

Unit R

Review of Basic Mathematics

Topic A: Basic math skills

- Numbers and place value
- Prime / composite numbers
- Prime factorization
- Basic mathematical symbols and terms

Topic B: Percent, decimal and fraction

- Fractions
- More about fractions
- Decimals
- Operations with decimals
- Percent and conversion

Topic C: Operations with fractions

- Least common denominator (LCD)
- Operations with fractions
- Ratio and proportion

Unit R Summary

Unit R Self-test

Unit R is a review of basic math fundamentals. There is a self-test at the end of the unit that can test students' understanding of the material. Students can take the self-test before beginning the unit to determine how much they know about the topic. Those who do well may decide to move on to the next unit without reading the lesson.

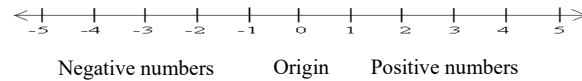
Topic A: Basic Math Skills

Numbers and Place Value

Numbers:

| Classify numbers | Definition | Numbers |
|------------------|--|---------------------------------|
| The ten digits | a symbol for numeral below 10 | 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 |
| Whole numbers | the numbers used for counting | 0, 1, 2, 3, 4, 5, 6, 7 ... |
| Integers | all the whole numbers and their negatives | ... -3, -2, -1, 0, 1, 2, 3 ... |
| Odd numbers | any integer that cannot be evenly divided by 2 | 1, 3, 5, 7, 9 ... |
| Even numbers | any integer that can be evenly divided by 2 | 0, 2, 4, 6, 8, 10 ... |

Number line is a straight line on which every point corresponds to an integer.



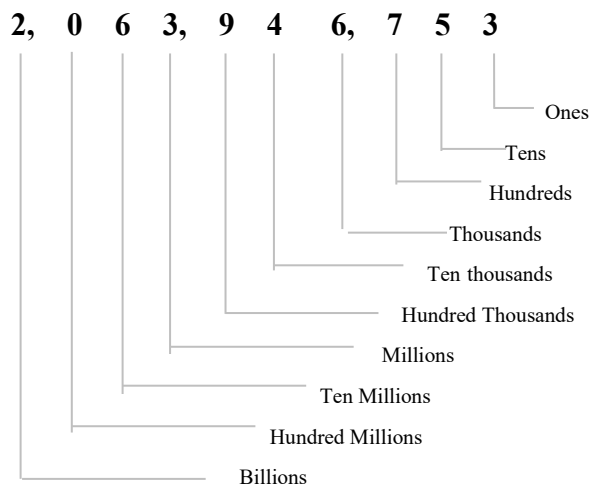
Place value: the value of the position of a digit in a number.

- Each digit in a number has a place value.
- The location in a number determines the value of a digit.

| Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
|-----------|------|------|----------|------|------|----------|------|------|-----------|------|------|----------|------|------|
| Trillions | | | Billions | | | Millions | | | Thousands | | | Ones | | |

(Read from right to left)

Example: 2,063,946,753



Prime / Composite Numbers

Factor: a number you multiply with others to get another number.

Example: $3 \times 4 = 12$ 3 and 4 are factors.

- Some numbers can be factored in many ways:

Example: $2 \times 4 = 8$ or $4 \times 2 = 8$ or $1 \times 8 = 8$ or $8 \times 1 = 8$

- Factors for some numbers:

| Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---------|---|------|------|---------|------|------------|------|------------|---------|-------------|
| Factors | 1 | 1, 2 | 1, 3 | 1, 2, 4 | 1, 5 | 1, 2, 3, 6 | 1, 7 | 1, 2, 4, 8 | 1, 3, 9 | 1, 2, 5, 10 |

Prime number: a whole number that only has two factors, 1 and itself.

Example: 2, 3, 5, and 7 are prime numbers.

(7 has two factors: 1 and 7, $1 \times 7 = 7$)

Composite number: a whole number that has more than two factors, and can be evenly divided.

Example: 4, 6, 8, 9 and 10 are composite numbers.

(6 has four factors: 1, 2, 3 and 6. $1 \times 6 = 6$, $2 \times 3 = 6$)

Rules for testing a prime / composite number:

- A prime number is always an odd number, except for 2 (but an odd number is not necessarily a prime number).

Example: The prime numbers 1, 3, 5, and 7 are odd numbers.

The odd number 9 is a composite number.

- An even number (ends in a 0, 2, 4, 6, and 8) is always a composite number (except number 2).

