

Topic C: Signed Numbers and Absolute Value

Signed Numbers

Signed number: a positive number is written with a plus sign (or without sign) in front and a negative number is written with a minus sign in front.

Example: Positive number: $+5$ (or 5), $7x$, $4y^2$
 Negative number: -3 , -2 , $-9x$

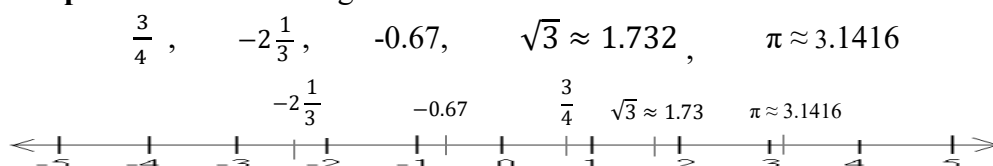
Positive and negative numbers in real life:

	Meaning	Example
Temperature	$+^{\circ}\text{C}$: above 0 degree $-^{\circ}\text{C}$: below 0 degree	$+20^{\circ}\text{C}$ -5°C
Money	$+\$$: gain or own $-\$$: loss or owe	Own: $+\$10000$ Owe: $-\$500$
Sports	$+$ points: gain $-$ points: loss	Gain 3 points: $+3$ Lost 2 points: -2

Positive and negative numbers: positive numbers are greater than zero; negative numbers are less than zero.

The real number line: a straight line on which every point corresponds to a real number.

Example: Put the following numbers on the real number line.



The number on the right is greater than the number on the left on the number line.

Example: $-5 < -3$, $-1 < 4$, $0 > -2$, $2 > \frac{1}{3}$, $-\frac{4}{5} < -\frac{2}{5}$

big $>$ small, small $<$ big

Example: Arrange the following numbers from the smallest to the largest number.

a) -17 , 3 , -3 , -6 , 11 , 0

$$-17 < -6 < -3 < 0 < 3 < 11$$

b) $-\frac{1}{2}$, $\frac{2}{3}$, $-\frac{1}{4}$, $2\frac{2}{3}$

$$-\frac{1}{2} = -0.5, \quad \frac{2}{3} \approx 0.67, \quad -\frac{1}{4} = -0.25, \quad 2\frac{2}{3} = \frac{8}{3} \approx 2.67$$

$$-0.5 < -0.25 < 0.67 < 2.67$$

$$-\frac{1}{2} < -\frac{1}{4} < \frac{2}{3} < 2\frac{2}{3}$$

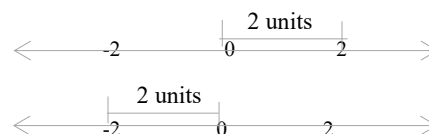
Absolute Value

Absolute value: geometrically, it is the distance of a number x from zero on the number line. It is symbolized “ $|x|$ ”.

Example: $|5|$ is 5 units away from 0.
 $|18|$ is 18 units away from 0.

No negatives for absolute value: the distance is always positive, and absolute value is distance, so the absolute value is never negative.

Example: $|2|$ is 2 units away from 0.
 $|-2|$ is also 2 units away from 0.



Example:

- a) $|-8| = 8$
- b) $|12 - 2| = 10$
- c) $|0.8 - 0.6| = 0.2$
- d) $-|-5| = -(5) = -5$
- e) $-|-6^2| = -(36) = -36$

Order of operations:

Order of operations	
Clear the brackets or parentheses and absolute values (innermost first).	$()$, $[]$, $\{ \}$ and $ $
Calculator exponents (power) and radicals.	a^n and $\sqrt{\quad}$
Perform multiplication or division (from left-to-right).	\times and \div
Perform addition or subtraction (from left-to-right).	$+$ and $-$

Example: 1) $3 [7 - 4 + (10 - 2)] = 3 [7 - 4 + 8]$
 $= 3 [3 + 8]$
 $= 3 \cdot 11$
 $= 33$

Parentheses

Brackets / subtraction

Brackets / addition

Multiplication

2) $\frac{|-8|}{2^2} - (4 - 3) = \frac{8}{2^2} - 1$
 $= \frac{8}{4} - 1$
 $= 2 - 1$
 $= 1$

Parentheses and absolute value

Exponent

Division

Subtraction

Topic D: Operations with Signed Numbers

Adding and Subtracting Signed Numbers

Adding signed numbers

- Add two numbers with the same sign: add their values and keep their common sign.

Example: 1) $5 + 4 = 9$ Add and keep the (+) sign.

