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Contents

Preface

If you are looking for a quick exam, homework guide, and review book in intermediate mathematics, *"Key Concepts of Intermediate Level Math"* is an excellent source. Skip the lengthy and distracting books and instead use this concise book as a guideline for your studies, quick reviewing and tutoring.

This unique and well-structured book is an excellent supplement and convenient reference book for intermediate mathematics. It provides concise, understandable and effective guide on intermediate level mathematics.

Key Features

As an aid to readers, the book provides some noteworthy features:

- A concise study guide, quickly getting to the heart of each particular topic, helping students with a quick review before doing mathematics homework as well as preparation for tests.
- Each topic, concept, term and phrase has a clear definition followed by examples on each page.
- Key terms, definitions, properties, phrases, concepts, formulae, rules, equations, etc. are easily located. Clear step-by-step procedures for applying theorems.
- Clear and easy-to-understand written format and style. Materials presented in visual and gray scale format with less text and more outlines, tables, boxes, charts, etc.
- Tables that organize and summarize procedures, methods, and equations; clearly presenting information and making studying more effective.
- Procedures and strategies for solving word problems, using realistic real-world application examples.
- Summary at the end of each unit to emphasize the key points and formulas in the unit, which is convenient for students reviewing before exams.
- Self-test at the end of each unit tests student's understanding of the material. Students can take the self-test before beginning the unit to determine how much they know about the topic. Those who do well may decide to move on to the next unit.

Suitable Readers

This book can be used for:

- Adult Basic Education programs at colleges.
- Students in community colleges, high schools, tutoring, or resource rooms.
- Self-study readers, including new teachers to brush up on their mathematics.
- Professionals as a quick review of some basic mathematic formulas and concepts, or parents to help their children with homework.

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About the Author

Meizhong Wang has been an instructor at the College of New Caledonia (CNC) in Canada for more than 25 years. She currently teaches mathematical and computer courses and has lectured in physics, electronics, electric circuits, etc. at the CNC and various other colleges and universities in Canada and China.

Meizhong is also the author of several books, including:

- *Algebra I & II – Key Concepts, Practice, and Quizzes* (The Critical Thinking Co. – U.S., 2013).
- *Math Made Easy* (CNC Press, Canada, 2011, second edition 2013).
- *Understandable Electric Circuits* (Michael Faraday House of the IET – Institution of Engineering and Technology – U.K., 2010).
- *Legends of Four Chinese Sages* – coauthor (Lily S.S.C Literary Ltd. – Canada, 2007).
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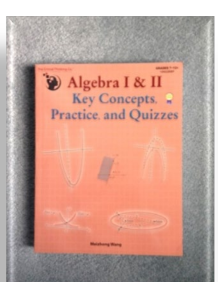
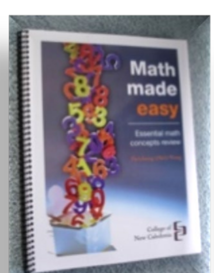
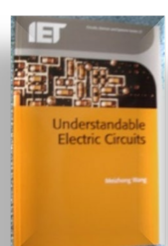






Unit R

Review of Basic Mathematics

Topic A: Basic math skills

- Numbers and place value
- Prime / composite numbers
- Prime factorization
- Basic mathematical symbols and terms

Topic B: Percent, decimal and fraction

- Fractions
- More about fractions
- Decimals
- Operations with decimals
- Percent and conversion

Topic C: Operations with fractions

- Least common denominator (LCD)
- Operations with fractions
- Ratio and proportion

Unit R Summary

Unit R Self-test

Unit R is a review of basic math fundamentals. There is a self-test at the end of the unit that can test students' understanding of the material. Students can take the self-test before beginning the unit to determine how much they know about the topic. Those who do well may decide to move on to the next unit without reading the lesson.

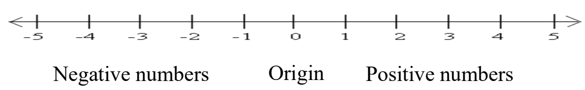
Topic A: Basic Math Skills

Numbers and Place Value

Numbers:

Classify numbers	Definition	Numbers
The ten digits	a symbol for numeral below 10	0, 1, 2, 3, 4, 5, 6, 7, 8 and 9
Whole numbers	the numbers used for counting	0, 1, 2, 3, 4, 5, 6, 7 …
Integers	all the whole numbers and their negatives	… -3, -2, -1, 0, 1, 2, 3 …
Odd numbers	any integer that cannot be evenly divided by 2	1, 3, 5, 7, 9 …
Even numbers	any integer that can be evenly divided by 2	0, 2, 4, 6, 8, 10 …

Number line is a straight line on which every point corresponds to an integer.



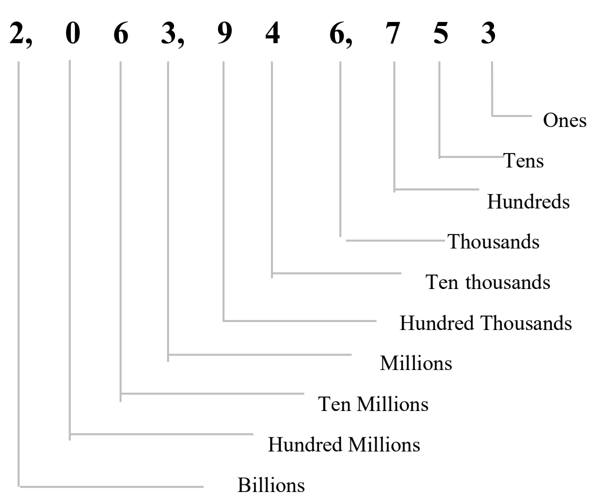
Place value: the value of the position of a digit in a number.

- Each digit in a number has a place value.
- The location in a number determines the value of a digit.

Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
Trillions			Billions			Millions			Thousands			Ones		

(Read from right to left)

Example: **2,063,946,753**



Prime / Composite Numbers

Factor: a number you multiply with others to get another number.

Example: $3 \times 4 = 12$ 3 and 4 are factors.

- Some numbers can be factored in many ways:

  Example: $2 \times 4 = 8$ or $4 \times 2 = 8$ or $1 \times 8 = 8$ or $8 \times 1 = 8$

- Factors for some numbers:

Number	1	2	3	4	5	6	7	8	9	10
Factors	1	1, 2	1, 3	1, 2, 4	1, 5	1, 2, 3, 6	1, 7	1, 2, 4, 8	1, 3, 9	1, 2, 5, 10

Prime number: a whole number that only has two factors, 1 and itself.

Example: 2, 3, 5, and 7 are prime numbers.

(7 has two factors: 1 and 7, $1 \times 7 = 7$)

Composite number: a whole number that has more than two factors, and can be evenly divided.

Example: 4, 6, 8, 9 and 10 are composite numbers.

(6 has four factors: 1, 2, 3 and 6. $1 \times 6 = 6$, $2 \times 3 = 6$)

Rules for testing a prime / composite number:

- A prime number is always an odd number, except for 2 (but an odd number is not necessarily a prime number).

  Example: The prime numbers 1, 3, 5, and 7 are odd numbers.
  The odd number 9 is a composite number.

- An even number (ends in a 0, 2, 4, 5, 6, and 8) is always a composite number (except number 2).

  Example: 14, 28, 376, and 5372 are composite numbers.

- All numbers that end with five and are greater than five are composite numbers.

  Example: 15, 65, and 345 are composite numbers.

Tip: The Prime Tester in the following website can determine if a number is a prime or a composite number.

http://www.murderousmaths.co.uk/games/primcal.htm

Prime Factorization

Prime factorization is finding which prime numbers can be used to multiply to get the original number.

Prime factorization: the product of all the prime numbers for a given number.

Example: $30 = 2 \times 3 \times 5$

"Product" – the keyword for multiplication
2, 3, and 5 are prime numbers (or prime factors).

Find the prime factorization:

- Method 1: do ***repeated division*** (or upside down division) by prime numbers, and multiply all the prime numbers around the outside to get the prime factorization.

  Example: Find the prime factorization of 24.

  $$\begin{array}{r|l} \mathbf{2} & 24 \\ \hline \mathbf{2} & 12 \\ \hline \mathbf{2} & 6 \\ \hline & \mathbf{3} \end{array}$$

  $24 \div 2 = 12$
  $12 \div 2 = 6$
  $6 \div 2 = 3$

  Note: Stop dividing until you reach a prime number.

  The outside numbers are 2, 2, 2, 3.

  The prime factorization for 24 is: $24 = 2 \times 2 \times 2 \times 3 = \boxed{2^3 \times 3}$

- Method 2: ***factor tree*** method - split the number into two factors, then split non-prime factors until all the factors are prime, and multiply all the prime numbers.

  Example: Find the prime factorization of 24.

  24 → **2**, 12 $(24 \div 2 = 12)$
  12 → **2**, 6 $(12 \div 2 = 6)$
  6 → **2**, **3** $(6 \div 2 = 3)$

  The prime numbers are 2, 2, 2, 3.