Example: 14, 28, 376, and 5372 are composite numbers.

- All numbers that end with five and are greater than five are composite numbers.

Example: 15, 65, and 345 are composite numbers.

Tip: The Prime Tester in the following website can determine if a number is a prime or a composite number.

<http://www.murderousmaths.co.uk/games/primcal.htm>

Prime Factorization

Prime factorization is finding which prime numbers can be used to multiply to get the original number.

Prime factorization: the product of all the prime numbers for a given number.

Example: $30 = 2 \times 3 \times 5$

“Product” – the keyword for multiplication
2, 3, and 5 are prime numbers (or prime factors).

Find the prime factorization:

- Method 1: do **repeated division** (or upside down division) by prime numbers, and multiply all the prime numbers around the outside to get the prime factorization.

Example: Find the prime factorization of 24.

$$\begin{array}{rcl}
 2 & \overline{) 24} & \\
 2 & \overline{) 12} & \text{———— } 24 \div 2 = 12 \\
 2 & \overline{) 6} & \text{———— } 12 \div 2 = 6 \\
 & 3 & \text{———— } 6 \div 2 = 3
 \end{array}$$

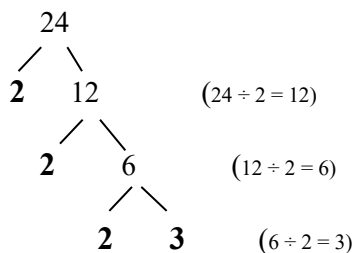
Note: Stop dividing until you reach a prime number.

The outside numbers are 2, 2, 2, 3.

The prime factorization for 24 is: $24 = 2 \times 2 \times 2 \times 3 = \boxed{2^3 \times 3}$

- Method 2: **factor tree** method - split the number into two factors, then split non-prime factors until all the factors are prime, and multiply all the prime numbers.

Example: Find the prime factorization of 24.



The prime numbers are 2, 2, 2, 3.

The prime factorization for 24 is: $24 = 2 \times 2 \times 2 \times 3 = \boxed{2^3 \times 3}$

Basic Mathematical Symbols and Terms

Basic mathematical symbols summary:

| Symbol | Meaning | Example |
|---|-----------------------------|--|
| = | equal | $3 = 3$ |
| \neq | not equal | $2 \neq 3$ |
| \approx | approximately | $4 \approx 3.89$ |
| $>$ | is greater than | $4 > 2$ |
| $<$ | is less than | $1 < 3$ |
| \geq | is greater than or equal to | $x \geq 4$ |
| \leq | is less than or equal | $x \leq 8$ |
| \pm | plus or minus | 3 ± 2 means: $3 + 2$ or $3 - 2$ |
| + | addition | $3 + 2$ |
| - | subtraction | $7 - 3$ |
| \times or \cdot or $()$ | multiplication | $6 \times 3 = 18$ or $6 \cdot 3 = 18$ or $(6)(3) = 18$ |
| \div or $/$ or — or $\overline{)}$ | division | $4 \div 2 = 2$, $4 / 2$, $\frac{4}{2}$, $2 \overline{)4}$ |

Arithmetic terms:

| Operation | Term |
|-----------------------|---|
| Addition | Addend + addend = sum $2 + 1 = 3$ |
| Subtraction | Subtrahend – minuend = difference $5 - 2 = 3$ |
| Multiplication | Multiplicand \times multiplier = product (factor) (factor) $2 \times 4 = 8$ |
| Division | Dividend \div divisor = quotient & remainder (factor) $7 \div 2 = 3 \text{ R } 1$ |

Properties of zero

| Property | Example |
|---|---|
| Any number multiplied by 0 will always equal to 0. | $3 \times 0 = 0$ |
| The number 0 divided by any nonzero number is zero. | $\frac{0}{6} = 0$ (0 apples divided by 6 kids, each kid gets 0 apples.) |
| A number divided by 0 is not defined (not allowed). | $\frac{6}{0}$ is undefined. (6 apples shared by zero kids has no meaning.) |

Writing whole numbers in words:

- Do not use ‘and’ when writing or reading whole numbers.
- Do not use ‘s’ at the end of trillion, million, thousand, hundred, etc.

Example: Write the following number in words: 12, 023, 476

Twelve million, twenty-three thousand, four hundred seventy-six.

Topic B: Percent, Decimal and Fraction

Fractions

Fraction: a fraction is a part of a whole. It is expressed in the form of $\frac{a}{b}$. (Example: $\frac{2}{5}$)

Numerator $\frac{a}{b}$ Denominator

- **Numerator:** the number that represents how many *parts* are being dealt with.
- **Denominator:** the number of parts the *whole* is being divided into.