2) $(-6) + (-2) = -8$ Add and keep the (−) sign.

3) $-\frac{1}{2} + (-1\frac{1}{2}) = -\frac{1}{2} + (-\frac{3}{2}) = -\frac{4}{2} = -2$ Add and keep the (−) sign.

- Add two numbers with different signs: subtract their values and keep the sign of the larger absolute value.

Example: 1) $2 + (-5) = -3$ Subtract and keep the sign of -5, since $|-5| > |2|$.

2) $(-3) + 7 = 4$ Subtract and keep the sign of 7, since $|7| > |-3|$.

3) $3.2 + (-0.2) = 3$ Subtract and keep the sign of 3.2, since $|3.2| > |-0.2|$.

Subtracting signed numbers

- Subtract a number by adding its opposite (additive inverse), i.e. $a - b = a + (-b)$

(Change the sign of b and then follow the rules for adding signed numbers.)

Example: 1) $(-3) - (-4) = (-3) + (4) = 1$ Change the sign of the (-4), then add (-3) and 4.

2) $(-7) - 2 = (-7) + (-2) = -9$ Change the sign of the 2, then add (-7) and (-2).
 $\quad \quad \quad - (+2)$

3) $-\frac{1}{3} - \frac{2}{3} = -\frac{1}{3} + (-\frac{2}{3}) = -\frac{3}{3} = -1$
 $\quad \quad \quad - (+\frac{2}{3})$

4) $|\frac{3}{5} - 1\frac{1}{2}| = |\frac{3}{5} - \frac{3}{2}| = |\frac{6}{10} - \frac{15}{10}| = |-\frac{9}{10}| = \frac{9}{10}$

- Opposite (or additive inverse): the opposite of a number (two numbers whose sum is 0).

Example: 1) The additive inverse of 7 is -7 $7 + (-7) = 0$

2) The additive inverse of $-\frac{2}{5}$ is $\frac{2}{5}$ $-\frac{2}{5} + \frac{2}{5} = 0$

Multiplying Signed Numbers

Multiplying two numbers with the same sign: the product is positive.

$$a \cdot b = c$$

Example: $4 \cdot 5 = \boxed{20}$

$$(-3)(-5) = \boxed{15}$$

Multiplying two numbers with different signs: the product is negative.

Example: $(-5)(6) = \boxed{-30}$

$$(0.3)(-3) = \boxed{-0.9}$$

$$(-4)^2 = (-4)(-4) = \boxed{16}$$

Multiplying by -1: $-1 \cdot a = \boxed{-a}$

Example: $-1(6x) = \boxed{-6x}$

$$-4^2 = -1 \cdot 4^2 = \boxed{-16}$$

Signs of multiplication:

Multiplication		Example
Positive \times Positive = Positive	$(+)(+) = (+)$	$4 \cdot 3 = 12$
Negative \times Positive = Negative	$(-)(+) = (-)$	$(-4)(3) = -12$
Positive \times Negative = Negative	$(+)(-) = (-)$	$(4)(-3) = -12$
Negative \times Negative = Positive	$(-)(-) = (+)$	$(-4)(-3) = 12$

Multiplying two or more numbers:

Multiplying

- If the two signs are the same, the result is positive.
- If the two signs are different, the result is negative.
- The product of an *even* number of negative numbers is always *positive*.
- The product of an *odd* number of negative numbers is always *negative*.

Example

$$(-3)(-4) = \boxed{12}$$

$$(-0.5)(0.6) = \boxed{-0.3}$$

$$(-4)(-2)(-5)(-1) = \boxed{40}$$

$$(-1)^4 = \boxed{1}$$

$$\left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{4}\right) = \boxed{-\frac{1}{4}}$$

$$(-1)^7 = \boxed{-1}$$

Evaluating expressions:

Example: Evaluate $a^4 - b + c$ if $a = -1$, $b = -2$, $c = 4$.

$$\begin{aligned} a^4 - b + c &= (-1)^4 - (-2) + 4 \\ &= 1 + 2 + 4 = \boxed{7} \end{aligned}$$

Substitute a for -1, b for -2 (add parentheses), and c for 4.

Dividing Signed Numbers

Dividing signed numbers

- Dividing two numbers with the same sign: the quotient is positive.

Example: 1) $-9 \div (-3) = \boxed{3}$ $a \div b = c$

2) $\frac{1.8}{2} = \boxed{0.9}$

3) $\frac{-8}{4} \div \left(\frac{-1}{4}\right) = \frac{-8}{4} \times \left(\frac{4}{-1}\right) = \boxed{8}$

- Dividing two numbers with different signs: the quotient is negative.

