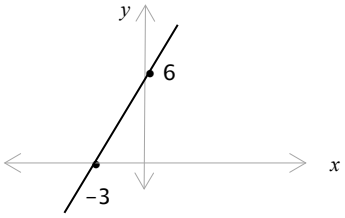
a) $4x - y = 8$

b) $y = \frac{-1}{2}x + 3$

Topic D

12. Write the slope intercept equation of the given line.

a)



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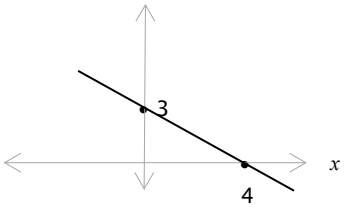
b)



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13. Write the equation for the following lines:

a) The line with a slope of -4 passing the point (-2, 5).

b) The line with a slope of $\frac{3}{5}$ passing the point (5, -7).

14. Write an equation of the line that passes through each pair of points.

a) (3, 2) and (4, -7).

b) (-3, 0) and (0, 6).

c) (0, 5) and (5, 3).