Three types of fractions

- **Proper fraction:** has a numerator smaller than ($<$) the denominator.

Example: $\frac{1}{2}$, $\frac{3}{8}$, $\frac{16}{237}$

- **Improper fraction:** has a numerator larger than or equal to (\geq) the denominator.

Example: $\frac{7}{6}$, $\frac{56}{31}$, $\frac{9}{9}$

- **Mixed fraction (or mixed number):** contains an *integer* and a *proper fraction*.

Example: $2\frac{1}{4}$, $3\frac{2}{5}$, $5\frac{4}{7}$

Conversion between a mixed number and an improper fraction

- **To convert a mixed number to an improper fraction:**

$$\text{Improper fraction} = \frac{\text{whole number} \times \text{denominator} + \text{numerator}}{\text{Denominator}}$$

Example: $2\frac{1}{4} = \frac{2 \times 4 + 1}{4} = \frac{9}{4}$

- **To convert an improper fraction to a mixed number:**

$$\text{Mixed number} = \text{Numerator} \div \text{Denominator}, \quad \text{Quotient} \frac{\text{Reminder}}{\text{Denominator}}$$

Example: $\frac{9}{2} = 9 \div 2 = 4 \text{ R } 1 = 4\frac{1}{2}$

Quotient

$$\begin{array}{r} 4 \\ 2 \overline{) 9} \\ - 8 \\ \hline 1 \end{array}$$

Remainder

More about Fractions

Equivalent fractions: different fractions that have the same value.

To find the equivalent fraction: divide or multiply the numerator and denominator by the same number.

- Divide by the same number (for a larger fraction):

To simplify (or reduce) fractions: divide the numerator and denominator by the same number until their only common factor is 1.

$$\frac{\text{Numerator} \div n}{\text{Denominator} \div n}$$

“ n ” is any whole number that does not equal to 0.

Example: Simplify $\frac{18}{36}$.

$$\frac{18}{36} = \frac{9}{18} = \frac{3}{6} = \frac{1}{2}$$

The simplest fraction of $\frac{18}{36}$ is $\frac{1}{2}$.

- Multiply by the same number (for a smaller fraction):

$$\frac{\text{Numerator} \times n}{\text{Denominator} \times n}$$

Example: $\frac{1}{3} = \frac{3}{9} = \frac{6}{18}$

Like and unlike fractions:

- Like fractions:** fractions that have the *same* denominators. **Examples:** $\frac{2}{7}$, $\frac{5}{7}$, $\frac{4}{7}$
- Unlike fractions:** fractions that have *different* denominators. **Examples:** $\frac{2}{3}$, $\frac{3}{5}$, $\frac{7}{10}$

Classifying fractions:

| Classifying fractions | | Examples |
|---|--|--|
| Proper fraction | numerator < denominator | $\frac{1}{2}$, $\frac{3}{8}$, $\frac{16}{237}$ |
| Improper fraction | numerator \geq denominator | $\frac{7}{6}$, $\frac{56}{31}$, $\frac{9}{9}$ |
| Mixed fraction (or mixed number) | A number made up of an integer and a fraction. | $2\frac{1}{4}$, $3\frac{2}{5}$, $5\frac{4}{7}$ |
| Like fractions | Fractions that have the same denominators. | $\frac{2}{7}$, $\frac{5}{7}$, $\frac{4}{7}$ |
| Unlike fractions | Fractions that have different denominators. | $\frac{2}{3}$, $\frac{3}{5}$, $\frac{7}{10}$ |

Decimals

Decimal number: a number contains a decimal point.

- The number to the left of the decimal is the integer part.
- The number to the right of the decimal is the fractional part.

Example:

Example: 34.8

Integer part + decimal point + fractional part (8 tenth)

Decimal place: a place of a digit to the right of a decimal point.