  The prime factorization for 24 is: $24 = 2 \times 2 \times 2 \times 3 = \boxed{2^3 \times 3}$

Basic Mathematical Symbols and Terms

Basic mathematical symbols summary:

Symbol	Meaning	Example
$=$	equal	$3 = 3$
$\neq$	not equal	$2 \neq 3$
$\approx$	approximately	$4 \approx 3.89$
$>$	is greater than	$4 > 2$
$<$	is less than	$1 < 3$
$\geq$	is greater than or equal to	$x \geq 4$
$\leq$	is less than or equal	$x \leq 8$
$\pm$	plus or minus	3 ± 2 means: $3 + 2$ or $3 - 2$
$+$	addition	$3 + 2$
$-$	subtraction	$7 - 3$
$\times$ or $\cdot$ or $(\ )$	multiplication	$6 \times 3 = 18$ or $6 \cdot 3 = 18$ or $(6)(3) = 18$
$\div$ or $/$ or $—$ or $\overline{)\ \ }$	division	$4 \div 2 = 2$, $4 / 2$, $\frac{4}{2}$, $2\overline{)4}$

Arithmetic terms:

Operation	Term
Addition	Addend + addend = sum 2 + 1 = 3
Subtraction	Subtrahend − minuend = difference 5 − 2 = 3
Multiplication	Multiplicand × multiplier = product (factor) (factor) 2 × 4 = 8
Division	Dividend ÷ divisor = quotient & remainder (factor) 7 ÷ 2 = 3 R 1

Properties of zero

Property	Example
▪ Any number multiplied by 0 will always equal to 0.	$3 \times 0 = 0$
▪ The number 0 divided by any nonzero number is zero.	$\frac{0}{6} = 0$ (0 apples divided by 6 kids, each kid gets 0 apples.)
▪ A number divided by 0 is not defined (not allowed).	$\frac{6}{0}$ is undefined. (6 apples shared by zero kids has no meaning.)

Writing whole numbers in words:

- Do not use ‘and’ when writing or reading whole numbers.
- Do not use ‘s’ at the end of trillion, million, thousand, hundred, etc.

Example: Write the following number in words: 12, 023, 476

Twelve million, twenty-three thousand, four hundred seventy-six.

Topic B: Percent, Decimal and Fraction

Fractions

Fraction: a fraction is a part of a whole. It is expressed in the form of $\frac{a}{b}$. (Example: $\frac{2}{5}$)

Numerator $\frac{a}{b}$ Denominator

- Numerator: the number that represents how many ***parts*** are being dealt with.
- Denominator: the number of parts the ***whole*** is being divided into.

Three types of fractions

- Proper fraction: has a numerator smaller than (<) the denominator.

Example: $\frac{1}{2}$, $\frac{3}{8}$, $\frac{16}{237}$

- Improper fraction: has a numerator larger than or equal to (≥) the denominator.

Example: $\frac{7}{6}$, $\frac{56}{31}$, $\frac{9}{9}$

- Mixed fraction (or mixed number): contains an ***integer*** and a ***proper fraction***.

Example: $2\frac{1}{4}$, $3\frac{2}{5}$, $5\frac{4}{7}$

Conversion between a mixed number and an improper fraction

- **To convert a mixed number to an improper fraction:**

$$\text{Improper fraction} = \frac{\text{whole number} \times \text{denominator} + \text{numerator}}{\text{Denominator}}$$

Example: $2\frac{1}{4} = \frac{2 \times 4 + 1}{4} = \frac{9}{4}$

- **To convert an improper fraction to a mixed number**:

$$\text{Mixed number} = \text{Numerator} \div \text{Denominator} \quad , \quad \text{Quotient}\ \frac{\text{Reminder}}{\text{Denominator}}$$

Example: $\frac{9}{2} = 9 \div 2 = 4\,R\,1 = 4\frac{1}{2}$

Quotient

$$\begin{array}{r} 4 \\ 2)\ 9 \\ -\ \underline{8} \\ 1 \end{array}$$

Remainder

More about Fractions

Equivalent fractions: different fractions that have the same value.

To find the equivalent fraction: divide or multiply the numerator and denominator by the same number.

- Divide by the same number (for a larger fraction):

To simplify (or reduce) fractions: divide the numerator and denominator by the same number until their only common factor is 1.

$$\frac{\text{Numerator} \div n}{\text{Denominator} \div n}$$

"n" is any whole number that does not equal to 0.

Example: Simplify $\frac{18}{36}$.

$$\frac{18}{36} \underset{\div 2}{\overset{\div 2}{=}} \frac{9}{18} \underset{\div 3}{\overset{\div 3}{=}} \frac{3}{6} \underset{\div 3}{\overset{\div 3}{=}} \boxed{\frac{1}{2}}$$

The simplest fraction of $\frac{18}{36}$ is $\frac{1}{2}$.

- Multiply by the same number (for a smaller fraction):

$$\frac{\text{Numerator} \times n}{\text{Denominator} \times n}$$

Example: $\frac{1}{3} \underset{\times 3}{\overset{\times 3}{=}} \frac{3}{9} \underset{\times 2}{\overset{\times 2}{=}} \boxed{\frac{6}{18}}$

Like and unlike fractions:

- Like fractions: fractions that have the ***same*** denominators. **Examples:** $\frac{2}{7}$, $\frac{5}{7}$, $\frac{4}{7}$
- Unlike fractions: fractions that have ***different*** denominators. **Examples:** $\frac{2}{3}$, $\frac{3}{5}$, $\frac{7}{10}$

Classifying fractions:

Classifying fractions		Examples
Proper fraction	numerator < denominator	$\frac{1}{2}$, $\frac{3}{8}$, $\frac{16}{237}$
Improper fraction	numerator ≥ denominator	$\frac{7}{6}$, $\frac{56}{31}$, $\frac{9}{9}$
Mixed fraction (or mixed number)	A number made up of an integer and a fraction.	$2\frac{1}{4}$, $3\frac{2}{5}$, $5\frac{4}{7}$
Like fractions	Fractions that have the same denominators.	$\frac{2}{7}$, $\frac{5}{7}$, $\frac{4}{7}$
Unlike fractions	Fractions that have different denominators.	$\frac{2}{3}$, $\frac{3}{5}$, $\frac{7}{10}$

Decimals

Decimal number: a number contains a decimal point.

- The number to the left of the decimal is the integer part.
- The number to the right of the decimal is the fractional part.

Example: 34 **. 8**

Integer part + decimal point + fractional part (8 tenth)

Decimal place: a place of a digit to the right of a decimal point.

- Each digit in a decimal number has a decimal place.
- The location in a number determines the value of a digit.

(Smaller)

Hundreds	Tens	Ones	•	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths

Example: 5.40378

5 . 4 0 3 7 8

Hundred thousandths

Ten thousandths

Thousands

Hundredths

Tenths

Ones

Write decimals in words: Integer part + ***and*** + fractional part

Decimal point

Example: **1)** 35. 348

Thirty-five and three hundred forty-eight thousandths

2) 6.038

Six and thirty-eight hundredths

Operations with Decimals

Operations with decimals:

+ or - decimals	- Line up the decimal points. - + or − as whole numbers. - Insert a decimal point in the answer (in the same line as above).	$\begin{array}{r} 0.3725 \\ 3.404 \\ +\ 2.13 \\ \hline 5.9065 \end{array}$
× decimals	- × as whole numbers. - Count the numbers of the decimal places in both factors. - Insert a decimal point in the product so that it matches the number of decimal places of factors (start at the far right).	$\begin{array}{r} 2.14 \\ \times\ 2.2 \\ \hline 428 \\ +\ 428\ \ \\ \hline 4.708 \end{array}$ (Two decimal places) (One decimal place) (Three decimal places)
÷ decimals	- Move the decimal point of the divisor to the right end. - Move the decimal point of the dividend the same number of places to the right (insert zeros if necessary). - ÷ as whole numbers. - Insert a decimal point in the quotient (directly above the decimal point in the dividend).	$4.86 \div 1.2 = ?$ $12.\overline{)48.60}$ quotient 4.05 $-\ 48$ 60 $-\ 60$ 0 $\text{Divisor}\overline{)\text{Dividend}}$ with Quotient above

Convert decimals to mixed numbers or fractions:

- Whole number does not change.
- Write the original term as a fraction.
  - Numerator = the fractional part (The digits on the right of the decimal point).
  - Denominator = a multiple of 10 (The number of zeros = The number of decimal places)
- Simplify (reduce) if possible.

Example: **1)** $5.25 = 5\frac{25}{100} = 5\frac{1}{4}$ ($\div 25$ numerator and denominator)

The fractional part = 25

The number of decimal places = 2

2) $0.045 = \frac{45}{1000} = \frac{9}{200}$

The number of decimal places = 3

3) $384.3645 = 384\frac{3645}{10000}$

The number of decimal places = 4

Percent and Conversion

Percent (%): one part per hundred, or per one hundred.

