Topic C: Scientific Notation and Square Roots

Scientific Notation

Scientific notation is a special way of concisely expressing very *large* and *small* numbers.

Example: $300{,}000{,}000 = 3 \times 10^{8}$ m/sec — The speed of light.

$0.00000000000000000016 = 1.6 \times 10^{-19}$ C — An electron.

Scientific notation: a product of a number between 1 and 10 and power of 10.

$N \times 10^{\pm n}$

Scientific notation	Example
$N \times 10^{\pm n}$ $1 \le N < 10$ n - integer	$67504.3 = 6.75043 \times 10^{4}$ Standard form — Scientific notation

Scientific vs. non-scientific notation:

Scientific notation	Not scientific notation		
7.6×10^{3}	76×10^{2}	$76 > 10$	76 is not between 1 and 10.
8.2×10^{13}	0.82×10^{14}	$0.82 < 1$	0.82 is not between 1 and 10.
5.37×10^{7}	53.7×10^{6}	$53 > 10$	53 is not between 1 and 10.

Writing a number in scientific notation:

Step	Example
▪ Move the decimal point ***after*** the ***first nonzero digit***.	0.0079 — 37213000.
▪ Determine n (the power of 10) by counting the number of places you moved the decimal.	$n = 3$ — $n = 7$
▪ If the decimal point is moved to the ***right***: $\times 10^{-n}$	$0.0079 = 7.9 \times 10^{-3}$ 3 places to the right
▪ If the decimal point is moved to the ***left***: $\times 10^{n}$	$37213000. = 3.7213 \times 10^{7}$ 7 places to the left

Example: Write in scientific notation.

1) **2340000** = 2340000. = 2.34×10^{6} — 6 places to the left, $\times 10^{n}$

2) **0.000000439** = 4.39×10^{-7} — 7 places to the right, $\times 10^{-n}$

Example: Write in standard (or ordinary) form.

1) $\mathbf{6.4275 \times 10^{4}} = 64275$ **2)** $2.9 \times 10^{-3} = 0.0029$

Example: Simplify and write in scientific notation.

1) $(\mathbf{4.9 \times 10^{-3}})(\mathbf{3.82 \times 10^{8}}) = (4.9 \times 3.82)(10^{-3+8})$ — Multiply coefficients of $10^{\pm n}$, $a^m a^n = a^{m+n}$

$= (18.718 \times 10^{5})$ — 18 > 10, this is not in scientific notation.

$= (1.8718 \times 10^{6})$ — N = 1, this is in scientific notation.

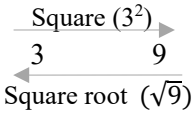
2) $\frac{(\mathbf{5 \times 10^{5}})(\mathbf{2.3 \times 10^{-2}})}{\mathbf{4.5 \times 10^{7}}} = \frac{5 \times 2.3}{4.5} \times \frac{(10^{5} \times 10^{-2})}{10^{7}}$ — Regroup coefficients of $10^{\pm n}$

$\approx 2.556 \times 10^{-4}$

$a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$

Square Roots

Square root ($\sqrt{\ }$): a number with the symbol $\sqrt{\ }$ that is the opposite of the square of a number, such as $\sqrt{9} = 3$ and $3^2 = 9$, respectively.



Perfect square: a number that is the exact square of a whole number.

Perfect square	Square root
$1\times1 = 1^2 =$ **1**	$\sqrt{1} = 1$
$2\times2 = 2^2 =$ **4**	$\sqrt{4} = 2$
$3\times3 = 3^2 =$ **9**	$\sqrt{9} = 3$
$4\times4 = 4^2 =$ **16**	$\sqrt{16} = 4$
$5\times5 = 5^2 =$ **25**	$\sqrt{25} = 5$
$6\times6 = 6^2 =$ **36**	$\sqrt{36} = 6$
$7\times7 = 7^2 =$ **49**	$\sqrt{49} = 7$
$8\times8 = 8^2 =$ **64**	$\sqrt{64} = 8$
$9\times9 = 9^2 =$ **81**	$\sqrt{81} = 9$
…	…

Examples:

Square root	Square
$\sqrt{100} = 10$	$10^2 = 100$
$\sqrt{49} = 7$	$7^2 = 49$
$\sqrt{121} = 11$	$11^2 = 121$
$\sqrt{169} = 13$	$13^2 = 169$
$\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$	$4^2 = 16$ $5^2 = 25$

Using a calculator: $\sqrt{81} = ?$

$\boxed{\sqrt{\ }}$ 81 $\boxed{=}$ (The display reads 9) Or 81 $\sqrt{\ }$ $\boxed{=}$ for some calculators.