- Each digit in a decimal number has a decimal place.
- The location in a number determines the value of a digit.

| | | | | | | | | (Smaller) |
|----------|------|------|---|--------|------------|-------------|-----------------|---------------------|
| Hundreds | Tens | Ones | . | Tenths | Hundredths | Thousandths | Ten thousandths | Hundred thousandths |

Example:

5.40378

5 . 4 0 3 7 8

Hundred thousandths

Ten thousandths

Thousands

Hundredths

Tenths

Ones

Write decimals in words:

Integer part + *and* + fractional part

Example: 1) 35.348

Decimal point

Thirty-five and three hundred forty-eight thousandths

2) 6.038

Six and thirty-eight hundredths

Operations with Decimals

Operations with decimals:

| | | |
|------------------------|--|--|
| + or - decimals | <ul style="list-style-type: none"> - Line up the decimal points. - + or - as whole numbers. - Insert a decimal point in the answer (in the same line as above). | $ \begin{array}{r} 0.3725 \\ 3.404 \\ + 2.13 \\ \hline 5.9065 \end{array} $ |
| × decimals | <ul style="list-style-type: none"> - × as whole numbers. - Count the numbers of the decimal places in both factors. - Insert a decimal point in the product so that it matches the number of decimal places of factors (start at the far right). | $ \begin{array}{r} 2.14 \quad (\text{Two decimal places}) \\ \times 2.2 \quad (\text{One decimal place}) \\ \hline 428 \\ + 428 \\ \hline 4.708 \quad (\text{Three decimal places}) \end{array} $ |
| ÷ decimals | <ul style="list-style-type: none"> - Move the decimal point of the divisor to the right end. - Move the decimal point of the dividend the same number of places to the right (insert zeros if necessary). - ÷ as whole numbers. - Insert a decimal point in the quotient (directly above the decimal point in the dividend). | $ \begin{array}{r} 4.86 \div 1.2 = ? \\ 12 \overline{) 48.60} \\ \underline{48} \\ 60 \\ \underline{60} \\ 0 \end{array} $ <div style="text-align: right; margin-top: 10px;"> Quotient Divisor) Dividend </div> |

Convert decimals to mixed numbers or fractions:

- Whole number does not change.
- Write the original term as a fraction.
 - Numerator = the fractional part (The digits on the right of the decimal point).
 - Denominator = a multiple of 10 (The number of zeros = The number of decimal places)
- Simplify (reduce) if possible.

Example: 1) $5.25 = 5 \frac{25}{100} = 5 \frac{1}{4}$

The fractional part = 25

The number of decimal places = 2

2) $0.045 = \frac{45}{1000} = \frac{9}{200}$

The number of decimal places = 3

3) $384.3645 = 384 \frac{3645}{10000}$

The number of decimal places = 4

Percent and Conversion

Percent (%): one part per hundred, or per one hundred.

Example: $5\% = \frac{5}{100}$

The standard form of percent proportion:

(With the word "is")

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}$$

or

$$\frac{\text{"is" number}}{\text{"of" number}} = \frac{\%}{100}$$

(With the word "of")

Use percent proportion method to solve % problems:

- Identify the part, whole, and percent.
- Set up the proportion equation.
- Solve for the unknown.

Example

8 percent *of* what number *is* 4 ?

$$\begin{array}{l} \text{Percent} \quad \text{Whole (x)} \quad \text{Part} \\ \frac{8}{x} = \frac{8}{100} \quad \frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100} \\ x = \frac{(8)(100)}{8} = \boxed{50} \end{array}$$

Converting between percent, decimal and fraction:

| Conversion | Step | Example |
|----------------------------|---|--|
| Percent to Decimal | Move the decimal point two places to the left, then remove %. | $31\% = 31. = 0.31$ |
| Decimal to Percent | Move the decimal point two places to the right, then insert %. | $0.317 = 0.317 = 31.7\%$ |
| Percent to Fraction | Remove %, divide by 100, then simplify. | $15\% = \frac{15}{100} = \frac{3}{20}$ (% = per one hundred.) |
| Fraction to Percent | Divide, move the decimal point two places to the right, then insert %. | $\frac{1}{4} = 1 \div 4 = 0.25 = 25\%$ |
| Decimal to Fraction | Convert the decimal to a percent, then convert the percent to a fraction. | $0.35 = 35\% = \frac{35}{100} = \frac{7}{20}$ |

Converting repeating decimals to fractions:

- Let x equals the repeating decimal:
- Multiply both sides by 100:
- Subtract the first equation from the second:

- Solve for x :

Example: $0.\overline{6} \rightarrow$ Fraction

$$\begin{array}{ll} x = 0.66 & (1) \\ 100x = 66 & (2) \\ 100x = 66 & (2) - (1) \\ \underline{- x = 0.66} & \\ 99x = 65.34 & \\ x = \frac{65.34}{99} = \frac{1.98}{3} \approx \frac{2}{3} & \end{array}$$

Topic C: Operations with Fractions

Least Common Denominator (LCD)

Least common multiple (LCM): the lowest number that is divisible by each given number without a remainder.