Example: $5\% = \frac{5}{100}$

The standard form of percent proportion:

(With the word "is")

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100} \quad \text{or} \quad \frac{\text{"is"number}}{\text{"of"number}} = \frac{\%}{100}$$

(With the word "of")

Use percent proportion method to solve % problems:

- Identify the part, whole, and percent.
- Set up the proportion equation.
- Solve for the unknown.

Example

8 percent ***of*** what number ***is 4*** ?

Percent Whole (x) Part

$$\frac{4}{x} = \frac{8}{100} \qquad \frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}$$

$$x = \frac{(4)(100)}{8} = \boxed{50}$$

Converting between percent, decimal and fraction:

Conversion	Step	Example
Percent to Decimal	Move the decimal point two places to the left, then remove %.	$31\% = .31.\% = 0.31$
Decimal to Percent	Move the decimal point two places to the right, then insert %.	$0.317 = 0.\,31.7 = 31.7\,\%$
Percent to Fraction	Remove %, divide by 100, then simplify.	$15\% = \frac{15}{100} = \frac{3}{20}$ (% = per one hundred.)
Fraction to Percent	Divide, move the decimal point two places to the right, then insert %.	$\frac{1}{4} = 1 \div 4 = 0.25. = 25\,\%$
Decimal to Fraction	Convert the decimal to a percent, then convert the percent to a fraction.	$0.35 = 35\% = \frac{35}{100} = \frac{7}{20}$

Converting repeating decimals to fractions:

Example: $0.\overline{6} \rightarrow$ Fraction

- Let x equals the repeating decimal: $x = 0.66$ (1)
- Multiply both sides by 100: $100x = 66$ (2)
- Subtract the first equation from the second: (2) – (1)

$$\begin{aligned} 100x &= 66 \\ -\,x &= 0.66 \\ \hline 99x &= 65.34 \end{aligned}$$

- Solve for x:

$$x = \frac{65.34 \div 33}{99 \div 33} = \frac{1.98}{3} \approx \boxed{\frac{2}{3}}$$

Topic C: Operations with Fractions

Least Common Denominator (LCD)

Least common multiple (LCM): the lowest number that is divisible by each given number without a remainder.

Example: The LCM of 2 and 3 is 6.

- Multiples of 2: 0. 2, 4, ***6***, 8, 10, ***12*** …
- Multiples of 3: 0. 3, ***6***, 9, ***12***, 15 …
- Common multiples of 2 and 3 are 6 and 12 …
- The ***least*** common multiple (LCM) of 2 and 3 is $\boxed{6}$.

The common multiple 12 is not the smallest (least).

Find the LCM: Use repeated division (or upside-down division). The product of all the prime numbers around the outside is the LCM.

Example: Find the LCM of 30 and 45.

$$\begin{array}{r|ll} \mathbf{5} & 30 & 45 \\ \hline \mathbf{3} & 6 & 9 \\ \hline & \mathbf{2} & \mathbf{3} \end{array}$$

30 ÷ 5 = 6 45 ÷ 5 = 9

6 ÷ 3 = 2 9 ÷ 3 = 3

(Stop dividing since 2 and 3 are prime numbers.)

LCM = $5 \times 3 \times 2 \times 3 = \boxed{90}$ Multiply all the prime numbers around the outside.

Least common denominator (LCD): the least common multiple (LCM) of the ***denominators*** of two or more given fractions.

Find the LCD: Use repeated division to find the LCM for all ***denominators*** of given fractions.

Example: Find the LCD for $\frac{4}{8}$, $\frac{5}{16}$ and $\frac{2}{42}$

$$\begin{array}{r|lll} \mathbf{2} & 8 & 16 & 42 \\ \hline \mathbf{2} & 4 & 8 & 21 \\ \hline \mathbf{2} & 2 & 4 & 21 \\ \hline & \mathbf{1} & \mathbf{2} & \mathbf{21} \end{array}$$

8 ÷ 2 = 4, 16 ÷ 2 = 8, 42 ÷ 2 = 21

4 ÷ 2 = 2, 8 ÷ 2 = 4, bring down 21.

2 ÷ 2 = 1, 4 ÷ 2 = 2, bring down 21.

LCD = $2 \times 2 \times 2 \times 1 \times 2 \times 21 = \boxed{336}$

Operations with Fractions

Operation	Steps	Example
Adding and subtracting *like* fractions	- Add / subtract the numerators. - Denominators do not change. - Simplify if necessary.	$\frac{3}{13}+\frac{5}{13}=\frac{3+5}{13}=\frac{8}{13}$ $\frac{7}{12}-\frac{3}{12}=\frac{7-3}{12}=\frac{4}{12}=\frac{1}{3}$
Adding and subtracting *unlike* fractions	- Determine the LCD. - Rewrite fractions with the LCD, and add or subtract the numerators. - Simplify if necessary.	$\frac{5^{\times 2}}{12_{\times 2}}+\frac{3^{\times 3}}{8_{\times 3}}=\frac{10}{24}+\frac{9}{24}=\frac{10+9}{24}=\frac{19}{24}$ (LCD = 24) $\frac{4^{\times 2}}{9_{\times 2}}-\frac{2^{\times 3}}{6_{\times 3}}=\frac{8}{18}-\frac{6}{18}=\frac{8-6}{18}=\frac{2}{18}=\frac{1}{9}$ (LCD = 18)
Adding and subtracting mixed numbers with *like* denominators	- Add / subtract integers. - Add / subtract as fractions. - Simplify if necessary.	$2\frac{3}{5}+5\frac{1}{5}=(2+5)\frac{3+1}{5}=7\frac{4}{5}$ $5\frac{9}{14}-3\frac{5}{14}=(5-3)\frac{9-5}{14}=2\frac{4}{14}=2\frac{2}{7}$
Adding and subtracting mixed numbers with *unlike* denominators	- Rewrite fractions with the LCD. - Add / subtract as factions. - If the sum/difference created an improper fraction → a mixed number.	$3\frac{5}{12}-2\frac{3}{8}=3\frac{10}{24}-2\frac{9}{24}$ $=(3-2)\frac{10-9}{24}=1\frac{1}{24}$
Multiplying fractions	- Cross simplify if the fraction is not in lowest terms. - Multiply the numerators. - Multiply the denominators. - Simplify the result if necessary.	$\frac{2}{\cancel{9}_{3}}\times\frac{\cancel{3}^{1}}{5}=\frac{2\times1}{3\times5}=\frac{2}{15}$
Multiplying mixed numbers	- Convert mixed numbers to improper fractions. - Cross simplify if the fractions is not in lowest terms. - Multiply the numerators. - Multiply the denominators. - Simplify the result if necessary.	$1\frac{1}{5}\times2\frac{1}{2}=\frac{\cancel{6}^{3}}{\cancel{5}_{1}}\times\frac{\cancel{5}^{1}}{\cancel{2}_{1}}=\frac{3\times1}{1\times1}=\frac{3}{1}=3$
Dividing fractions	- Change the divisor to its reciprocal (switch the numerator and denominator). - Multiply the resulting fractions.	$\frac{2}{7}\div\frac{3}{5}=\frac{2}{7}\times\frac{5}{3}=\frac{2\times5}{7\times3}=\frac{10}{21}$
Dividing mixed numbers	- Convert mixed numbers to improper fractions. - Divide fractions.	$8\div3\frac{1}{5}=\frac{8}{1}\div\frac{16}{5}=\frac{\cancel{8}^{1}}{1}\times\frac{5}{\cancel{16}_{2}}=\frac{1\times5}{1\times2}=\frac{5}{2}=2\frac{1}{2}$

Ratio and Proportion

Ratio, rate and proportion:

	Notation	Unit	Example
Ratio	a to b or $a{:}b$ or $\frac{a}{b}$	with the same unit.	5 to 9 or 5:9 or $\frac{5\text{ m}}{9\text{ m}}$
Rate	a to b or $a{:}b$ or $\frac{a}{b}$	with different units.	3 to 7 or 3:7 or $\frac{3\text{ cm}}{7\text{ m}}$
Proportion	$\frac{a}{b}=\frac{c}{d}$	an equation with a ratio on each side.	$\frac{3\text{ cm}}{7\ m}=\frac{1\text{ cm}}{5\text{ m}}$
Note: the units for both numerators must match and the units for both denominators must match. **Example**: $\frac{\text{in}}{\text{ft}}=\frac{\text{in}}{\text{ft}}$, $\frac{\text{minutes}}{\text{hours}}=\frac{\text{minutes}}{\text{hours}}$			

Solving a proportion:

- Cross multiply: multiply along two diagonals. $\frac{a}{b}=\frac{c}{d}$
- Solve for the unknown.

