Unit 9

Ratio, Proportion, and Percent

Topic A: Ratio and rate

- Ratio
- Rate

Topic B: Proportion

- Solving proportion

Topic C: Percent

- Percent review
- Solving percent problems

Topic D: Similar triangles

- Similar triangles
- Solving similar triangles

Unit 9 Summary

Unit 9 Self-test

Topic A: Ratio and Rate

Ratio

Ratio

- Ratio: a relationship between two numbers, expressed as the quotient with the ***same unit*** in the denominator and the numerator.
- Express a ratio: there are three ways to write a ratio.

  The ratio of a and b is: a to b or $a : b$ or $\frac{a}{b}$

  Example: Write the ratio of 5 cents to 9 cents.

  5 to 9 or 5 : 9 or $\frac{5}{9}$

- Write a ratio in lowest terms (simplify):
  - Write the ratio in a fractional form.
  - Simplify and drop the units if given (as they cancel each other out).

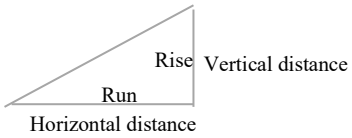
Example: $4:28 = \frac{4}{28} \overset{\div 4}{\underset{\div 4}{}} = \frac{1}{7}$

Example: 0.75 meters to 0.25 meters $\frac{0.75m}{0.25m} \overset{\times 100 \quad \div 25}{\underset{\times 100 \quad \div 25}{}} = \frac{75}{25} = \frac{3}{1} = 3$

Grade and pitch

- Grade (or slope, pitch, incline etc.): the slope of a straight line is the rate of change in height over a distance. It is a measure of the "steepness" "or incline" of a line.
- **The grade or slope formula:**

Formula
$\text{Grade or slope} = \frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{\text{rise}}{\text{run}}$



Example: Determine the grade (%) of a road that has a length of 75 m and a vertical height of 3 m.

$$\textbf{Grade} = \frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{3\text{ m}}{75\text{ m}} = 0.04 = \boxed{4\%}$$

Rate

Rate

- Rate: a ratio of two quantities with different units.

  Example: teachers to students; money to time; distance to time, etc.

  $$\frac{2 \text{ teachers}}{83 \text{ students}}, \qquad \frac{24 \text{ dollars}}{3 \text{ hours}}, \qquad \frac{85 \text{ miles}}{2 \text{ hours}}$$

- Write a rate in lowest terms (simplify the rate):

  Example: 80 kilometres per 320 minutes: $\frac{80 \text{ km}}{320 \text{ min}} \overset{\div 80}{=} \frac{1 \text{km}}{4 \text{ min}}$

Unit rate: a rate in which the number in the ***second term (denominator)*** is 1.

Example: 15 dollars per hours: $\frac{\$15}{1 \text{ h}} = \$ 15 \text{ per h}$

- Some unit rates:
  - Miles (or kilometres) per hour (or minute).
  - Cost (dollars/cents) per item or quantity.
  - Earnings (dollars) per hour (or week).
- Unit price and the best buy.

  Example: Find the best buy.

  12 eggs for $ 3.19; 18 eggs for $4.91; 30 eggs for $7.13.

  $$\frac{\$3.19}{12 \text{ eggs}} \approx \$ 0.266 \text{ per egg}$$

  $$\frac{\$4.91}{18 \text{ eggs}} \approx \$ 0.273 \text{ per egg}$$

  $$\frac{\$7.13}{30 \text{ eggs}} \approx \$ 0.238 \text{ per egg}$$

  So the best buy is 30 eggs for $7.13 (the lowest price). $0.238 < 0.266 < 0.273$

Topic B: Proportion

Solving Proportion

Proportion: an equation with a ratio (or rate) on two sides ($\frac{a}{b} = \frac{c}{d}$), in which the two ratios are equal.

Example: Write the following sentence as a proportion.