Example: The LCM of 2 and 3 is 6.

- Multiples of 2: 0, 2, 4, **6**, 8, 10, **12** ...
- Multiples of 3: 0, 3, **6**, 9, **12**, 15 ...
- Common multiples of 2 and 3 are 6 and 12 ...
- The *least* common multiple (LCM) of 2 and 3 is **6**.

The common multiple 12 is not the smallest (least).

Find the LCM: Use repeated division (or upside-down division). The product of all the prime numbers around the outside is the LCM.

Example: Find the LCM of 30 and 45.

$$\begin{array}{r|rr}
 5 & 30 & 45 \\
 3 & 6 & 9 \\
 & 2 & 3
 \end{array}
 \quad
 \begin{array}{l}
 \text{———— } 30 \div 5 = 6 \quad 45 \div 5 = 9 \\
 \text{———— } 6 \div 3 = 2 \quad 9 \div 3 = 3
 \end{array}$$

(Stop dividing since 2 and 3 are prime numbers.)

$$\text{LCM} = 5 \times 3 \times 2 \times 3 = \boxed{90} \quad \text{Multiply all the prime numbers around the outside.}$$

Least common denominator (LCD): the least common multiple (LCM) of the *denominators* of two or more given fractions.

Find the LCD: Use repeated division to find the LCM for all *denominators* of given fractions.

Example: Find the LCD for $\frac{4}{8}$, $\frac{5}{16}$ and $\frac{2}{42}$

$$\begin{array}{r|rrr}
 2 & 8 & 16 & 42 \\
 2 & 4 & 8 & 21 \\
 2 & 2 & 4 & 21 \\
 & 1 & 2 & 21
 \end{array}
 \quad
 \begin{array}{l}
 \text{———— } 8 \div 2 = 4, \quad 16 \div 2 = 8, \quad 42 \div 2 = 21 \\
 \text{———— } 4 \div 2 = 2, \quad 8 \div 2 = 4, \quad \text{bring down } 21. \\
 \text{———— } 2 \div 2 = 1, \quad 4 \div 2 = 2, \quad \text{bring down } 21.
 \end{array}$$

$$\text{LCD} = 2 \times 2 \times 2 \times 1 \times 2 \times 21 = \boxed{336}$$

Operations with Fractions

| Operation | Steps | Example |
|---|--|--|
| Adding and subtracting <i>like</i> fractions | <ul style="list-style-type: none"> - Add / subtract the numerators. - Denominators do not change. - Simplify if necessary. | $\frac{3}{13} + \frac{5}{13} = \frac{3+5}{13} = \frac{8}{13}$ $\frac{7}{12} - \frac{3}{12} = \frac{7-3}{12} = \frac{4}{12} = \frac{1}{3}$ |
| Adding and subtracting <i>unlike</i> fractions | <ul style="list-style-type: none"> - Determine the LCD. - Rewrite fractions with the LCD, and add or subtract the numerators. - Simplify if necessary. | $\frac{5}{12} + \frac{3}{8} = \frac{5 \times 2}{12 \times 2} + \frac{3 \times 3}{8 \times 3} = \frac{10}{24} + \frac{9}{24} = \frac{10+9}{24} = \frac{19}{24}$ <p style="text-align: center;">(LCD = 24)</p> $\frac{4}{9} - \frac{2}{6} = \frac{4 \times 2}{9 \times 2} - \frac{2 \times 3}{6 \times 3} = \frac{8}{18} - \frac{6}{18} = \frac{8-6}{18} = \frac{2}{18} = \frac{1}{9}$ <p style="text-align: center;">(LCD = 18)</p> |
| Adding and subtracting mixed numbers with <i>like</i> denominators | <ul style="list-style-type: none"> - Add / subtract integers. - Add / subtract as fractions. - Simplify if necessary. | $2\frac{3}{5} + 5\frac{1}{5} = (2+5)\frac{3+1}{5} = 7\frac{4}{5}$ $5\frac{9}{14} - 3\frac{5}{14} = (5-3)\frac{9-5}{14} = 2\frac{4}{14} = 2\frac{2}{7}$ |
| Adding and subtracting mixed numbers with <i>unlike</i> denominators | <ul style="list-style-type: none"> - Rewrite fractions with the LCD. - Add / subtract as fractions. - If the sum/difference created an improper fraction → a mixed number. | $3\frac{5}{12} - 2\frac{3}{8} = 3\frac{10}{24} - 2\frac{9}{24}$ $= (3-2)\frac{10-9}{24} = 1\frac{1}{24}$ |
| Multiplying fractions | <ul style="list-style-type: none"> - Cross simplify if the fraction is not in lowest terms. - Multiply the numerators. - Multiply the denominators. - Simplify the result if necessary. | $\frac{2}{9} \times \frac{3}{5} = \frac{2 \times 1}{3 \times 5} = \frac{2}{15}$ |
| Multiplying mixed numbers | <ul style="list-style-type: none"> - Convert mixed numbers to improper fractions. - Cross simplify if the fractions is not in lowest terms. - Multiply the numerators. - Multiply the denominators. - Simplify the result if necessary. | $1\frac{1}{5} \times 2\frac{1}{2} = \frac{6}{5} \times \frac{5}{2} = \frac{3 \times 1}{1 \times 1} = \frac{3}{1} = 3$ |
| Dividing fractions | <ul style="list-style-type: none"> - Change the divisor to its reciprocal (switch the numerator and denominator). - Multiply the resulting fractions. | $\frac{2}{7} \div \frac{3}{5} = \frac{2}{7} \times \frac{5}{3} = \frac{2 \times 5}{7 \times 3} = \frac{10}{21}$ |
| Dividing mixed numbers | <ul style="list-style-type: none"> - Convert mixed numbers to improper fractions. - Divide fractions. | $8 \div 3\frac{1}{5} = \frac{8}{1} \div \frac{16}{5} = \frac{8}{1} \times \frac{5}{16} = \frac{1 \times 5}{1 \times 2} = \frac{5}{2} = 2\frac{1}{2}$ |