Example

$$\frac{x}{9}=\frac{2}{6}$$

$$6\cdot x=2\cdot 9$$

(x is the unknown.)

$$x=\frac{2\cdot 9}{6}=\frac{18}{6}=3$$

Example: John's height is 1.75 meters, and his shadow is 1.09 meters long. A building's shadow is 10 meters long at the same time. How tall is the building?

- Facts and ***unknown***:

John's height = 1.75 m	Let x = Building's height (unknown)
John's shadow = 1.09 m	Building's shadow = 10 m

- Equation: $$\frac{1.75\text{ m}}{1.09\text{ m}}=\frac{x\text{ m}}{10\text{ m}}$$ $\frac{a}{b}=\frac{c}{d}$

- Cross multiply: $$\frac{1.75\text{ m}}{1.09\text{ m}}=\frac{x\text{ m}}{10\text{ m}}$$ $(1.75)\ (10)=(1.09)\ (x)$

- Solve for x: $$x=\frac{(1.75)(10)}{1.09}=\boxed{16.055\text{ m}}$$ Divide 1.09 both sides.

The building's height is 16.055m.

Unit R: Summary

Review of Basic Mathematics

Numbers:

Classify Numbers	Numbers
Whole numbers	0, 1, 2, 3, 4, 5, 6,7, 8, 9, 10, 11 …
Odd numbers	1, 3, 5, 7, …
Even numbers	0, 2, 4, 6, 8, …
Digits	0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
Expanded form	345 = 300 + 40 + 5
Prime number	A whole number that only has two factors, 1 and itself.
Composite number	A whole number that has more than two factors.

Place value: the value of the position of a digit in a number.

Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
Trillions			Billions			Millions			Thousands			Ones		

Read from right to left

Factor: a number you multiply with others to get another number.

Prime factorization: the product of all the prime factors for a given number.

Find the prime factorization: do repeated division (or upside-down division) by prime numbers, and multiply all the prime numbers around the outside to get the prime factorization.

Properties of zero:

- Any number multiplied by 0 will always equal to 0.
- The number 0 divided by any nonzero number is zero.
- A number divided by 0 is not defined (not allowed).

Basic mathematical symbol summary:

Symbol	Meaning
$=$	equal
$\neq$	not equal
$\approx$	approximately
$>$	is greater than
$<$	is less than
$\geq$	is greater than or equal to
$\leq$	is less than or equal to
$\pm$	plus or minus
$+$	addition
$-$	subtraction
$\times$ or $\cdot$ or ( )	multiplication
$\div$ or / or — or $\overline{)\quad}$	division

Writing whole numbers in words:

- Do not use ‘and’ when writing or reading whole numbers.
- Do not use ‘s’ at the end of trillion, million, thousand, hundred, etc.

Fraction: a fraction is a part of a whole. It is expressed in the form of $\frac{a}{b}$.

$$\text{Fraction: } \frac{a}{b} = \frac{\text{Numerator}}{\text{Denominator}}$$

Decimal number: a number contains a decimal point.

Integer part + decimal point + fractional part

Decimal place: a place of a digit to the right of a decimal point.

Hundreds	Tens	Ones	•	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred thousandths

Write decimals in words: Integer part + ***and*** + fractional part

Decimal point

Convert decimals to mixed numbers or fractions:

- Whole number does not change.
- Write the original term as a fraction.
  - Numerator = the fractional part
  - Denominator = a multiple of 10 (The number of zeros = The number of decimal places)
- Simplify if possible.

Classifying fractions: $\text{Fraction: } \frac{a}{b} = \frac{\text{Numerator}}{\text{Denominator}}$

Classifying fractions	
Proper fraction	numerator $<$ denominator
Improper fraction	numerator $\geq$ denominator
Mixed fraction (or mixed number)	A number made up of an integer and a fraction.
Like fractions	Fractions that have the same denominators.
Unlike fractions	Fractions that have different denominators.

Arithmetic terms:

Operation	Term
Addition	Addend + addend = sum
Subtraction	Subtrahend − minuend = difference
Multiplication	Multiplicand × multiplier = product (factor) (factor)
Division	Dividend ÷ divisor = quotient & remainder (factor)

To convert a mixed number to an improper fraction:

$$\text{Improper fraction} = \frac{\text{whole number} \times \text{denominator} + \text{numerator}}{\text{Denominator}}$$

To convert an improper fraction to a mixed number:

$$\text{Mixed number} = \text{Numarator} \div \text{Denominator} \Rightarrow \text{Quotient}\frac{\text{Remainder}}{\text{Denominator}}$$

The standard form of percent proportion:

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100} \quad \text{or} \quad \frac{\text{"is"number}}{\text{"of"number}} = \frac{\%}{100}$$

Converting between percent, decimal and fraction:

Conversion	Steps
Percent to Decimal	Move the decimal point two places to the left, then remove %.
Decimal to Percent	Move the decimal point two places to the right, then insert % .
Percent to Fraction	Remove % , divide by 100, then simplify.
Fraction to Percent	Divide, move the decimal point two places to the right, then insert % .
Decimal to Fraction	Convert the decimal to a percent, then convert the percent to a fraction.

Least common multiple (LCM): the lowest number that is divisible by each given number without a remainder.

Least common denominator (LCD): the least common multiple (LCM) of the denominators of two or more given fractions.

Find the LCD: Use repeated division to find the LCM for all denominators of given fractions.

Ratio, rate and proportion:

	Notation	Unit
Ratio	a to b or $a{:}b$ or $\frac{a}{b}$	With the same unit.
Rate	a to b or $a{:}b$ or $\frac{a}{b}$	With different units.
Proportion	$\frac{a}{b} = \frac{c}{d}$	The units for both numerators must match and the units for both denominators must match.

Solving a proportion:

- Cross multiply: multiply along two diagonals.
- Solve for the unknown.

To find the equivalent fraction: divide or multiply the numerator and denominator by the same number.

To simplify (or reduce) fractions: divide the numerator and denominator by the same number until their only common factor is 1.

$$\frac{\text{Numerator} \div n}{\text{Denominator} \div n}$$

"n" is any whole number that does not equal to 0.

Operations with fractions:

Operation	Steps
Adding and subtracting *like* fractions	- Add / subtract the numerators. - Denominators do not change. - Simplify if necessary.
Adding and subtracting *unlike* fractions	- Determine the LCD. - Rewrite fractions with the LCD, and add or subtract the numerators. - Simplify if necessary.
Adding and subtracting mixed numbers with *like* denominators	- Add / subtract whole numbers. - Add / subtract as fractions. - Simplify if necessary.
Adding and subtracting mixed numbers with *unlike* denominators	- Rewrite fractions with the LCD. - Add / subtract as factions. - If the sum/difference created an improper fraction → a mixed number.
Multiplying fractions	- Cross - simplify if the fraction is not in lowest terms. - Multiply the numerators. - Multiply the denominators. - Simplify the result if necessary.
Multiplying mixed numbers	- Convert mixed numbers to improper fractions. - Cross - simplify if the fractions is not in lowest terms. - Multiply the numerators. - Multiply the denominators. - Simplify the result if necessary.
Dividing fractions	- Change the divisor to its reciprocal (switch the numerator and denominator). - Multiply the resulting fractions.
Dividing mixed numbers	- Convert mixed numbers to improper fractions. - Divide fractions.

Unit R: Self-Test

Review of Basic Mathematics

Topic A

1. Find the prime factorization of 36.

2. **a)** Write the number in words: 10, 024, 526

b) Write the decimal in words: 47.268

3. Calculate the following without using a calculator:

a) $0.463 + 2.456 + 3.52$

b) 3.21×2.5

c) $6.48 \div 2.4$

Topic B

4. **a)** Convert a mixed number to an improper fraction: $4\,\frac{2}{7}$

b) Convert an improper fraction to a mixed number: $\frac{9}{5}$

5. Reduce to lowest terms: $\frac{12}{48}$

6. 12 percent of what number is 48 ?

7. Convert between percent, decimal and fraction:

a) 45% to decimal

b) 0.436 to %

c) 25% to fraction

d) $\frac{5}{25}$ to %

e) 0.4 to fraction

f) $0.\overline{3}$ to Fraction

Topic C

8. **a)** Find the LCM of 24 and 64.

b) Find the LCD for $\frac{2}{5}$, $\frac{3}{15}$ and $\frac{24}{35}$

9. Calculate:

a) $\frac{1}{6}+\frac{4}{6}$

b) $\frac{11}{14}-\frac{4}{14}$

c) $\frac{3}{8}+\frac{5}{4}$

d) $\frac{6}{7}-\frac{4}{21}$

e) $2\,\frac{3}{7}+4\,\frac{2}{7}$

f) $7\,\frac{8}{12}-5\,\frac{7}{12}$

g) $4\,\frac{9}{12}-3\,\frac{2}{4}$

h) $\frac{8}{10}\times\frac{5}{2}$

i) $2\,\frac{1}{4}\times\;4\,\frac{4}{3}$

j) $\frac{4}{9}\div\frac{8}{3}$

k) $3\div\;2\,\frac{5}{2}$

Unit 1

Basic Statistics and Calculator Use

Topic A: Average

- Mean and range
- Median and mode

Topic B: Graphs

- Bar or column graph
- Line graph
- Circle or pie graph
- Create a circle graph

Topic C: Using a calculator and estimating

- Scientific calculator
- Basic functions of a scientific calculator
- Estimating and rounding

Unit 1 Summary

Unit 1 Self-test

Topic A: Average

Mean and Range

Statistics: the mathematical branch that deals with data collection, organization, description, and analysis to draw conclusions.

