Unit 8

Formulas

Topic A: Substitution into formulas

- Geometry formulas
- Substituting into formulas

Topic B: Solving formulas

- Solving for a specific variable
- More examples for solving formulas

Topic C: Pythagorean theorem

- Pythagorean theorem
- Applications of the Pythagorean theorem

Unit 8 Summary

Unit 8 Self-test

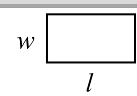
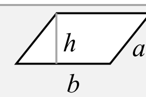
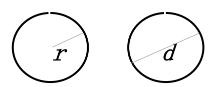
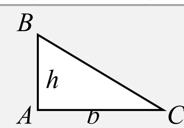
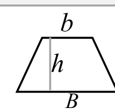
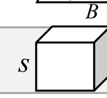
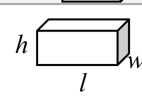
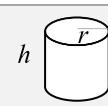
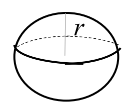
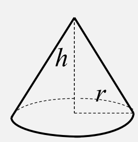
Topic A: Substitution into Formulas

Geometry Formulas

Formula: an equation that contains more than one variable and is used to solve practical problems in everyday life.

Geometry formulas review:

s – side, P – perimeter, C – circumference, A – area, V – volume

Name of the figure	Formula	Figure
Rectangle	$P = 2\,w + 2\,l$ $A = w\,l$ (w = width, l = length)	
Parallelogram	$P = 2\,a + 2\,b$ $A = b\,h$ (a and b = sides, h = height)	
Circle	$C = \pi d = 2\pi r$ $A = \pi r^2$ (r = radius, d = diameter)	
Triangle	$< A + < B + < C = 180^0$ $A = \frac{1}{2} bh$ (b = base, h = height)	
Trapezoid	$A = \frac{1}{2} h\,(b + B)$ (b = top base, B = bottom base, h = height)	
Cube	$V = s^3$ (s = side)	
Rectangular solid	$V = w\,l\,h$ (w = width, l = length, h = height)	
Cylinder	$V = \pi\, r^2\, h$ (r = radius, h = height)	
Sphere	$V = \frac{4}{3} \pi\, r^3$ (r = radius)	
Cone	$V = \frac{1}{3} \pi\, r^2 h$ (r = radius, h = height)	

Substituting into Formulas

- **Examples of formula:**

Application	Formula	Components
Distance	$d = v\,t$	d – distance v – velocity t – time
Simple interest	$I = P\,r\,t$	I – interest P – principle r – interest rate (%) t – time (years)
Compound interest	$B = P\,(100\% + r)^{t}$	B – *balance* P – principle r – interest rate (%) t – time (years)
Percent increase	$\frac{N - O}{O}$	N – new value O – original value
Percent decrease	$\frac{O - N}{O}$	N – new value O – original value
Sale price and discount	$S = P - d\,P$ $D = d\,P$	S – sale price P – price (original or regular price) d – discount rate D – discount
Original price and markup	$P = C + m\,C$ $M = m\,C$	P – price (original or selling price) C – cost m – markup rate M – markup
Intelligence quotient (I.Q.)	$I = \frac{100m}{c}$	I – I.Q. m – mental age c – chronological age
Cost of running electrical appliances	$C = \frac{Wtr}{1000}$	C – Cost (in cents) W – power in watts (watts used) t – time (hours) r – rate (per kilowatt-hour)

Substitution into formula: "substitution" means replacing numbers with variables (letters).

Example: Find the IQ of a 10-year-old girl with a mental age of 12.

- Formula: $I = \frac{100m}{c}$
- Facts: m = 12 years, c = 10 years
- Substituting: $I = \frac{100m}{c}$

$= \frac{100\,(12\,y)}{10\,y}$ Substitute m for 12 y and c for 10 y.

$I = \boxed{120}$

The 10-year-old girl has an IQ of 120.

Example: Find the distance travelled by a train which has a velocity of 83 km per hour for 3 hours.

- Formula: $d = v\,t$
- Facts: $v = 83$ km/h, $t = 3$ h
- Substituting: $d = v\,t = (83 \text{ km/h})\ (3 \text{ h})$ — Substitute v for 83 km/h and t for 3h.