Ratio and Proportion

Ratio, rate and proportion:

| | Notation | Unit | Example |
|---|--------------------------------------|--|---|
| Ratio | a to b or $a:b$ or $\frac{a}{b}$ | with the same unit. | 5 to 9 or 5:9 or $\frac{5 \text{ m}}{9 \text{ m}}$ |
| Rate | a to b or $a:b$ or $\frac{a}{b}$ | with different units. | 3 to 7 or 3:7 or $\frac{3 \text{ cm}}{7 \text{ m}}$ |
| Proportion | $\frac{a}{b} = \frac{c}{d}$ | an equation with a ratio on each side. | $\frac{3 \text{ cm}}{7 \text{ m}} = \frac{1 \text{ cm}}{5 \text{ m}}$ |
| <p>Note: the units for both numerators must match and the units for both denominators must match.</p> <p>Example: $\frac{\text{in}}{\text{ft}} = \frac{\text{in}}{\text{ft}}$, $\frac{\text{minutes}}{\text{hours}} = \frac{\text{minutes}}{\text{hours}}$</p> | | | |

Solving a proportion:

- Cross multiply: multiply along two diagonals.
- Solve for the unknown.

$$\frac{a}{b} = \frac{c}{d}$$

Example

$$\frac{x}{9} = \frac{2}{6}$$

$$6 \cdot x = 2 \cdot 9$$

$$x = \frac{2 \cdot 9}{6} = \frac{18}{6} = 3$$

(x is the unknown.)

Example: John's height is 1.75 meters, and his shadow is 1.09 meters long. A building's shadow is 10 meters long at the same time. How tall is the building?

- Facts and **unknown**:

| | |
|------------------------|---------------------------------------|
| John's height = 1.75 m | Let x = Building's height (unknown) |
| John's shadow = 1.09 m | Building's shadow = 10 m |

Equation: $\frac{1.75 \text{ m}}{1.09 \text{ m}} = \frac{x \text{ m}}{10 \text{ m}}$ $\frac{a}{b} = \frac{c}{d}$

Cross multiply: $\frac{1.75 \text{ m}}{1.09 \text{ m}} = \frac{x \text{ m}}{10 \text{ m}}$ $(1.75)(10) = (1.09)(x)$

Solve for x : $x = \frac{(1.75)(10)}{1.09} = \boxed{16.055 \text{ m}}$ Divide 1.09 both sides.

The building's height is 16.055m.