Average: it refers to the statistical mean, median, mode, or range of a group of numbers or a set of data.

- Mean = average.
- Median = middle number.
- Mode = the number that occurs most often.
- Range = the difference between the largest and smallest values.

Mean (or arithmetic mean): the standard average value of a group of numbers or a set of data.

It is the most common expression for the average.

- Determine the mean: add up all the numbers in the group and divide by the number of values.

$$\boxed{\text{Mean} = \frac{\text{Sum of numbers}}{\text{Number of values}}}$$

- **Example:** Find the mean of 2, 3, 4, 0, 1.

$$\text{Mean} = \frac{2+3+4+0+1}{5} = \frac{10}{5} = \boxed{2}$$

There are 5 numbers.

Range: the difference between the highest and lowest values in a group of numbers.

- **Determine the range:**

$$\boxed{\text{Range} = \text{highest value} - \text{lowest value}}$$

- **Example:** Find the range: 3, 5, 2, **9**, 4, 8, **1**

Range = 9 − 1 = $\boxed{8}$

Median and Mode

Mode: the value(s) that occurs most frequently in a group of numbers.

Example: Find the mode:

2, ***4***, 5, 3, 7, 8, ***4***, 1 Mode = $\boxed{4}$ The value that occurs most frequently is 4.

- If no value is repeated, the mode does not exist.

Example: 13, 27, 30, 49, 47 $\boxed{\text{No mode.}}$ No value is repeated.

- A bimodal has 2 modes in a group of numbers.

Example: 1, ***3***, ***8***, 17, 9, ***8***, 4, 6, 11, ***3*** Modes = $\boxed{\text{3 and 8}}$ It has two modes.

- If more than one value occurs the same number of times, each value is a mode.

Median: the ***middle*** number of an ordered group of numbers.

Example: 1, 3, ***5***, 7, 9

- Determine the median: arrange the values in order (ascending or descending).
  - Ascending order: numbers are arranged from the smallest to the largest number.
  - Descending order: numbers are arranged from the largest to the smallest number.
- If the total number of terms in the group is ***odd***, the median is the middle number.

Example: Find the median of 2, 8, 7, 1, 6, 5, 3, 4, 8, 1, 9 11 numbers (odd)

- Ascending order: 1, 1, 2, 3, 4, ***5***, 6, 7, 8, 8, 9
- Median = $\boxed{5}$ 5 is the middle number.

- If the total number of terms in the group is ***even***, the median is the average of the two values in the middle (add two middle numbers and divide by 2):

$$\text{Median} = \frac{\text{Add two middle values}}{2}$$

Example: Find the median of 5, 4, 9, 0, 2, 6 6 numbers (even)

- Ascending order: 0, 2, ***4***, ***5***, 6, 9
- Median = $\frac{4+5}{2} = \boxed{4.5}$ 4 and 5 are the middle numbers.
- Or descending order: 9, 6, ***5***, ***4***, 2, 0
- Median = $\frac{5+4}{2} = \boxed{4.5}$

Topic B: Graphs

Bar or Column Graph

Bar or column graph: a chart with rectangular bars whose heights or lengths display the values. (It used to compare information between different groups.)

A bar graph can be vertical (column graph) or horizontal (bar graph).

Create a bar (or column) graph:

- Put data into tabular form (make a table).
- Label each axis and make up a title for the graph. Example $\begin{cases} \text{horizontal axis} - \text{ student names} \\ \text{vertical axis} - \text{test scores} \end{cases}$
- Create a scale (number) for each axis starting from zero.

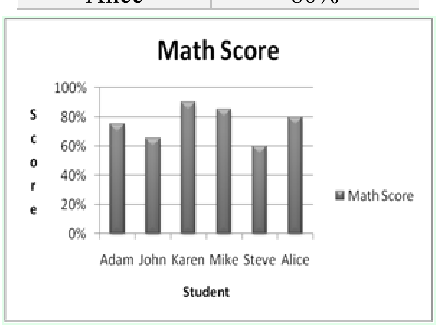
  Example $\begin{cases} \text{horizontal axis} - \text{ Adam, John, Karen, Mike, Steve ...} \\ \text{vertical axis} - 0\%, 20\%, 40\%, 60\%, 80\% \text{ ...} \end{cases}$
- Draw bars or columns (use the data from the table). Example: Bar's height displays the student score.

Table: a group of numbers arranged in a condensed form of columns and rows. It is a more effective way to present information.

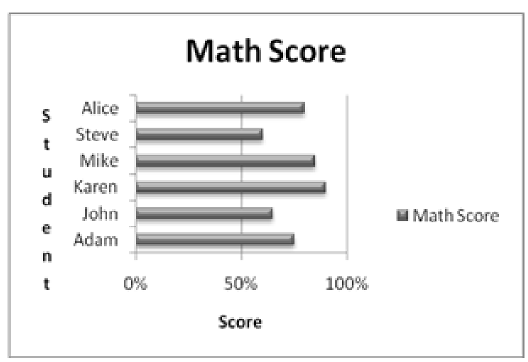
Interpolate and extrapolate from the information provided:

Example: Make a graph from the table and answer questions.

Student	Test score
Adam	75%
John	65%
Karen	90%
Mike	85%
Steve	60%
Alice	80%



Column graph



Bar Graph

a) How many students earned 80% or greater? 3 students (80, 85, 90)

b) How many students earned 60%? 1 student (60)

c) How many more students earned between 59% and 81%? 4 students (60, 65, 75, 80)

Line Graph

Line graph: a chart that displays information by connecting lines between data points.

It is used to track changes over periods of time.

A line graph consists of a horizontal x-axis and a vertical y-axis.

- Horizontal x-axis: represents the independent variable (such as time).
- Vertical y-axis: represents the dependent variable (such as temperature, population, sales, rainfall, etc.).

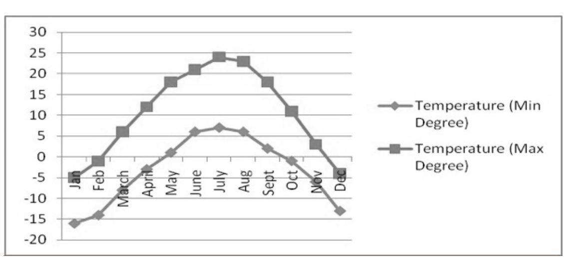
Create a line graph:

- Put data into tabular form (make a table).
- Label each axis and make up a title for the graph. Example $\begin{cases} \text{horizontal axis} - \text{ months of the year} \\ \text{vertical axis} - \text{ temeparature} \end{cases}$
- Create a scale for each axis. Example $\begin{cases} \text{horizontal axis} - \text{ Jan., Feb., Mar., April ...} \\ \text{vertical axis} - 0^0\text{C},\ 5^0\text{C},\ 10^0\text{C},\ 15^0\text{C ...} \end{cases}$
- Plot the data points (use the data from the table).
- Draw a curve (or a line) that best fits the data points (connect the points).

Example of a line graph:

Average temperatures in Prince George

Month	Temperature ^{0}C (Low)	Temperature ^{0}C (High)
Jan	-16	-5
Feb	-14	-1
March	-8	6
April	-3	12
May	1	18
June	6	21
July	7	24
Aug	6	23
Sept	2	18
Oct	-1	11
Nov	-6	3
Dec	-13	-4



Average Temperatures in Prince George (^{0}C)

Circle or Pie Graph

Circle (or pie) graph: a chart made by dividing a circle into sections (parts) that each represent a percentage of the total. It is used to compare parts of a whole.

- Entire pie: represents the total amount (360^0).
- Sectors: represent percentages of the total. Example $\begin{cases}\text{entire pie} - \text{the final grade of a class} \\ \text{sectors} - \text{percentage of students who get A, B, C} \ldots\end{cases}$

Create a circle graph:

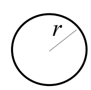
- Put data into tabular form (make a table).
- Calculate the total amount.
- Determine the percentage of each sector or part.

$$\boxed{\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}} \quad \text{or} \quad \boxed{\text{Percent} = \frac{\text{Part}}{\text{Whole}} \cdot 100}$$

- Determine the angle of each sector (convert the percent to a decimal first).

$$\boxed{\text{Angle for each part} = (\text{Decimal})\ (360^0)}$$

- Draw a circle (use a compass) and a radius (r).