Example: 1) $8 \div (-2) = \boxed{-4}$

2) $\frac{-49}{7} = \boxed{-7}$

3) $\frac{3}{9} \div \left(-\frac{6}{3}\right) = \overset{1}{\cancel{\frac{3}{9}}} \times \left(\overset{1}{\cancel{-\frac{3}{6}}}\right) = \boxed{\frac{-1}{6}}$

Signs of division:

Division	Sign	Example
Positive \div Positive = Positive	$\frac{+}{+} = +$	$\frac{28}{7} = 4$
Negative \div Positive = Negative	$\frac{-}{+} = -$	$\frac{-9}{3} = -3$
Positive \div Negative = Negative	$\frac{+}{-} = -$	$\frac{4.9}{-0.7} = -7$
Negative \div Negative = Positive	$\frac{-}{-} = +$	$\frac{-72}{-8} = 9$

Properties of zero:

Property

- The number 0 divided by any nonzero number is zero.
- A number divided by 0 is undefined (not allowed).

Example

$$\frac{0}{6} = 0$$

$$\frac{4}{0} \text{ is undefined.}$$

Evaluating expressions:

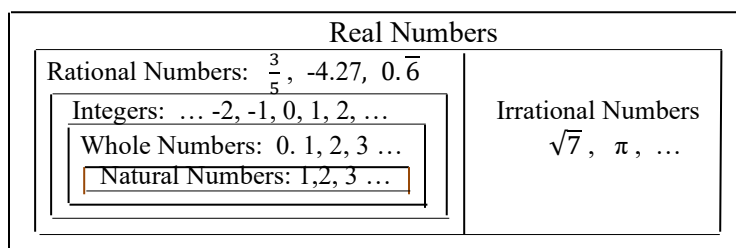
Example: Evaluate $a^2 - \frac{a}{abc}$ if $a = -2$, $b = 1$, $c = (-1)$, and $d = 0$.

$$\begin{aligned}
 a^2 - \frac{a}{abc} + \frac{d}{c} &= (-2)^2 - \frac{-2}{(-2)(1)(-1)} + \frac{0}{-1} && \text{Substitute } a \text{ for } -2, b \text{ for } 1, c \text{ for } -1 \text{ and } d \text{ for } 0. \\
 &= 4 - \frac{-2}{2} + 0 \\
 &= \boxed{5}
 \end{aligned}$$

Unit 5: Summary

The Real Number System

The real number system:



Properties of addition and multiplication:

Name	Additive properties	Multiplicative properties
Commutative property	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative property	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity property	$a + 0 = a$	$a \cdot 1 = a$
Closure property	If a and b are real numbers, then $a + b$ is a real number.	If a and b are real numbers, then $a \cdot b$ is a real number.
Inverse property	$-a + a = 0$	$a \cdot \frac{1}{a} = 1$
Distributive property		$a(b + c) = ab + ac$ $a(b - c) = ab - ac$
Property of zero		$a \cdot 0 = 0$

Signed number: a positive number is written with a plus sign (or without sign) in front and a negative number is written with a minus sign in front.

Positive and negative numbers: positive numbers are greater than zero; negative numbers are less than zero.

The real number line: a straight line on which every point corresponds to a real number.

The number on the right is greater than the number on the left on the number line.

Absolute value: geometrically, it is the distance of a number x from zero on the number line. It is symbolized “ $|x|$ ”.

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Order of operations with absolute value:

Order of operations	
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Calculator exponents (power) and absolute value.	a^n and $\sqrt{\quad}$
Perform multiplication or division (from left-to-right).	\times and \div
Perform addition or subtraction (from left-to-right).	$+$ and $-$

Signed numbers summary:

Operation	Method
Adding signed numbers	<ul style="list-style-type: none"> Add two numbers with the <i>same</i> sign: Add their values, and keep their common sign. Add two numbers with <i>different</i> signs: Subtract their values, and keep the sign of the larger number.
Subtracting signed numbers	Subtract a number by adding its opposite.
Multiplying signed numbers	$(+)(+) = (+)$, $(-)(-) = (+)$, $(-)(+) = (-)$, $(+)(-) = (-)$
Dividing signed numbers	$\frac{+}{+} = +$, $\frac{-}{-} = +$, $\frac{+}{-} = -$, $\frac{-}{+} = -$ Note: $\frac{0}{A} = 0$, $\frac{A}{0}$ is undefined

Multiplying two or more numbers:

- If the two signs are the same, the result is positive.
- If the two signs are different, the result is negative.
- The product of an **even** number of negative numbers is always **positive**.
- The product of an **odd** number of negative numbers is always **negative**.