Unit R: Summary

Review of Basic Mathematics

Numbers:

| Classify Numbers | Numbers |
|------------------|---|
| Whole numbers | 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 ... |
| Odd numbers | 1, 3, 5, 7, ... |
| Even numbers | 0, 2, 4, 6, 8, ... |
| Digits | 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. |
| Expanded form | $345 = 300 + 40 + 5$ |
| Prime number | A whole number that only has two factors, 1 and itself. |
| Composite number | A whole number that has more than two factors. |

Place value: the value of the position of a digit in a number.

| Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones | Hundreds | Tens | Ones |
|-----------|------|------|----------|------|------|----------|------|------|-----------|------|------|----------|------|------|
| Trillions | | | Billions | | | Millions | | | Thousands | | | Ones | | |



Read from right to left

Factor: a number you multiply with others to get another number.


Prime factorization: the product of all the prime factors for a given number.

Find the prime factorization: do repeated division (or upside-down division) by prime numbers, and multiply all the prime numbers around the outside to get the prime factorization.

Properties of zero:

- Any number multiplied by 0 will always equal to 0.
- The number 0 divided by any nonzero number is zero.
- A number divided by 0 is not defined (not allowed).

Basic mathematical symbol summary:

| Symbol | Meaning |
|--|-----------------------------|
| = | equal |
| ≠ | not equal |
| ≈ | approximately |
| > | is greater than |
| < | is less than |
| ≥ | is greater than or equal to |
| ≤ | is less than or equal to |
| ± | plus or minus |
| + | addition |
| − | subtraction |
| × or ▪ or () | multiplication |
| ÷ or / or — or  | division |

Writing whole numbers in words:

- Do not use ‘and’ when writing or reading whole numbers.
- Do not use ‘s’ at the end of trillion, million, thousand, hundred, etc.

Fraction: a fraction is a part of a whole. It is expressed in the form of $\frac{a}{b}$.

$$\text{Fraction: } \frac{a}{b} = \frac{\text{Numerator}}{\text{Denominator}}$$

Decimal number: a number contains a decimal point.

Integer part + decimal point + fractional part

Decimal place: a place of a digit to the right of a decimal point.

| | | | | | | | | |
|----------|------|------|---|--------|------------|-------------|-----------------|---------------------|
| Hundreds | Tens | Ones | . | Tenths | Hundredths | Thousandths | Ten thousandths | Hundred thousandths |
|----------|------|------|---|--------|------------|-------------|-----------------|---------------------|

Write decimals in words: Integer part + *and* + fractional part
Decimal point

Convert decimals to mixed numbers or fractions:

- Whole number does not change.
- Write the original term as a fraction.
 - Numerator = the fractional part
 - Denominator = a multiple of 10 (The number of zeros = The number of decimal places)
- Simplify if possible.

Classifying fractions:

$$\text{Fraction: } \frac{a}{b} = \frac{\text{Numerator}}{\text{Denominator}}$$

| Classifying fractions | |
|---|--|
| Proper fraction | numerator < denominator |
| Improper fraction | numerator ≥ denominator |
| Mixed fraction (or mixed number) | A number made up of an integer and a fraction. |
| Like fractions | Fractions that have the same denominators. |
| Unlike fractions | Fractions that have different denominators. |

Arithmetic terms:

| Operation | Term |
|-----------------------|--|
| Addition | Addend + addend = sum |
| Subtraction | Subtrahend – minuend = difference |
| Multiplication | Multiplicand × multiplier = product (factor) (factor) |
| Division | Dividend ÷ divisor = quotient & remainder (factor) |

To convert a mixed number to an improper fraction:

$$\text{Improper fraction} = \frac{\text{whole number} \times \text{denominator} + \text{numerator}}{\text{Denominator}}$$

To convert an improper fraction to a mixed number:

$$\text{Mixed number} = \text{Numarator} \div \text{Denominator} \Rightarrow \text{Quotient} \frac{\text{Remainder}}{\text{Denominator}}$$

The standard form of percent proportion:

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100} \quad \text{or} \quad \frac{\text{"is" number}}{\text{"of" number}} = \frac{\%}{100}$$

Converting between percent, decimal and fraction:

| Conversion | Steps |
|----------------------------|---|
| Percent to Decimal | Move the decimal point two places to the left, then remove %. |
| Decimal to Percent | Move the decimal point two places to the right, then insert % . |
| Percent to Fraction | Remove % , divide by 100, then simplify. |
| Fraction to Percent | Divide, move the decimal point two places to the right, then insert % . |
| Decimal to Fraction | Convert the decimal to a percent, then convert the percent to a fraction. |

Least common multiple (LCM): the lowest number that is divisible by each given number without a remainder.

Least common denominator (LCD): the least common multiple (LCM) of the denominators of two or more given fractions.