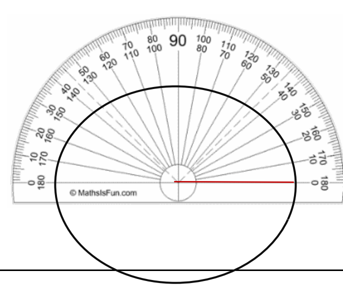


- Draw in the sectors of the circle (use a protractor), and add colors to the sectors (this will help to make them easier to distinguish).
- Label the sectors and make up a title for the graph.

How to use a protractor:

- Place the protractor on the circle so that the center mark of the protractor at the center of the circle.
- Ensure that the radius of the circle is lined up on the zero line at the end of the protractor.
- Draw the sector by using the calculated angle.

Each time you add a sector the radius changes to the line you just drew.



Create a Circle Graph

Example: Create a circle graph using the following table – final grades in a math class.

Final grades in a math class	Number of students
D	1
C	2
B	4
A	3
Total number of students:	10

- The total number of students: $1 + 2 + 4 + 3 = 10$ There are 10 students in the class.
- Determine the percentage of each sector (convert the percent to a decimal):

- First sector in the circle chart: $\frac{1}{10} = \frac{\text{Percent}}{100}$, $\% = \frac{1\times100}{10} = 10\% = 0.1$ $\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}$
- Second sector in the circle chart: $\frac{2}{10} = \frac{\text{Percent}}{100}$, $\% = \frac{2\times100}{10} = 20\% = 0.2$
- Third sector in the circle chart: $\frac{4}{10} = \frac{\text{Percent}}{100}$, $\% = \frac{4\times100}{10} = 40\% = 0.4$
- Fourth sector in the circle chart: $\frac{3}{10} = \frac{\text{Percent}}{100}$, $\% = \frac{3\times100}{10} = 30\% = 0.3$

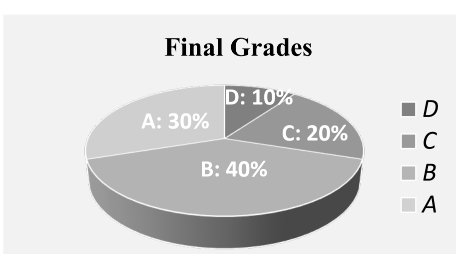
- Determine the angle of each sector: Angle for each part = (Decimal) (360^0)

- First sector in the circle chart: (Decimal) $(360^0) = (0.1)\ (360^0) = 36^0$
- Second sector in the circle chart: (Decimal) $(360^0) = (0.2)\ (360^0) = 72^0$
- Third sector in the circle chart: (Decimal) $(360^0) = (0.4)\ (360^0) = 144^0$
- Fourth sector in the circle chart: (Decimal) $(360^0) = (0.3)\ (360^0) = 108^0$

Percent	Decimal	Angle
10%	(0.1)	36^0
20%	(0.2)	72^0
40%	(0.4)	144^0
30%	(0.3)	108^0
Total: 100%	(1)	Total: 360^0

Check: The sum of the percentages = 100%. The sum of all the degrees should be = 360^0.

- Draw the circle graph:



# Topic C:	Using a Calculator and Estimating

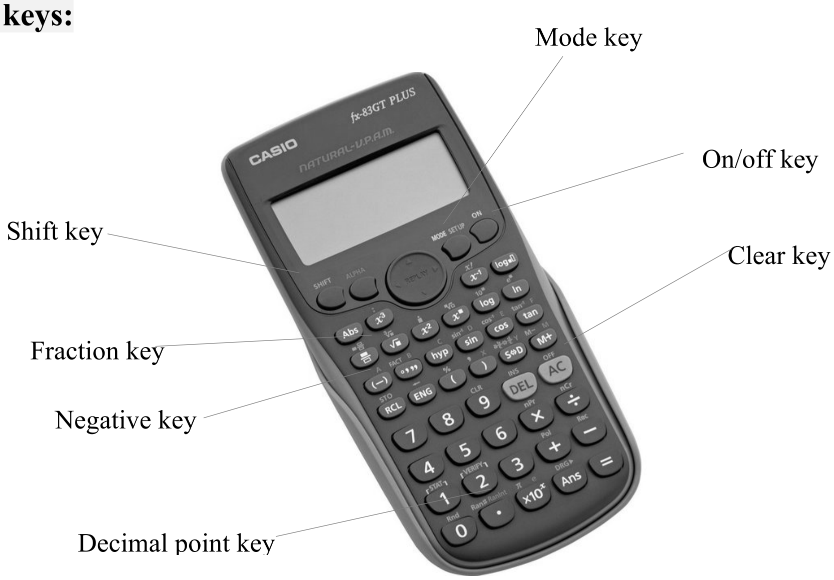
Scientific Calculator

Scientific calculator: a calculator with advanced functions that can solve mathematics, science, and engineering problems.

Basic functions of a scientific calculator

- Basic functions (+, $-$, $\times$, $\div$)
- Parentheses
- Absolute values (abs)
- Order of operations
- Exponents or powers
- Pi problems ($\pi = 3.141592654...$)
- Fractions
- Scientific notation
- Trigonometry functions (sine, cosine, tangent)
- Etc.

Identify main keys:



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Basic Functions of a Scientific Calculator

Basic features:

Operation	Function
$+$	Addition
$-$	Subtraction
$\times$	Multiplication
$\div$	Division
(−) or neg	Negative number
x^2	Squaring
x^y **or** y^x **or** ^	Exponent or power
√ **or Sqrt**	Square root
$\sqrt[3]{}$	Cube root
$\sqrt[x]{}$	nth root
( )	Parentheses
π	Pi
Mode	Converting between degrees and radians
Shift or 2nd F or INV	Converting between main and upper symbols
$\frac{a}{b}$ **or d/c**	Fraction
$a\frac{b}{c}$ **or a b/c**	Mixed number
Exp or $\times 10^x$	Scientific notation
sin, cos, tan	Trigonometry functions
sin^{-1}, cos^{-1}, tan^{-1}	Inverse trigonometry functions

Determine what order you need to press the keys (it may vary with different calculators).

Examples:

1) $21 + 34 \times 5 = ?$

21 $\boxed{+}$ 34 $\boxed{\times}$ 5 $\boxed{=}$ Display: 191

2) $\frac{432}{6} + \pi = ?$

432 $\boxed{\div}$ 6 $\boxed{+}$ $\boxed{\pi}$ $\boxed{=}$ Display: 75.14159…

3) $27^2 + 38 \times 17 = ?$

27 $\boxed{x^2}$ $\boxed{+}$ 38 $\boxed{\times}$ 17 $\boxed{=}$ Display: 1375

4) $\sqrt[3]{27} + 2^3 = ?$

$\boxed{\text{Shift}}$ $\boxed{\sqrt[3]{}}$ 27 $\boxed{+}$ 2 $\boxed{x^y}$ 3 $\boxed{=}$ Display: 11

or $\boxed{\text{2nd F}}$ 27 $\boxed{\sqrt[3]{}}$ $\boxed{+}$ 2 $\boxed{x^y}$ 3 $\boxed{=}$

Rounding and Estimating

Rounding whole numbers: choose an approximation for a whole number (making a number simpler).

The method of rounding:

- If the rounding digit (next digit) is ≥ 5 (greater than or equals to), round-up (add 1 to the left digit of the rounding digit and replace all the digits to the right of the rounding digit with 0).
- If the rounding digit is < 5 (less than), round down (do not change the left digit of the rounding digit, replace the rounding digit and all the digits to the right of it with 0).

Example:

			Rounding digit (*next digit*)		
1)	Round to the nearest ***largest place***.	$3{,}459{,}567 \approx \boxed{3{,}000{,}000}$	4	$4 < 5$	round down
2)	Round to the nearest ***ten***.	$345 \approx \boxed{350}$	5	$5 \geq 5$	round-up
3)	Round to the nearest ***hundred***.	$3{,}429 \approx \boxed{3{,}400}$	2	$2 < 5$	round down
4)	Round to the nearest ***thousand***.	$27{,}656 \approx \boxed{28{,}000}$	6	$6 > 5$	round-up

Estimate: find a value that can be used to check if an answer is reasonable (approximating).

Method of estimating: round to the largest place value.

- If the next digit is ≥ 5, round-up.
- If the next digit is < 5, round down.

Example: Estimate the following.

1)

$$\begin{array}{rcr} 7656 & \approx & 8000 \\ +\ 4358 & \approx & +\ 4000 \\ \hline & \approx & \boxed{12000} \end{array}$$

The next digit of 7 is 6 (6 > 5, round-up).
The next digit of 4 is 3 (3 < 5, round down).

2)

$$\begin{array}{rcr} 8756 & \approx & 9000 \\ -\ 5432 & \approx & -\ 5000 \\ \hline & \approx & \boxed{4000} \end{array}$$

The next digit of 8 is 7 (7 > 5, round-up).
The next digit of 5 is 4 (4 < 5, round down).