Opposite (or additive inverse): the opposite of a number.

Properties of zero

- The number 0 divided by any nonzero number is zero. $\frac{0}{A} = 0$
- A number divided by 0 is undefined (not allowed). $\frac{A}{0}$ is undefined.

Unit 5: Self-Test

The Real Number System

Topic A

1. Give two examples of rational numbers that are not integers.
2. Given the set of numbers:

$-3, 4.7, 0, 8, \frac{3}{5}, 2.\overline{56}, 5.4259\dots, \pi, \sqrt{5}$

Determine which of the numbers above are

- a) natural numbers?
- b) integers?
- c) rational numbers?
- d) irrational numbers?

Topic B

3. Name the properties.
 - a) $12a + 0 = 12a$
 - b) $(3x + 11y) + 7 = 7 + (3x + 11y)$
 - c) $(4 + x) + 11 = 4 + (x + 11)$
 - d) $(6a + 5) + [-(6a + 5)] = 0$
 - e) $7(3y + 4) = 7 \cdot 3y + 7 \cdot 4$
 $= 21y + 28$
 - f) $(0.5a)b = 0.5(ab)$
 - g) $(4x)(7y) = (4 \cdot 7)(xy)$
 - h) $-(8y) \cdot \frac{1}{-(8y)} = 1$
 - i) $(4 - 7y)3 = 12 - 21y$
 - j) $\frac{1}{23+7x} \cdot 0 = 0$
 - k) $(199 + 36) + 1 = (199 + 1) + 36$
 - l) $(1000 \cdot 8) \cdot 9 = 1000(8 \cdot 9)$

4. Regroup and simplify the calculations using properties.
- $12 + (45 + 88)$
 - $9 (1000 \cdot 8)$ [L] [SEP]
 - $3 + (2997 + 56)$
5. Use the distributive property to write an equivalent expression without parentheses.
- $4y (y + 0.3)$
 - $(2 - 3y^2) 5$
 - $\frac{1}{3} (\frac{2}{3} - \frac{1}{2} x)$

Topic C

6. Compare these numbers using either $<$ or $>$ [L] [SEP]
- 6 8
 - 0 -6
 - 4 - 2
 - $-\frac{3}{7}$ $\frac{1}{7}$
 - 0.6 -0.8
 - $1\frac{1}{2}$ $\frac{3}{8}$
7. Arrange the following numbers from the smallest to the largest number (using $<$ to order them).
- 8, -9, -4, 23, 0, -17
 - 0.05, -8, $\frac{2}{5}$, $\frac{3}{5}$, -3.24
 - $-\frac{1}{3}$, $\frac{2}{5}$, $-\frac{1}{7}$, $1\frac{3}{4}$
8. Perform the indicated operation.
- $|-67|$
 - $|35 - 14|$
 - $|-0.45 + 0|$
 - $-|-7^2|$
 - $|- \frac{1}{8}|$
9. Perform the indicated operation.
- $4 [7 - 3 + (30 - 5)]$
 - $\frac{|-9|}{3^2} + (27 - 3)$

Topic D

10. Perform the indicated operation.

a) $13 + 24$

b) $(-7) + (-8)$

c) $-\frac{1}{5} + (-2\frac{2}{5})$

d) $9 + (-4)$

e) $(-25) + 12$

f) $8.4 + (-0.9)$

g) $(-7) - (6)$

h) $(-5) - (-7)$

i) $-\frac{3}{7} - \frac{2}{7}$

j) $|\frac{1}{7} - 1\frac{3}{4}|$

k) $-45 \div (-9)$

l) $\frac{-3.6}{6}$

m) $\frac{-9}{5} \div (\frac{-1}{15})$

n) $-72 \div 9$

o) $\frac{0}{1789}$

p) $\frac{3.78}{0}$

11. Write the additive inverse (opposite) of each number.

a) -45

b) $\frac{5}{8}$

c) -1

12. If $x = -2$, $y = 5$, $z = 4$ and $w = 0$, evaluate each of the following.

a) $zy + x^3$

b) $x^2 - 2xy + y^2 + \frac{w}{3xyz}$

c) $(x + y)(x - y) - 5z$

d) $4(\frac{2xy}{3w})$