  $d = \boxed{249 \text{ km}}$ — $\left(\frac{83 \text{ km}}{\cancel{\text{h}}}\right)(3\cancel{\text{h}}) = 249$ km

  The distance is 249 km.

Example: Find the volume of a cylinder with a radius of 2.3 cm and a height of 4.2 cm.

- Formula: $V = \pi\, r^2\, h$
- Facts: $r = 2.3$ cm, $h = 4.2$ cm
- Substituting: $V = \pi r^2 h = \pi\ (2.3 \text{ cm})^2\ (4.2 \text{ cm})$ — Substitute r for 2.3cm and h for 4.2 cm.

  $V \approx \boxed{69.8 \text{ cm}^3}$ — $(\text{cm}^2)\ (\text{cm}) = \text{cm}^3$

  The volume of the cylinder is 69.8 cm^3.

Example: Find the area of a triangle with a base of 12 ft and a height of 34 ft.

- Formula: $A = \frac{1}{2}bh$
- Facts: $b = 12$ ft , $h = 34$ ft
- Substituting: $A = \frac{1}{2}bh = \frac{1}{2}\ (12 \text{ ft})\ (34 \text{ ft})$ — Substitute b for 12 ft and h for 34 ft.

  $A = \boxed{204 \text{ ft}^2}$ — $(\text{ft})\ (\text{ft}) = \text{ft}^2$

  The area of the triangle is 204 ft^2.

Example: An electric stove top burner runs for 2 hours and uses 750 watts of electricity at a cost of 10 cents per kilowatt-hour. What is the total cost of running the stove top burner?

- Formula: $C = \frac{Wtr}{1000}$
- Facts: $t = 2$ h , $W = 750$ w , $r = 10¢$ / kwh
- Substituting: $C = \frac{Wtr}{1000} = \frac{(750\text{w})(2\text{h})(10¢/kwh)}{1000}$ — Substitute W, t and r.

  $= \boxed{15\ ¢}$

  The cost of running the stove top burner is 15 cents.

Topic B: Solving Formulas

Solving for a Specific Variable

To solve for a variable in a formula: isolate the unknown or desired variable so that it is by itself on one side of the equals sign and all the other terms are on the other side.

- Use the same process as you would for regular linear equations, the only difference is that you will be working with more variables.
- Remember to always do the same thing to both sides of the formula (add, subtract, multiply or divide the same variable or number to both sides of a formula).

Rearrange the formula so that the unknown or desired variable is by itself on one side of the equals sign. You can reverse the sides of the formula if you want.

Example: Solve each formula for the given variable.

1) Solve $\boldsymbol{d = r\,t}$ **for** $\boldsymbol{t}$. — Isolate t (t is the desired variable).

$\frac{d}{\boldsymbol{r}} = \frac{\cancel{r}t}{\cancel{\boldsymbol{r}}}$ — Divide both sides by r.

$\frac{d}{r} = t$ or $\boxed{t = \frac{\boldsymbol{d}}{\boldsymbol{r}}}$ — Reverse the sides of the formula.

Tip: solve a formula for a given letter by isolating the given letter on one side of the formula.

2) Solve $\boldsymbol{I = P\,r\,t}$ **for** $\boldsymbol{r}$ **and** $\boldsymbol{P}$. — Isolate r (r is the desired variable).

$\boldsymbol{r}$: $\frac{I}{\boldsymbol{P\,t}} = \frac{\cancel{P}\,r\,\cancel{t}}{\cancel{\boldsymbol{P}}\,\cancel{\boldsymbol{t}}}$ — Divide both sides by Pt.

$\frac{I}{Pt} = r$ or $\boxed{r = \frac{\boldsymbol{I}}{\boldsymbol{P\,t}}}$ — Reverse the sides of the formula.

$\boldsymbol{P}$: $\frac{I}{\boldsymbol{r\,t}} = \frac{P\,\cancel{r}\,\cancel{t}}{\cancel{\boldsymbol{r}}\,\cancel{\boldsymbol{t}}}$ — Divide both sides by $r\,t$.

$\frac{I}{r\,t} = P$ or $\boxed{P = \frac{\boldsymbol{I}}{\boldsymbol{r\,t}}}$ — Reverse the sides of the formula.

3) Solve $\boldsymbol{P = 2\,w + 2\,l}$ **for** $\boldsymbol{w}$. — Isolate $2w$ (w is the desired variable).

