

## Topic D: Similar Triangles

### Similar Triangles

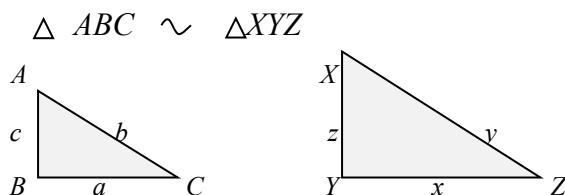
**Similar triangles:** triangles that have the same shape and proportions, but may have different sizes.

The symbol “ $\triangle$ ” is used for triangle; the symbol “ $<$ ” is used for angle.

**Sides and angles in a triangle  $\triangle$  :**

- Sides are labeled with lower case letters.
- Angles ( $<$ ) are labeled with uppercase letters.

**Corresponding (matching) angles and corresponding sides** of two similar triangles:



“ $\sim$ ” means “is similar to”.

- The corresponding angles of two similar triangles are equal.

$$<A = <X \qquad <B = <Y \qquad <C = <Z$$

- The corresponding sides of two similar triangles are proportional in length.
  - Side **a** corresponds to side **x**.
  - Side **b** corresponds to side **y**.
  - Side **c** corresponds to side **z**.

**The formula for similar triangles:**

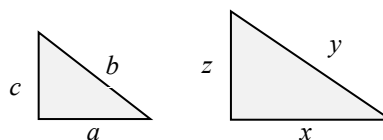
$$\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$$

This includes three proportions:

$$\frac{a}{x} = \frac{b}{y}$$

$$\frac{a}{x} = \frac{c}{z}$$

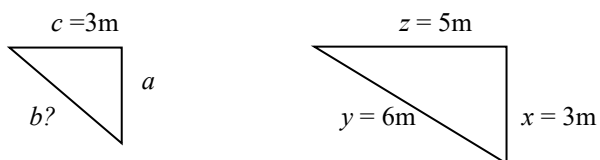
$$\frac{b}{y} = \frac{c}{z}$$



## Solving Similar Triangles

**Example:** Find the value of the missing side in the following figures (the two triangles are similar).

1)



$$\frac{b}{y} = \frac{c}{z} \quad \text{or} \quad \frac{b}{6m} = \frac{3m}{5m}$$

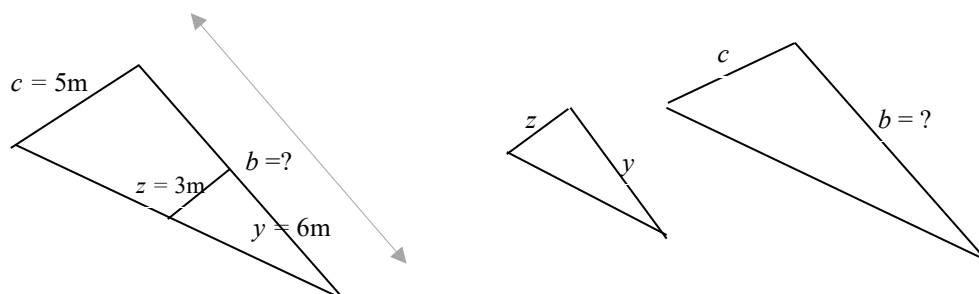
$b$  and  $y$  are corresponding sides.

$c$  and  $z$  are corresponding sides.

Multiply both sides by  $6m$ .

$$b = \frac{(3m)(6m)}{5m} = \boxed{3.6m}$$

2)



$$\frac{b}{y} = \frac{c}{z} \quad \text{or} \quad \frac{b}{6m} = \frac{5m}{3m}$$

$b$  and  $y$  are corresponding sides.

$c$  and  $z$  are corresponding sides.

Multiply both sides by  $6m$ .

$$b = \frac{(5m)(6m)}{3m} = \boxed{10m}$$

3)



$$\frac{a}{4cm} = \frac{6cm}{7cm}$$

$a$  and  $4cm$  are corresponding sides.