3 printers is to 18 computers as 2 printers is to 12 computers. $\frac{3 \text{ printers}}{18 \text{ computers}} = \frac{2 \text{ printers}}{12 \text{ computers}}$

Review ratio, rate and proportion:

	Representation		Example
Ratio	a to b or a:b or $\frac{a}{b}$	with the same unit.	5 to 9 or 5:9 or $\frac{5 \text{ km}}{9 \text{ km}}$
Rate	a to b or a:b or $\frac{a}{b}$	with different units.	3 to 7 or 3:7 or $\frac{3 \text{ cm}}{7 \text{ km}}$
Proportion	$\frac{a}{b} = \frac{c}{d}$	an equation with a ratio/rate on each side.	$\frac{x \text{ m}}{5 \text{ km}} = \frac{3\text{m}}{8 \text{ km}}$, $\frac{x \text{ m}}{7 \text{ m}} = \frac{2\text{m}}{5\text{m}}$
Note: the units for both numerators must match and the units for both denominators must match. **Example**: $\frac{\text{in}}{\text{ft}} = \frac{\text{in}}{\text{ft}}$, $\frac{\text{minutes}}{\text{hours}} = \frac{\text{minutes}}{\text{hours}}$			

Solving a proportion:

- Cross multiply: multiply along two diagonals. $\frac{a}{b} = \frac{c}{d}$
- Solve for the unknown.

Example

$\frac{x}{9} = \frac{2}{6}$

$6 \cdot x = 2 \cdot 9$

$x = \frac{2 \cdot 9}{6} = \frac{18}{6} = 3$

Application

Example: 4 liters of milk cost \$4.38, how much does a 2 liters of milk cost?

- Facts and unknown:

4 L milk	2 L milk
\$4.38	\$ x = ?

- Proportion: $\frac{4\text{L}}{\$4.38} = \frac{2\text{L}}{\$x}$ (The same units. / The same units.)
- Cross multiply: $\frac{4\text{L}}{\$4.38} = \frac{2\text{L}}{\$x}$

(4) (x) = (2) (4.38)

- Solve for x: $\frac{4x}{4} = \frac{2(4.38)}{4}$ Divide both sides by 4.

  $x = \frac{(2)(4.38)}{4} = 2.19$

  2 liters of milk cost $2.19.

- Check: $\frac{4\text{L}}{\$4.38} = \frac{2\text{L}}{\$2.19}$ Replace x with 2.19.

  $(4)\ (2.19) \overset{?}{=} (2)\ (4.38)$

  $\sqrt{}$

  $8.76 = 8.76$ Correct! (LS = RS)

Example: Tom's height is 1.75 meters, and his shadow is 1.09 meters long. A building's shadow is 10 meters long at the same time. How high is the building?

- Facts and unknown:

Tom's height = 1.75 m	Building's height (x) = ?
Tom's shadow = 1.09 m	Building's shadow = 10m

- Proportion: $\frac{1.75\,\text{m}}{1.09\,\text{m}} = \frac{x\,\text{m}}{10\,\text{m}}$ $\frac{\text{Tom's height}}{\text{Tom's shadow}} = \frac{\text{Building's height}}{\text{Building's shadow}}$

- Cross multiply: $\frac{1.75\,\text{m}}{1.09\,\text{m}} = \frac{x\,\text{m}}{10\,\text{m}}$

  $(1.75)\ (10) = (1.09)\ (x)$

- Solve for x: $\frac{(1.75)(10)}{1.09} = \frac{(1.09)x}{1.09}$ Divide both sides by 1.09.

  $x = \frac{(1.75)(10)}{1.09} \approx 16.055$ The building's height is 16.055m.

- Check: $\frac{1.75\,\text{m}}{1.09\,\text{m}} = \frac{16.055\,\text{m}}{10\,\text{m}}$ Replace x with 16.055.

  $(1.75)\ (10) \overset{?}{=} (16.055)\ (1.09)$

  $\sqrt{}$

  $17.5 = 17.5$ Correct! (LS = RS)

Example: If 15 mL of medicine must be mixed with 180 mL of water, how many mL of medicine must be mixed in 230 mL of water?