$P - \boldsymbol{2\,l} = 2\,w + \cancel{2\,l} - \cancel{\boldsymbol{2\,l}}$ — Subtract $2l$ from both sides.

$P - 2\,l = 2w$ — Divide both sides by 2.

$\frac{P-2l}{\boldsymbol{2}} = \frac{\cancel{2}w}{\cancel{\boldsymbol{2}}}$

$\frac{p-2l}{2} = w$ or $\boxed{w = \frac{p-2l}{2}}$ — Reverse the sides of the formula.

More Examples for Solving Formulas

Example: Solve each formula for the given variable.

1) **a)** Solve $\boldsymbol{F = \frac{9}{5}C + 32}$ **for** $\boldsymbol{C}$**.** **b)** If $\boldsymbol{F = 68}$**,** $\boldsymbol{C = ?}$

Solution:

a) $F - \mathbf{32} = \frac{9}{5}C + \cancel{32} \ \cancel{-\mathbf{32}}$ Subtract 32 from both sides.

$F - \mathbf{32} = \frac{9}{5}C$

$\mathbf{\frac{5}{9}}\,(F - 32) = \cancel{\mathbf{\frac{5}{9}}} \cdot \cancel{\frac{9}{5}}C$ Multiply both sides by $\frac{5}{9}$.

$\frac{5}{9}(F - 32) = C$ or $\boxed{C = \frac{5}{9}(F - 32)}$ Reverse the sides of the formula.

b) If $F = 68$, $C = \frac{5}{9}(68 - 32)$ Substitute 68 for F in the formula.

$\boldsymbol{C} = \frac{5}{9}(36)$

$\boxed{C = 20}$

2) Solve $\boldsymbol{P = C + m\,C}$ **for** $\boldsymbol{C}$.

$P = C\,(1 + m)$ Factor out C.

$\frac{P}{1+m} = \frac{C\,\cancel{(1+m)}}{\cancel{1+m}}$ Divide both sides by $(1 + m)$.

$\frac{P}{1+m} = C$ or $\boxed{C = \frac{P}{1+m}}$ Reverse the sides.

3) Solve $\boldsymbol{p = 35\,q^2 + s\,q}$ *for* $\boldsymbol{s}$.

$p - 35\,q^2 = \cancel{35\,q^2} + s\,q - \cancel{35\,q^2}$ Subtract $35q^2$ from both sides.

$p - 35\,q^2 = s\,q$

$\frac{P - 35q^2}{q} = \frac{s\cancel{q}}{\cancel{q}}$ Divide both sides by q.

$\boxed{s = \frac{P - 35q^2}{q}}$ Reverse the sides.

4) Solve $\boldsymbol{x = \frac{y - z}{t}}$ **for** $\boldsymbol{y}$. Multiply both sides by t.

$x\,t = \frac{y - z}{\cancel{t}} \cdot \cancel{t}$

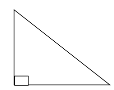
$x\,t + z = y - \cancel{z} + \cancel{z}$ Add z to both sides.

$\boxed{y = x\,t + z}$ Reverse the sides.

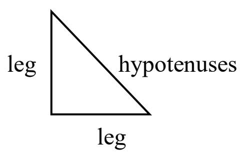
Topic C: Pythagorean Theorem

Pythagorean Theorem

Right triangle: a triangle containing a 90° angle.



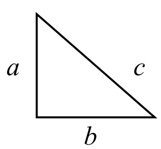
Pythagorean theorem: a relation among the three sides of a right triangle which states that the square of the hypotenuse is equal to the sum of the squares of the other two sides (legs).



$\boxed{\text{hypotenuse}^2 = \text{leg}^2 + \text{leg}^2}$

$\boxed{\text{hypotenuse} = \sqrt{\text{leg}^2 + \text{leg}^2}}$, $\boxed{\text{leg} = \sqrt{\text{hypotenuse}^2 - \text{leg}^2}}$

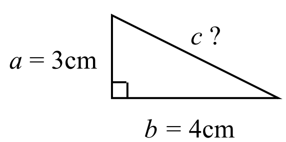
Using the Pythagorean theorem can find the length of the missing side in a right triangle.