3) $5378 \times 367 \approx 5000 \times 400 = \boxed{2{,}000{,}000}$

4) $7576 \div 237 \approx 8000 \div 200 = \boxed{40}$

Unit 1: Summary

Basic Statistics and Calculator Use

Graphs

- Bar or column graph: a chart with rectangular bars whose heights or lengths display the values. (It used to compare values between different groups.)

Construct a bar or column graph: page 23.

- Line graph: a chart that displays information by connecting lines between data points. (It is used to track changes over periods of time). Construct a line graph: page 24.

- Circle graph: a chart made by dividing a circle into sections (parts) that each represent a percentage of the total. (It is used to compare parts of a whole.)

Construct a circle graph: page 25-26.

- **Average:**

Average/Range	Description / Formula
Mean	The "standard" average value of a group of numbers or a set of data. $\text{Mean} = \dfrac{\text{Sum of numbers}}{\text{Number of values}}$
Median	The middle number of an ordered group of numbers. - Arrange the values in order. - If the total number of terms in the group is **odd**, the median is the middle number. - If the total number of terms in the sample is **even**: $\text{Median} = \dfrac{\text{Add two middle values}}{2}$
Mode	The value(s) that occurs most frequently in a group of numbers. - If no value is repeated, the mode does not exist. - If more than one value occurs with the same frequency, each value is a mode. - A bimodal has 2 modes in a group of numbers.
Range	The difference between the highest and lowest values in a group of numbers. Range = highest value - lowest value

Scientific calculator

- Scientific calculator: a calculator with advanced functions that can solve mathematics, science, and engineering problems.

- **Basic functions of a scientific calculator:**

Operation	Function
$+$	Addition
$-$	Subtraction
$\times$	Multiplication
$\div$	Division
(−) or neg	Negative number
x^2	Squaring
x^y **or** y^x	Exponent or power
$\sqrt{\ }$ **or Sqrt**	Square root
$\sqrt[3]{\ }$	Cube root
$\sqrt[x]{\ }$	nth root
( )	Parentheses
π	Pi
Mode	Converting between degrees and radians
Shift or 2nd F or INV	Converting between main and upper symbols
— or d/c	Fraction
or a b/c	Mixed number
Exp or $\times 10^x$	Scientific notation
sin, cos, tan	Trigonometry functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$	Inverse trigonometry functions
…	…

Rounding

- Rounding whole numbers: choose an approximation for a number.
- The method of rounding:
  - If the rounding digit (next digit) is ≥ 5, round-up.
  - If the rounding digit is < 5 (less than), round down.

Estimating

- Estimate: find a value that can be used to check if an answer is reasonable.
- Method of estimating: round to the largest place value.
  - If the next digit is ≥ 5, round-up.
  - If the next digit is < 5, round down.

Unit 1: Self-Test

Basic Statistics and Calculator Use

Topic A

1. Find the mean: 4, 0, 5, 10, 9, 2
2. Find the range: 11, 7, 2, 6, 9, 13, 3
3. Find the mode:
   - **a)** 12, 4, 7, 3, 9, 51, 6, 7
   - **b)** 21, 13, 4, 16, 54, 100
4. Find the median:
   - **a)** 4, 6, 7, 10, 9, 11, 3, 8, 5, 1, 14, 2, 23
   - **b)** 6, 14, 10, 11, 0, 19, 5, 4

Topic B

5. Create a column graph from the table and answer the following questions:

Student	Test score
Evan	85%
Jon	75%
Alice	90%
Tom	65%
Damon	95%
Steve	70%

   - **a)** How many students earned 85% or greater?
   - **b)** How many students earned 75%?
   - **c)** How many more students earned between 64% and 91%?

6. Create a line graph from the table (average temperatures in Vancouver):

Month	Temperature ^{0}C (High)	Temperature ^{0}C (Low)
Jan	7	7
Feb	8	2
March	10	3
April	13	6
May	17	9
June	20	12
July	22	14
Aug	22	14
Sept	19	11
Oct	14	7
Nov	9	3
Dec	6	1

7. Create a circle graph from the table (Tom's monthly expenses):

Tom	Monthly Expenses
Rent	\$600
Food	\$300
Transportation	\$60
Utilities	\$80
Clothing	\$85
Entertainment	\$165
Miscellaneous	\$35

Topic C

8. Complete the following with your calculator:

a) $78 + 43 \times 11$

b) $\frac{2468}{8} + \pi$

c) $42^2 + 43 \times 25$

d) $4\frac{1}{6} + 3\frac{4}{7}$

e) $\sqrt[3]{125} + 3^5$

9. Rounding:

a) Round to the nearest largest place. 6,345,789

b) Round to the nearest ***ten***. 567

c) Round to the nearest ***hundred***. 8, 649

d) Round to the nearest ***thousand***. 47,567

10. Estimate the following:

a) $79{,}215 + 784$

b) $11{,}345 - 372$

c) $4{,}738 \times 624$

d) $8{,}345 \div 382$

Unit 2

Introduction to Algebra

Topic A: Algebraic Expressions

Basic Algebraic Terms

Algebra: a branch of mathematics containing numbers, letters and arithmetic operators (+, $-$, $\times$, $\div$, etc.) with the letters used to represent unknown quantities (variables).

Example: $3 + 2 = 5$ in algebra may look like $x + 2 = 5$ — x represents 3.

Constant: a ***number*** stands for a fixed value that does not change.

Example: $\mathbf{2}$ in $x + \mathbf{2}$ is a constant.

Variable: a ***letter*** that can be assigned different values (it represents an unknown quantity).

Example: $\boldsymbol{x} + 2$ when $x = 0$, $\boldsymbol{x} + 2 = 0 + 2 = \mathbf{2}$

when $x = 3$, $\boldsymbol{x} + 2 = 3 + 2 = \mathbf{5}$

Coefficient: the ***number*** that in front of a letter (variable).

Example:

$\mathbf{9}\,x$	coefficient:	$\boxed{9}$
$-\frac{2}{7}x$	coefficient:	$\boxed{-\frac{2}{7}}$
x	coefficient:	$\boxed{1}$

$x = 1 \cdot x$

Algebraic expression: a mathematical phrase that contains numbers, letters, grouping symbols (parentheses) and arithmetic operations (+, $-$, $\times$, $\div$, etc.)

Example: $5x + 2$, $\frac{2y}{3} + 4$, $(3x - 4y^2) + 6$

Term: a term can be a number, letter, or the product (multiplication) of a number and letter. (Terms are separated by addition or subtraction signs.)

Example: **a)** $3x - 4 + \frac{2}{5} + y$ has $\boxed{\text{four}}$ terms: $3x$, -4, $\frac{2}{5}$, and y.

b) $7xyz + 12 - \frac{4}{19}z^2$ has $\boxed{\text{three}}$ terms: $7xyz$, 12, and $-\frac{4}{19}z^2$.

Like terms: the terms that have the same variables and exponents.

Example: $2x - 3y^2 - \frac{6}{7} + 5x + 9 + 4y^2$

$\boxed{\text{Like terms:}}$ $2x$ and $5x$	The same variable: x
$-3y^2$ and $4y^2$	The same variable raised to the same power: y^2
$-\frac{6}{7}$ and 9	All constants are like terms.

Evaluating Algebraic Expressions

Evaluating an algebraic expression: substitute a specific value for a variable and perform the mathematical operations (+, −, ×, ÷, etc.).

Note:

- In algebra, a multiplication sign "×" is usually omitted to avoid confusing it with the letter x.
- If there is no symbol or sign between a number and letter, it means multiplication, such as $3x = 3 \cdot x$.

Steps to evaluate an algebraic expression:

- Replace the variable(s) with number(s).
- Calculate.

Example: Evaluate the following algebraic expressions.

1) $3x - 4$, given $x = 5$.

$3x - 4 = 3 \cdot 5 - 4$ Substitute x for 5.

$= 15 - 4$ Calculate.

$= \boxed{11}$

2) $\frac{x}{y} + 8$ given $x = -9$ and $y = 3$.   Substitute x for -9 and y for 3.

$\frac{x}{y} + 8 = \frac{-9}{3} + 8$

$= \boxed{5}$

3) $3a - 4 + 2$, given $a = 5$.

$3a - 4 + 2 = 3 \cdot 5 - 4 + 2$ Substitute a for 5.

$= 15 - 4 + 2$ Calculate.

$= \boxed{13}$

4) $\frac{6x}{y-3} + 7x - 2$, given $x = 1$ and $y = 9$.

$\frac{6x}{y-3} + 7x - 2 = \frac{6 \cdot \mathbf{1}}{\mathbf{9}-3} + 7 \cdot \mathbf{1} - 2$ Substitute x for 1 and y for 9.

$= \frac{6}{6} + 7 - 2$ Calculate.

