

Unit 5

The Real Number System

Topic A: Rational and irrational numbers

- Real numbers

Topic B: Properties of addition and multiplication

- Properties of addition
- Properties of multiplication
- Properties of addition & multiplication

Topic C: Signed numbers and absolute value

- Signed numbers
- Absolute value

Topic D: Operations with signed numbers

- Adding and subtracting signed numbers
- Multiplying signed numbers
- Dividing signed numbers

Unit 5 Summary

Unit 5: Self - test

Topic A: Rational and Irrational Numbers

Real Numbers

Natural numbers: the numbers used for counting. 1, 2, 3, 4, 5, 6 ...

Whole numbers: the natural numbers plus 0. 0, 1, 2, 3, 4, 5, 6 ...

Integers: all the whole numbers and their negatives. ... -4, -3, -2, -1, 0, 1, 2, 3, 4 ...

Rational number: a number that can be expressed as a fraction of two integers ($\frac{a}{b}$).

Examples of rational numbers:

$$\frac{3}{4}, \quad 4\frac{2}{3} (= \frac{14}{3}), \quad 11 (= \frac{11}{1}), \quad 0 (= \frac{0}{7}), \quad 0.52 (= \frac{52}{100}), \quad -4.5 (= \frac{-9}{2}), \quad \sqrt{4} (=2)$$

Rational numbers can be expressed as terminating decimals or repeating decimals.

Example: $\frac{3}{4} = 0.75$ _____ A terminating decimal.

$\frac{2}{3} = 0.66666... = 0.\overline{6}$ _____ A repeating decimal.

$0.232323... = 0.\overline{23}$ _____ A repeating decimal.

Irrational number: a number that *cannot* be represented by the fractions of two integers.

Examples of irrational numbers: π , $\sqrt{3}$, $\sqrt{19}$, $5\sqrt{13}$

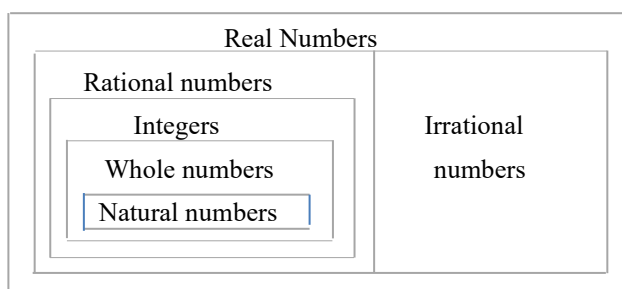
Irrational numbers *cannot* be expressed as terminating decimals or repeating decimals.

$\pi \approx 3.14159265358979323...$ _____ A non-terminating and non-repeating decimal.

$\sqrt{3} \approx 1.73205 ...$ _____ A non-terminating and non-repeating decimal.

Real numbers (R): rational numbers plus irrational numbers.

The real number system:



Topic B: Properties of Addition and Multiplication

Properties of Addition

Commutative property: changing the order of the numbers does not change the sum (order does not matter).

$$a + b = b + a$$

Example: $2 + 3 = 3 + 2$

$$5 = 5$$

Associative property: regrouping the numbers does not change the sum (it does not matter where you put the parenthesis).

$$(a + b) + c = a + (b + c)$$

Example: $(2 + 1) + 3 = 2 + (1 + 3)$

$$5 = 5$$

Additive identity property: the sum of any number and zero leaves that number unchanged.

$$a + 0 = a$$

Example: $100 + 0 = 100$

Closure property of addition: the sum of any two real numbers equals another real number.

Example: If 3 and 8 are real numbers, then $3 + 8 = 11$ is another real number.

Additive inverse property: the sum of any real number and its negative is always a zero.

$$-a + a = 0$$

Example: $7 + (-7) = 0$

A summary of properties of addition:

Additive Properties		Example
Commutative property (switch order)	$a + b = b + a$	$2 + 3 = 3 + 2$
Associative property (switch parentheses)	$(a + b) + c = a + (b + c)$	$(2 + 1) + 3 = 2 + (1 + 3)$
Identity property	$a + 0 = a$	$100 + 0 = 100$
Closure property	If a and b are real numbers, then $a + b$ is a real number.	2 and 5 are real numbers, so $2 + 5 = 7$ is a real number.
Inverse property	$-a + a = 0$	$-2 + 2 = 0$

Example: Name the properties.

- $7x + 0 = 7x$
- $(97 + 22) + 3 = (97 + 3) + 22$
- $(3 + 11x) + 7x = 3 + (11x + 7x)$
- $(4y + 3) + [-(4y + 3)] = 0$

Answer

- Identity property
- Commutative property (switch order)
- Associative property (switch parentheses)
- Inverse property of addition

Properties of Multiplication

Commutative property: changing the order of the numbers does not change the product (order does not matter).

$$a b = b a$$

Example: $2 \cdot 6 = 6 \cdot 2$

$$12 = 12$$

Associative property: regrouping the numbers does not change the product (it does not matter where you put the parenthesis).

$$(a b) c = a (b c)$$

Example: $(2 \cdot 4) \cdot 3 = 2 \cdot (4 \cdot 3)$

$$24 = 24$$

Multiplicative identity property: a number does not change when it is multiplied by 1.