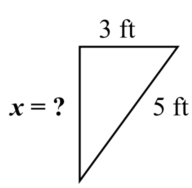
$\boxed{c = \sqrt{a^2 + b^2}}$ $\boxed{b = \sqrt{c^2 - a^2}}$ $\boxed{a = \sqrt{c^2 - b^2}}$

- c is the longest side of the triangle (hypotenuses).
- Other two sides (legs) of the triangle a and b can be exchanged.

Example: Find the missing side of the following triangles.



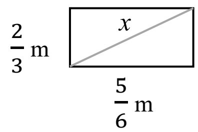
$c = \sqrt{a^2 + b^2} = = \sqrt{3^2 + 4^2} = \boxed{5 \text{ cm}}$ hypotenuse = $\sqrt{\text{leg}^2 + \text{leg}^2}$



$x = \sqrt{5^2 - 3^2} = \boxed{4 \text{ ft}}$ arm = $\sqrt{\text{hypotenus}^2 - \text{arm}^2}$

Applications of the Pythagorean Theorem

Example: Find the distance of the diagonal across the rectangle.



$x = \sqrt{(\frac{2}{3})^2 + (\frac{5}{6})^2} \approx 1.067$ m $\qquad c = \sqrt{a^2 + b^2}$

$x \approx \boxed{1.067 \text{ m}}$

The distance of the diagonal is 1.067 m.

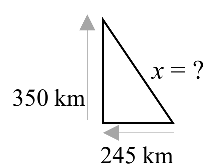
Example: What is the length of one leg of a right triangle whose hypotenuse measures 5.36 cm and the other leg measures 3.24 cm?

$x = \sqrt{5.36^2 - 3.24^2} \approx 4.27$ cm $\qquad a = \sqrt{c^2 - b^2}$

$x \approx \boxed{4.27 \text{ cm}}$

The length of one leg is 4.27 cm.

Example: A plane leaves the Vancouver airport and flies 245 km west, then 350 km north. How far is the plane from the airport?

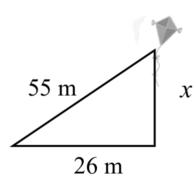


$x = \sqrt{245^2 + 350^2} \approx 427.23$ km

$x \approx \boxed{427.23 \text{ km}}$

The distance of the plane from the airport is 427.23 km.

Example: A kite at the end of a 55 m line is 26 m behind the runner. How high is the kite?



$x = \sqrt{55^2 - 26^2} \approx 48.47$ m

$x \approx \boxed{48.47 \text{ m}}$

The height of the kite is 48.47 m.

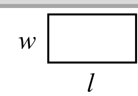
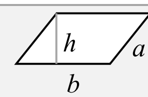
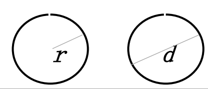
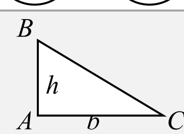
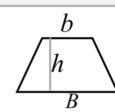
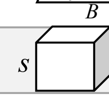
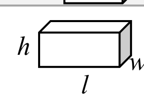
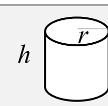
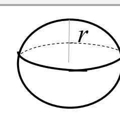
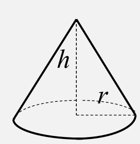
Unit 8: Summary

Formulas

Formula: an equation that contains more than one variable and is used to solve practical problems in everyday life.

Geometry formulas review:

s – side, P – perimeter, C – circumference, A – area, V – volume

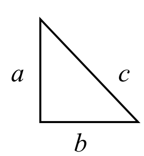
Name of the figure	Formula	Figure
Rectangle	$P = 2w + 2l$ $A = wl$ (w = width, l = length)	
Parallelogram	$P = 2a + 2b$ $A = bh$ (a and b = sides, h = height)	
Circle	$C = \pi d = 2\pi r$ $A = \pi r^2$ (r = radius, d = diameter)	
Triangle	$\angle A + \angle B + \angle C = 180^0$ $A = \frac{1}{2}bh$ (b = base, h = height)	
Trapezoid	$A = \frac{1}{2}h(b + B)$ (b = top base, B = bottom base, h = height)	
Cube	$V = s^3$ (s = side)	
Rectangular solid	$V = wlh$ (w = width, l = length, h = height)	
Cylinder	$V = \pi r^2 h$ (r = radius, h = height)	
Sphere	$V = \frac{4}{3}\pi r^3$ (r = radius)	
Cone	$V = \frac{1}{3}\pi r^2 h$ (r = radius, h = height)	

Substitution into formula: "substitution" means replacing numbers with variables (letters).