Example: Find the square roots.

1) $\sqrt{\mathbf{144}} = \sqrt{12^2} = \boxed{12}$ $\boxed{\sqrt{\ }}$ 144 $\boxed{=}$

2) $\frac{\sqrt{\mathbf{64}}}{\sqrt{\mathbf{225}}} = \frac{\sqrt{8^2}}{\sqrt{15^2}} = \boxed{\frac{8}{15}}$ $\boxed{\sqrt{\ }}$ 225 $\boxed{=}$

Simplifying Square Roots

Order of operations:

Order of operations	
1. the brackets or parentheses and absolute values (innermost first)	( ) , [ ] , { } , | |
2. exponent or square root (from left-to-right)	a^n , $\sqrt{\ }$
3. multiplication or division (from left-to-right)	× and ÷
4. addition or subtraction (from left-to-right)	+ and -

Memory aid - BEDMAS

B	E (R)	D M	A S
Brackets	**E**xponents or Square **R**oot	**D**ivide or **M**ultiply	**A**dd or **S**ubtract

Example: Calculate.

1) $\mathbf{6 - 2\sqrt{81}} = 6 - 2 \cdot 9$ $\quad 81 = 9^2$

$= 6 - 18 = \boxed{-12}$

2) $\mathbf{3.2^2 - 3\sqrt{2 + 3^2}} = 10.24 - 3\sqrt{11}$

$\approx 10.24 - 3\ (3.32)$

$= 10.24 - 9.96 = \boxed{0.28}$

3) $\dfrac{\sqrt{\mathbf{64}}}{\sqrt{\mathbf{250-249}}} = \dfrac{8}{\sqrt{1}} = \boxed{8}$ $\quad 64 = 8^2\ ,\ \sqrt{1} = \sqrt{1^2} = 1$

Simplifying square roots:

- Factor the number inside the square root sign.
  (Find the perfect square(s) that will divide the number).
- Rewrite the square root as a multiplication problem.
- Reduce the perfect squares ("pulling out" the integer(s)).

Example

$\sqrt{75}$ → 25, 3

$\sqrt{75} = \sqrt{25 \cdot 3}$

$\sqrt{75} = \sqrt{5^2 \cdot 3} = \boxed{5\sqrt{3}}$

Example: Simplify.

1) $\sqrt{\mathbf{180}} = \sqrt{45 \cdot 4} = \sqrt{9 \cdot 5 \cdot 4} = \sqrt{3^2 \cdot 5 \cdot 2^2} = 3 \cdot \sqrt{5} \cdot 2 = \boxed{6\sqrt{5}}$

45, 4; 9, 5

2) $\sqrt{\dfrac{\mathbf{92}}{\mathbf{64}}} = \dfrac{\sqrt{4 \times 23}}{\sqrt{8^2}} = \dfrac{2\sqrt{23}}{8} = \boxed{\dfrac{\sqrt{23}}{4}}$

Unit 11: Summary

Exponents, Roots & Scientific Notation

The degree of a term with one variable: the exponent of its variable.

The degree of a term with more variables: the sum of the exponents of its variables.

The degree of a polynomial with more variables: the highest degree of any individual term.

Descending order: the exponent of a variable decreases for each succeeding term.

Ascending order: the exponent of a variable increases for each succeeding term.

Properties of exponents:

Name	Rule	Example
Product rule	$a^m\, a^n = a^{m+n}$	$2^3\, 2^2 = 2^{3+2} = 2^5 = 32$
Quotient rule (the same base)	$\frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$	$\frac{y^4}{y^2} = y^{4-2} = y^2$
Power of a power	$(a^m)^n = a^{mn}$	$(x^3)^2 = x^{3\cdot 2} = x^6$
Power of a product (different bases)	$(a \cdot b)^n = a^n\, b^n$	$(2 \cdot 3)^2 = 2^2\, 3^2 = 4 \cdot 9 = 36$
	$(a^m \cdot b^n)^p = a^{mp}\, b^{np}$	$(t^3 \cdot s^4)^2 = t^{3\cdot 2}\, s^{4\cdot 2} = t^6\, s^8$
Power of a quotient (different bases)	$(\frac{a}{b})^n = \frac{a^n}{b^n}$ $(b \neq 0)$	$(\frac{2}{3})^2 = \frac{2^2}{3^2} = \frac{4}{9}$
	$(\frac{a^m}{b^n})^p = \frac{a^{mp}}{b^{np}}$ $(b \neq 0)$	$(\frac{q^2}{p^4})^3 = \frac{q^{2\cdot 3}}{p^{4\cdot 3}} = \frac{q^6}{p^{12}}$
Negative exponent a^{-n}	$a^{-n} = \frac{1}{a^n}$ $(a \neq 0)$	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
	$\frac{1}{a^{-n}} = a^n$ $(a \neq 0)$	$\frac{1}{4^{-2}} = 4^2 = 16$
Zero exponent a^0	$a^0 = 1$	$15^0 = 1$
One exponent a^1	$a^1 = a$ (But $1^n = 1$)	$7^1 = 7$, $1^{13} = 1$

Steps for simplifying exponential expressions:

- Remove parentheses using "power rule" if necessary. $(a^m \cdot b^n)^p = a^{mp}\, b^{np}$
- Regroup coefficients and variables.
- Use "product rule" and "quotient rule". $a^m\, a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$
- Simplify.
- Use "negative exponent" rule to make all exponents positive if necessary.

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Unit 11: Self-Test

Exponents, Roots & Scientific Notation

Topic A

1. Write the following exponential expressions in expanded form.

a) 7^4

b) $(-t)^3$

c) $(5a^4b^0)^2$

d) $\left(\frac{-7}{11}x\right)^3$

2. Write each of the following in the exponential form.

a) (0.5) (0.5) (0.5) (0.5)

b) (6*w*) (6*w*) (6*w*)

c) (7*u*) (3*v*) (*u*) (2*v*)

3. Evaluate.

a) $4x^2 + 5y$, for $x = 1$, $y = 4$

b) $(2a)^3 - 3b$, for $a = 5$, $b = 6$

4. What is the degree of the following term / polynomial?

a) $15ab^4$

b) $6xy^2z^4 + 5y^6 - xz + 2z^0$

5. Arranging polynomials in descending order:

a) $x^2 + 2 - 7x^3 - x + 9x^4$

b) $4v - 67 + 21uv^3 - uv^2$

6. Arranging polynomials in ascending order.

a) $26x^2 - 17x^3 - 5x + 43$

b) $4.3t^2w^2 + \frac{4}{7}tw + w^4 - 8w^3 - 9$

Topic B

7. Simplify (do not leave negative exponents in the answer).

a) $(-92)^1$

b) $(-38076)^0$

c) $(-0.4)^3$

d) -8^2

e) $y^4\,y^3$

f) $\frac{x^9}{x^6}$

g) $(t^4)^{-5}$

h) $13a^{-1}$

i) $[(-4) \cdot (0.2)]^3$

j) $(3a^2 \cdot b^3)^4$

k) $\frac{1}{4^{-3}}$

l) $(\frac{w}{u})^{-3}$

m) $(\frac{a^3}{b^{-4}})^2$

n) $(3^{-4})(3^4)$

o) $\frac{5x^5y^{-6}}{11^0 \cdot x^3y^4}$

p) $(\frac{u^{-2}v^3}{w^{-4}})^{-3}$

q) $(2x^2y^3)^3(3x^{-1}y^{-2})^2(-2345w^{-34})^0$

r) $(\frac{(3x^3)(y^4)}{4x^2y^3})^3$

8. Evaluate for $x = 3$, $y = 2$, $z = -2$.

a) $(-145x^{-6}y^5z^{-8})^0$

b) $\left(\frac{x}{y}\right)^{-3}$

c) $(x - y + 2z)^y$

Topic C

9. Write in scientific notation.

a) 45,600,000

b) 0.00000523

10. Write in standard (or ordinary) form.

a) 3.578×10^3

b) 4.3×10^{-5}

11. Simplify and write in scientific notation.

a) $(5.42 \times 10^{-2})(4.38 \times 10^7)$

b) $\frac{(5\times10^5)(2.4\times10^{-3})}{3.2\times10^8}$

12. Simplify.

a) $\sqrt{196}$

b) $\frac{\sqrt{121}}{\sqrt{225}}$

c) $\sqrt{320}$

d) $\sqrt{\frac{117}{81}}$