$6cm$  and  $7cm$  are corresponding sides.

Multiply both sides by  $4cm$ .

$$a = \frac{(4cm)(6cm)}{7cm} \approx \boxed{3.43cm}$$

## Unit 9: Summary

### Ratio, Proportion, and Percent

**Ratio, rate and proportion:**

	Representation		Example
<b>Ratio</b>	$a$ to $b$ or $a:b$ or $\frac{a}{b}$	with the same unit.	5 to 9 or 5:9 or $\frac{5 \text{ m}}{9 \text{ m}}$
<b>Rate</b>	$a$ to $b$ or $a:b$ or $\frac{a}{b}$	with different units.	3 to 7 or 3:7 or $\frac{3 \text{ cm}}{7 \text{ m}}$
<b>Proportion</b>	$\frac{a}{b} = \frac{c}{d}$	an equation with a ratio/rate on each side.	$\frac{x \text{ m}}{5 \text{ km}} = \frac{3 \text{ m}}{8 \text{ km}}$ , $\frac{x \text{ m}}{7 \text{ m}} = \frac{2 \text{ m}}{5 \text{ m}}$
<b>Note:</b> the units for both numerators must match and the units for both denominators must match.			

**Unit rate:** A rate in which the number in the **second term (denominator)** is 1.

**Solving a proportion:**

- Cross multiply: multiply along two diagonals.

$$\frac{a}{b} = \frac{c}{d}$$

- Solve for the unknown.

**Percent (%):** one part per hundred, or per one hundred.

**Converting between percent, decimal and fraction:**

Conversion	Steps	Example
<b>Percent → Decimal</b>	Move the decimal point two places to the left, then remove %.	$31\% = 31.\% = 0.31$
<b>Decimal → Percent</b>	Move the decimal point two places to the right, then insert %.	$0.317 = 0.317 = 31.7\%$
<b>Percent → Fraction</b>	Remove %, divide by 100, then simplify.	$15\% = \frac{15}{100} = \frac{3}{20}$
<b>Fraction → Percent</b>	Divide, move the decimal point two places to the right, then insert %.	$\frac{1}{4} = 1 \div 4 = 0.25 = 25\%$
<b>Decimal → Fraction</b>	Convert the decimal to a percent, then convert the percent to a fraction.	$0.35 = 35\% = \frac{35}{100} = \frac{7}{20}$

## Grade and pitch

- Grade (or slope, pitch, incline etc.): the slope of a straight line is the rate of change in height over a distance. It is a measure of the “steepness” or incline” of a line.
- The grade or slope formula:**

Formula
Grade or slope = $\frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{\text{rise}}{\text{run}}$



## Two methods to solve percent problems

- Percent proportion method
- Translation (translate the words into math symbols.)

### Percent proportion method:

With the word “is”

Part	=	Percent
Whole		100

or

"is" number	=	%
"of" number		100

With the word “of”

### Translation method (translate the words into mathematical symbols):

- What       $\_\_\_\_\_\_$        $x$  :      the word “what” represents an unknown quantity  $x$ .
- Is       $\_\_\_\_\_\_$        $=$  :      the word “is” represents an equal sign.
- of       $\_\_\_\_\_\_$        $\times$  :      the word “of” represents a multiplication sign.
- %      to      decimal:      always change the percent to a decimal.

### Percent increase or decrease:

Application	Formula
<b>Percent increase</b>	Percent increase = $\frac{\text{New value} - \text{Original value}}{\text{Original value}}$ , $x = \frac{N - O}{O}$
<b>Percent decrease</b>	Percent decrease = $\frac{\text{Original value} - \text{New value}}{\text{Original value}}$ , $x = \frac{O - N}{O}$

The symbol “ $\triangle$ ” is used for triangle; the symbol “ $<$ ” is used for angle.