- Proportion: $\frac{15\text{ mL}}{180\text{ mL}} = \frac{x\text{ mL}}{230\text{ mL}}$ $\frac{15\text{ mL medicine}}{180\text{ mL water}} = \frac{x\text{ mL medicine}}{230\text{ mL water}}$

- Cross multiply: $\frac{15\text{ mL}}{180\text{ mL}} = \frac{x\text{ mL}}{230\text{ mL}}$

- Solve for x: $x = \frac{(15\text{ mL})(230\text{ mL})}{180\text{ mL}} \approx 19.17\text{ mL}$

  19.17 mL of medicine must be mixed in 230 mL of water.

Topic C: Percent

Percent Review

Percent (%): one part per hundred, or per one hundred.

Review - converting between percent, decimal and fraction:

Conversion	Steps	Example
Percent $\Longrightarrow$ Decimal	Move the decimal point two places to the left, then remove %.	$31\% = 31.\% = 0.31$
Decimal $\Longrightarrow$ Percent	Move the decimal point two places to the right, then insert %.	$0.317 = 0.\,317 = 31.7\,\%$
Percent $\Longrightarrow$ Fraction	Remove %, divide by 100, then simplify.	$15\% = \frac{15}{100} = \frac{3}{20}$
Fraction $\Longrightarrow$ percent	Divide, move the decimal point two places to the right, then insert %.	$\frac{1}{4} = 1 \div 4 = 0.25 = 25\,\%$
Decimal $\Longrightarrow$ Fraction	Convert the decimal to a percent, then convert the percent to a fraction.	$0.35 = 35\% = \frac{35}{100} = \frac{7}{20}$ % = per one hundred

Two methods to solve percent problems

- Percent proportion method
- Translation (translate the words into mathematical symbols.)

Percent proportion method:

With the word "***is***"

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100} \quad \text{or} \quad \frac{\text{"is" number}}{\text{"of" number}} = \frac{\%}{100}$$

With the word "***of***"

Step

- Identify the part, whole, and percent.
- Set up the proportion equation.
- Solve for unknown (x).

Example

8 percent ***of*** what number **is** ***4*** ?

Percent Whole (x) Part

$$\frac{4}{x} = \frac{8}{100} \qquad \frac{\text{Part}}{\text{Whole}} = \frac{\%}{100}$$

$$x = \frac{(4)(100)}{8} = 50 \quad , \qquad \boxed{x = 50}$$

Solving Percent Problems

Translation method (translate the words into mathematical symbols):

Translation:

- What ——— x : the word "what" represents an unknown quantity x.
- Is ——— $=$: the word "is" represents an equal sign.
- of ——— $\times$: the word "of" represents a multiplication sign.
- % to decimal: always change the percent to a decimal.

Example:

1) What is 15% of 80?

$x = 0.15 \cdot 80$ $\qquad x = (0.15)(80) = \boxed{12}$

2) What percent of 90 is 45?

$x\,\% \cdot 90 = 45$ $\qquad x\% = \frac{45}{90} = 0.5 = \boxed{50\%}$ Divide both sides by 90.

3) 12 is 8% of what number?

$12 = 0.08 \cdot x$ $\qquad x = \frac{12}{0.08} = \boxed{150}$ Divide both sides by 0.08.

- **Percent increase or decrease:**

Application	Formula
Percent increase	Percent increase $= \frac{\text{New value} - \text{Original value}}{\text{Original value}}$, $x = \frac{N-O}{O}$
Percent decrease	Percent decrease $= \frac{\text{Original value} - \text{New value}}{\text{Original value}}$, $x = \frac{O-N}{O}$

Example: A product increased production from ***1500 last month*** to ***1650 this month***. Find the ***percent increase.***

- New value (N): 1650 This month.
- Original value (O): 1500 Last month.
- Percent increase: $x = \frac{N-O}{O} = \frac{1650-1500}{1500} = 0.1 = 10\%$ A 10% increase.

Example: A product was ***reduced*** from ***$33*** to ***$29.*** What percent ***reduction*** is this?

Percent decrease: $x = \frac{O-N}{O} = \frac{33-29}{33} \approx 0.12 = 12\%$ A 12 % decrease.