Example: $9 \cdot 1 = 9$

$$a \cdot 1 = a$$

Distributive property: multiply the number outside the parenthesis by each of the numbers inside the parenthesis.

$$a (b + c) = ab + ac$$

or

$$a (b - c) = ab - ac$$

Example: $2 (3 + 4) = 2 \cdot 3 + 2 \cdot 4$

$$14 = 14$$

$$5 (6 - 3) = 5 \cdot 6 - 5 \cdot 3$$

$$15 = 15$$

Multiplicative property of zero: any number multiplied by zero always equals zero.

Example: $100 \cdot 0 = 0$

$$a \cdot 0 = 0$$

Closure property of multiplication: the product of any two real numbers equals another real number.

Example: If 5 and 4 are real numbers, then $5 \cdot 4 = 20$ is another real number.

Multiplicative inverse property: the product of any nonzero real number and its reciprocal is always one.

$$a \cdot \frac{1}{a} = 1$$

Example: 1) $9 \cdot \frac{1}{9} = 1$

2) $(12x) \left(\frac{1}{12x}\right) = 1$

Recall reciprocal: $\text{Reciprocal} = \frac{1}{\text{number}}$

For example, the reciprocal of 4 is $\frac{1}{4}$

number its reciprocal

A summary of properties of multiplication:

Multiplicative properties		Example
Commutative property (Switch order)	$a b = b a$	$2 \cdot 3 = 3 \cdot 2$
Associative property (Switch parentheses)	$(a b) c = a (b c)$	$(2 \cdot 1) 3 = 2 (1 \cdot 3)$
Identity property of 1	$a \cdot 1 = a$	$100 \cdot 1 = 100$
Closure property	If a and b are real numbers, then ab is a real number.	3 and 4 are real numbers, so $3 (4) = \mathbf{12}$ is a real number
Distributive property	$a (b + c) = ab + ac$ $a (b - c) = ab - ac$	$2 (3 + 4) = 2 \cdot 3 + 2 \cdot 4$ $3 (4 - 2) = 3 \cdot 4 - 3 \cdot 2$
Property of zero	$a \cdot 0 = 0$	$35 \cdot 0 = 0$
Inverse property	$a \cdot \frac{1}{a} = 1$	$5 \cdot \frac{1}{5} = 1$

Example: Name the properties

1) $(3y)(5y) = (5 \cdot 3)(y \cdot y)$
 $= 15 y^2$

2) $(9x)x^2 = 9(x \cdot x^2)$
 $= 9x^3$

3) $\frac{1}{5}(10x - 15) = \frac{1}{5} \cdot 10x - \frac{1}{5} \cdot 15$
 $= 2x - 3$

4) $-(7 + 3x) \cdot \frac{1}{-(7+3x)} = 1$

5) $(2x - 3y)x = 2x^2 - 3xy$

6) $\frac{1}{4x} \cdot 0 = 0$

7) $(1000 \cdot 8) \cdot 9 = 1000(8 \cdot 9)$
 $= 1000(72) = 72000$

Answer

Commutative property of multiplication

Associative property of multiplication

Distributive property of multiplication

Inverse property of multiplication

Distributive property

Multiplicative property of zero

Associative property of multiplication

Properties of Addition & Multiplication

Properties of addition and multiplication:

Name	Additive properties	Multiplicative properties
Commutative property	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative property	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity property	$a + 0 = a$	$a \cdot 1 = a$
Closure property	If a and b are real numbers, then $a + b$ is a real number.	If a and b are real numbers, then $a \cdot b$ is a real number.
Inverse property	$-a + a = 0$	$a \cdot \frac{1}{a} = 1$
Distributive property		$a(b + c) = ab + ac$
Property of zero		$a \cdot 0 = 0$

Example: Regroup and simplify the calculations using properties.

1) $(43 + 1998) + 2 = ?$

$$43 + (1998 + 2) = \boxed{2043}$$

Associative property of addition

2) $(7 \cdot 1000) \cdot 9 = ?$

$$(7 \cdot 9) \cdot 1000 = \boxed{63,000}$$

Commutative property of multiplication

Example: Solving the problems in two ways.

1) $3(4 + 2) = ?$

a) $3 \cdot 6 = \boxed{18}$

b) $3 \cdot 4 + 3 \cdot 2 = \boxed{18}$

Distributive property

2) $\frac{1}{2} \left(\frac{1}{2} + 1\frac{2}{3} \right) = ?$

a) $\frac{1}{2} \left(\frac{1}{2} + \frac{5}{3} \right) = \frac{1}{2} \left(\frac{3}{6} + \frac{10}{6} \right)$

$$1\frac{2}{3} = \frac{5}{3}$$

$$= \frac{1}{2} \left(\frac{13}{6} \right) = \frac{13}{12} = \boxed{1\frac{1}{12}}$$

b) $\frac{1}{2} \left(\frac{1}{2} + \frac{5}{3} \right) = \frac{1}{2} \left(\frac{1}{2} \right) + \frac{1}{2} \left(\frac{5}{3} \right)$

Distributive property

$$= \frac{1}{4} + \frac{5}{6} = \frac{3}{12} + \frac{10}{12} = \frac{13}{12} = \boxed{1\frac{1}{12}}$$