**Similar ( $\sim$ ) triangles:** triangles that have the same shape and proportions, but may be of different sizes.

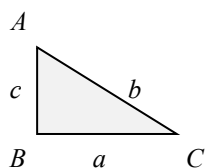
### Sides and angles in a triangle:

- Sides are labeled with lower case letters.
- Angles ( $\angle$ ) are labeled with uppercase letters.

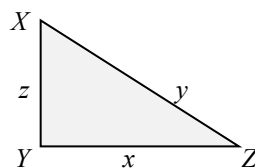
### Corresponding angles and corresponding (matching) sides:

$$\triangle ABC \sim \triangle XYZ$$

$\sim$  means “is similar to”,



1<sup>st</sup> triangle



2<sup>nd</sup> triangle

- The corresponding angles of two similar triangles are equal.

$$\angle A = \angle X \qquad \angle B = \angle Y \qquad \angle C = \angle Z$$

- The corresponding sides of two similar triangles are proportional in length.

- Side **a** corresponds to side **x**.
- Side **b** corresponds to side **y**.
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### Solve similar triangles:

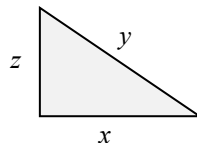
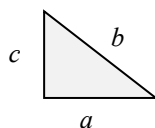
$$\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$$

This includes three proportions:

$$\frac{a}{x} = \frac{b}{y}$$

$$\frac{a}{x} = \frac{c}{z}$$

$$\frac{b}{y} = \frac{c}{z}$$



## Unit 9: Self-Test

### Ratio, Proportion, and Percent

## Topic A

1. Write the following as a ratio or rate in lowest terms.
  - a) 15 nickels to 45 nickels.
  - b) 24 kilometers to 88 kilometers.
  - c) 350 people for 1500 tickets. L  
SEP
  - d) 0.33 centimetres to 0.93 centimetres.
  - e) 160 kilometres per 740 minutes.
2. Determine the grade (%) of a road that has a length of 2,500 m and a vertical height of 3.5m.
3. What is the grade (%) of a river that drops 9 meters over a distance of 720 meters?
4. A train travelled 459 km in 6 hours. What is the unit rate? L  
SEP
5. A 4 L bottle of milk sells for \$4.47. A 2 L bottle of the same milk sells for \$3.43. What is the best buy?
6. An 8-pound bag of apples costs \$7.49. A 6-pound bag of the same apples costs \$5.99. What is the best buy?

## Topic B

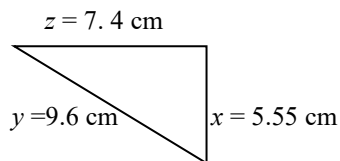
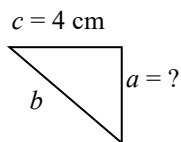
7. Write the following sentence as a proportion.
  - a) 5 teachers is to 110 students as 15 teachers is to 330 students. L  
SEP
  - b) 24 hours is to 1,940 kilometers as 12 hours is to 985 kilometers.
8. 4 liters of juice cost \$7.38, how much do 2 liters cost?
9. Todd's height is 5.44 feet, and his shadow is 8.5 feet long. A building's shadow is 25 feet long at the same time. How high is the building?
10. Sarah earns \$4,500 in 30 days. How much does she earn

in 120 days?  $\frac{1}{5}$

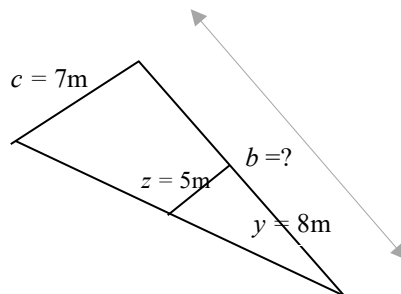
### Topic C

11. What is 45% of 260?
12. 36 is 12% of what number?
13. A product increased production from 2,800 last year to 3,920 this year. Find the percent increase.
14. A product was reduced from \$199 to \$159. What percent reduction is this?
15. Find the value of the missing side in the following figures (the two triangles are similar).

a)



b)



c)