Examples of formula:

Application	Formula	Components
Distance	$d = v\ t$	*d* – distance *v* – velocity *t* – time
Simple interest	$I = P\ r\ t$	*I* – interest *P* – principle *r* – interest rate (%) *t* – time (years)
Compound interest	$B = P\ (100\% + r)^{t}$	*B – balance* *P* – principle *r* – interest rate (%) *t* – time (years)
Percent increase	$\frac{N - O}{O}$	*N* – new value *O* – original value
Percent decrease	$\frac{O - N}{O}$	*N* – new value *O* – original value
Sale price and Discount	$S = P - d\ P$ $D = d\ P$	*S* – sale price *P* – price (original or regular price) *d* – discount rate *D* – discount
Original price and Markup	$P = C + m\ C$ $M = m\ C$	*P* – price (original or selling price) *C* – cost *m* – markup rate M – markup
Intelligence quotient (I.Q.)	$I = \frac{100m}{c}$	*I* – I.Q. *m* – mental age *c* – chronological age
Cost of running electrical appliances	$C = \frac{Wtr}{1000}$	*C* – Cost (in cents) *W* – power in watts (watts used) *t* – time (hours) *r* – rate (per kilowatt-hour)

Pythagorean theorem: a relation among the three sides of a right triangle which states that the square of the hypotenuse is equal to the sum of the squares of the other two sides (legs).



$$\boxed{c = \sqrt{a^2 + b^2}}$$

Using the Pythagorean theorem can find the length of the missing side in a right triangle.

- *c* is the longest side of the triangle (hypotenuses).
- Other two sides (legs) of the triangle *a* and *b* can be exchanged.

Unit 8: Self-Test

Formulas

Topic A

1. Find the IQ of a 70-year-old man with a mental age of 85.

2. Find the distance travelled by a train which has a velocity of 78 km per hour for 2.5 hours.

3. Steve rides his bicycle at a speed of 11 miles per hour. He goes on a 22-mile bike ride. How many hours does this ride take?

4. Find the volume of a cone with a radius of 4.6 cm and a height of 8.4 cm.

5. Find the area and perimeter of a rectangle with a width of 11 cm and a length of 35 cm.

6. Find the area of a triangle with a base of 24 ft and a height of 58 ft.

7. The diameter of a circle is 4.8 ft. Find the circumference and area of the circle.

8. Ann invests $15,000 at an annual interest rate of 0.75%. How much simple interest will she earn by the end of 3 years?

9. An electric stove top burner runs for 2.5 hours and uses 800 watts of electricity per hour at a cost of 9 cents per kilowatt-hour. What is the total cost of running the stove top burner?

Topic B

10. Solve each formula for the given variable.

a) Solve $d = r\,t$ for r.

b) Solve $I = P\,r\,t$ for t.

c) Solve $P = 2\,w + 2\,l$ for l.

d) Solve $C = \frac{5}{9}(F - 32)$ for F.

If $C = 24$, $F = ?$

e) Solve $P = C + m\,C$ for m.

f) Solve $x = 35\,y^2 + z\,y$ for z.

g) Solve $A = \frac{1}{2}\,bh^2$ for b.

h) Solve $x = \frac{y - z}{t}$ for z.

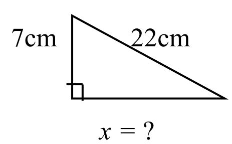
i) Solve $w = \frac{\pi r^2 h}{35}$ for h.

j) Solve $x = y - (2z + 3)\,w$ for w,

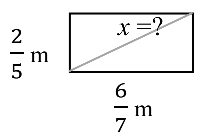
if $x = 2$, $y = 3$, $z = 4$

Topic C

11. Find the missing side of the following triangles.



12. Find the distance of the diagonal across the rectangle.



13. What is the length of one leg of a right triangle whose hypotenuse measures 21.34 ft and the other leg measures 15.27 ft?

14. A plane leaves the Calgary airport and flies 134 km east, then 250 km south. How far is the plane from the airport?

15. A kite at the end of a 89 ft line is 57 ft behind the runner. How high is the kite?