Find the LCD: Use repeated division to find the LCM for all denominators of given fractions.

Ratio, rate and proportion:

| | Notation | Unit |
|-------------------|--------------------------------------|--|
| Ratio | a to b or $a:b$ or $\frac{a}{b}$ | With the same unit. |
| Rate | a to b or $a:b$ or $\frac{a}{b}$ | With different units. |
| Proportion | $\frac{a}{b} = \frac{c}{d}$ | The units for both numerators must match and the units for both denominators must match. |

Solving a proportion:

- Cross multiply: multiply along two diagonals.
- Solve for the unknown.

To find the equivalent fraction: divide or multiply the numerator and denominator by the same number.

To simplify (or reduce) fractions: divide the numerator and denominator by the same number until their only common factor is 1.

| |
|---|
| $\frac{\text{Numerator} \div n}{\text{Denominator} \div n}$ |
|---|

“ n ” is any whole number that does not equal to 0.

Operations with fractions:

| Operation | Steps |
|---|--|
| Adding and subtracting <i>like</i> fractions | <ul style="list-style-type: none"> - Add / subtract the numerators. - Denominators do not change. - Simplify if necessary. |
| Adding and subtracting <i>unlike</i> fractions | <ul style="list-style-type: none"> - Determine the LCD. - Rewrite fractions with the LCD, and add or subtract the numerators. - Simplify if necessary. |
| Adding and subtracting mixed numbers with <i>like</i> denominators | <ul style="list-style-type: none"> - Add / subtract whole numbers. - Add / subtract as fractions. - Simplify if necessary. |
| Adding and subtracting mixed numbers with <i>unlike</i> denominators | <ul style="list-style-type: none"> - Rewrite fractions with the LCD. - Add / subtract as fractions. - If the sum/difference created an improper fraction → a mixed number. |
| Multiplying fractions | <ul style="list-style-type: none"> - Cross - simplify if the fraction is not in lowest terms. - Multiply the numerators. - Multiply the denominators. - Simplify the result if necessary. |
| Multiplying mixed numbers | <ul style="list-style-type: none"> - Convert mixed numbers to improper fractions. - Cross - simplify if the fractions is not in lowest terms. - Multiply the numerators. - Multiply the denominators. - Simplify the result if necessary. |
| Dividing fractions | <ul style="list-style-type: none"> - Change the divisor to its reciprocal (switch the numerator and denominator). - Multiply the resulting fractions. |
| Dividing mixed numbers | <ul style="list-style-type: none"> - Convert mixed numbers to improper fractions. - Divide fractions. |

Unit R: Self-Test

Review of Basic Mathematics

Topic A

1. Find the prime factorization of 36.
2.
 - a) Write the number in words: 10, 024, 526
 - b) Write the decimal in words: 47.268
3. Calculate the following without using a calculator:
 - a) $0.463 + 2.456 + 3.52$
 - b) 3.21×2.5
 - c) $6.48 \div 2.4$

Topic B

4.
 - a) Convert a mixed number to an improper fraction: $4\frac{2}{7}$
 - b) Convert an improper fraction to a mixed number: $\frac{9}{5}$
5. Reduce to lowest terms: $\frac{12}{48}$
6. 12 percent of what number is 48 ?
7. Convert between percent, decimal and fraction:
 - a) 45% to decimal
 - b) 0.436 to %
 - c) 25% to fraction

- d) $\frac{5}{25}$ to %
- e) 0.4 to fraction
- f) $0.\bar{3}$ to Fraction

Topic C

8.
 - a) Find the LCM of 24 and 64.
 - b) Find the LCD for $\frac{2}{5}$, $\frac{3}{15}$ and $\frac{24}{35}$
9. Calculate:
 - a) $\frac{1}{6} + \frac{4}{6}$
 - b) $\frac{11}{14} - \frac{4}{14}$
 - c) $\frac{3}{8} + \frac{5}{4}$
 - d) $\frac{6}{7} - \frac{4}{21}$
 - e) $2\frac{3}{7} + 4\frac{2}{7}$
 - f) $7\frac{8}{12} - 5\frac{7}{12}$
 - g) $4\frac{9}{12} - 3\frac{2}{4}$
 - h) $\frac{8}{10} \times \frac{5}{2}$
 - i) $2\frac{1}{4} \times 4\frac{4}{3}$
 - j) $\frac{4}{9} \div \frac{8}{3}$
 - k) $3 \div 2\frac{5}{2}$