$= \boxed{6}$

Topic B: Translating Words into Algebraic Expressions

Key Words in Word Problems

Identifying keywords:

- When trying to figure out the correct operation (+, −, ×, ÷, etc.) in the word problem it is important to pay attention to keywords (clues to what the problem is asking).
- Identifying keywords and pulling out relevant information that appear in the word problem are effective ways for solving mathematical word problems.

Key or clue words in word problems

Addition (+)	Subtraction (−)	Multiplication (×)	Division (÷)	Equals to (=)
add	subtract	times	divided by	equals
sum (of)	difference	product	quotient	is
plus	take away	multiplied by	over	was
total (of)	minus	double	split up	are
altogether	less (than)	twice	fit into	were
increased by	decreased by	triple	per	amounts to
gain (of)	loss (of)	of	each	totals
combined	(amount) left	how much (total)	goes into	results in
in all	savings	how many	as much as	the same as
greater than	withdraw		out of	gives
complete	reduced by		ratio /rate	yields
together	fewer (than)		percent	
more (than)	how much more		share	
additional	how long		average	

Examples:

1) Edward drove from Prince George to Williams Lake (235 km), then to Cache Creek (203 km) and finally to Vancouver (390 km). How many kilometers in ***total*** did Edward drive? 235km + 203 km + 390 km = $\boxed{828 \text{ km}}$ The key word: total (+)

2) Emma had \$150 in her purse on Friday. She bought a pizza for \$15, and a pair of shoes for \$35. How much money does she have ***left***?

\$150 – 15 – 35 = $\boxed{\$100}$ The key word: left (–)

3) Lucy received \$950 per month of rent from Mark for the months September to November. ***How much*** rent in ***total*** did she receive?

\$950 · 3 = $\boxed{\$2850}$ The key word: how much total (×)

4) Julia is going to buy a \$7500 used car from her uncle. She promises to pay \$500 ***per*** month, in how many months she can pay it off?

\$7500 ÷ \$500 = $\boxed{15 \text{ month}}$ The key word: per (÷)

Translating Phrases into Algebraic Expressions

Method to translate words into algebraic expression:

- Look for basic key words for translating word problems from English into algebraic expressions.
- Translate English words into mathematical symbols (the language of mathematics).

Translate words into algebraic expression:

Algebraic expression	Word phrases
$7 + y$	the **sum** of 7 and y
	7 **more than** y
	y **increased by** 7
	7 **plus** y

Algebraic expression	Word phrases
$t - 8$	8 **less than** t
	t **decreased** (or reduced) by 8
	subtract 8 from t
	the **difference** between t and 8

Algebraic expression	Word phrases
$2x$ or $2 \cdot x$	the **product** of 2 and x
	2 **multiplied** by x
	double (or twice) of x

Algebraic expression	Word phrases
$z \div 3$ or $\frac{z}{3}$	The **quotient** of z and 3
	z **divided** by 3
	One third of z

Algebraic expression	Word phrases
y^3	The **third power** of y
	y **cubed**
	y raised to the **third power**

Algebraic expression	Word phrases
$4y - 9$	9 **less than** 4 **times** y
$2(t - 5)$	**Twice** the **difference** of t and 5
$6 + \frac{2x}{3}$	6 **more than** the **quotient** of $2x$ by 3

Note:

- The order of the subtraction and division is important when translate words into algebraic expression.
- Place the numbers in the correct order for subtraction and division.

Example:

1) The difference between t and 7 means $t - 7$ not $7 - t$. t appears first.

2) 8 less than t means $t - 8$ not $8 - t$. 8 less than t not t lees than 8.

3) The quotient of z and 3 means $\frac{z}{3}$ not $\frac{3}{z}$. z appears first.

Writing Algebraic Expressions

Example: Write a mathematical equation for each of the following:

1) Five ***greater than*** four ***divided*** by a number ***is*** seventeen. **Equation**

$5 \quad + \quad 4 \quad \div \quad \boldsymbol{x} \quad = \quad \mathbf{17}$ $\boxed{5 + \frac{4}{x} = 17}$

(Let x = a number)

2) A ***number is*** 7 ***times*** the number y ***added*** to 23.

$x \quad = \quad 7 \quad \cdot \quad y \quad + \quad 23$ $\boxed{x = 7y + 23}$

(Let x = a number)

Example: Write an algebraic expression for each of the following: **Expression**

1) The difference of y and 3.45. $\boxed{y - 3.45}$

2) The difference of $\frac{4}{23}$ and w. $\boxed{\frac{4}{23} - w}$

3) z less than the number 67. $\boxed{67 - z}$

4) 27 minus the product of 18 and ***a number***. $\boxed{27 - 18x}$

(Let x = a number)

5) The sum of ***a number*** and 7 divided by 2. $\boxed{\frac{x+7}{2}}$

6) Steve has \$200 in his saving account. If he makes a deposit of x dollars, how much in total will he have in his account? $\boxed{200 + x}$

7) Ann weighs 150 pounds. If she loses y pounds, how much will she weigh? $\boxed{150 - y}$

8) A piece of wire 30 centimeters long was cut in two pieces and one piece is z centimeters long. How long is the other piece? $\boxed{30 - z}$

9) Alice made 3 dozen cupcakes. If it costs her y dollars, what was her cost per dozen cupcakes? What was her cost per cupcake? $\boxed{\frac{y}{3}}$, $\boxed{\frac{y}{36}}$

(1 dozen = 12 , $3 \cdot 12 = 36$)

Steps for Solving Word Problems

Steps for solving word problems:

Steps for solving word problems
- **Organize** the **facts** given from the problem (create a **table** or **diagram** if it will make the problem clearer). - Identify and label the unknown quantity (**let *x* = unknown**). - Convert words into mathematical symbols, and **determine the operation** – write an **equation** (looking for 'key' or 'clue' words). - **Estimate** and **solve** the equation and find the solution(s). - **Check** and state the **answer**. (Check the solution to the equation and check it back into the problem – is it logical?)

Example to illustrate the steps involved

Example: William bought 5 pairs of socks for \$4.35 each. The cashier charged him an additional \$2.15 in sales tax. He left the store with a measly \$5.15. How much money did William start with?

- Organize the facts (make a table):

5 socks	\$4.35 each
Sales tax	\$2.15
Money left	\$5.15

- Determine the unknown: How much did William start with? (***x* = ?**)
- Convert words into math symbols, and determine the operation (find ***key words***):
  - The ***total*** cost without the sales tax: $\$4.35 \times 5$
  - With an ***additional*** \$2.15 sales tax: $(\$4.35 \times 5) + \$2.15$
  - William started with: $x = [(\$4.35 \times 5) + \$2.15] + \mathbf{\$5.15}$
- Estimate and solve the unknown:
  - Estimate: $x = [(\$4 \times 5) + \$2] + \$5$
    $= \boxed{\$27}$
  - Actual solution: $x = [(\$4.35 \times 5) + \$2.15] + \$5.15$
    $= \boxed{\$29.05}$

- Check: If William started with \$29.05, and subtract 5 socks for \$4.35 each and sales tax in \$2.15 to see if it equals \$5.15.

$$\$29.05 - [(\$4.35 \times 5) + \$2.15] \stackrel{?}{=} \$5.15$$

$$\$29.05 - \$23.9 \stackrel{\surd}{=} \$5.15 \qquad \text{Correct!}$$

- State the answer: William started with $\boxed{\$29.05}$.

More examples:

Example: James had 96 toys. He sold 13 on first day, 32 on second day, 21 on third day, 14 on fourth day and 7 on the last day. What percentage of the toys were not sold?

- Organize the facts:

James had	96 toys
The total number of toys sold	13 + 32 + 21 + 14 + 7
The toys not sold	96 – the total number of toys sold

- Determine the unknown: Let x = percentage of the toys were not sold
- The total number of toys sold: 13 + 32 + 21 + 14 + 7 = 87
- The toys not sold: 96 – 87 = 9
- Percentage of the toys were not sold: $x = \frac{\text{Toys not sold}}{\text{Total number of toys}} = \frac{9}{96} \approx 0.094 = \boxed{9.4\%}$
- State the answer: 9.4% percentage of the toys were not sold.

Example: The 60-liter gas tank in Robert's car is 1/2 full. Kelowna is about 390 km from Vancouver and his car averages 7 liters per 100 km. Can Robert make his trip to Vancouver?

- Let x = liters of fuel are required to get to Vancouver.
- The 60-liter gas tank in Robert's car is 1/2 full:

$$60 \text{ L} \times \frac{1}{2} = 30 \text{ L}$$

Robert has 30 liters gas in his car.

- Robert's car averages 7 liters per 100 km, and Vancouver is about 390 km from Kelowna.

$$\frac{7\text{L}}{100 \text{ km}} = \frac{x}{390 \text{ km}}$$

Proportion: $\frac{a}{b} = \frac{c}{d}$

$$(x)(100\text{km}) = (7 \text{ L})\ (390 \text{ km})$$

Cross multiply and solve for x.

$$x = \frac{(7 \text{ L})(390\text{km})}{100 \text{ km}} = \boxed{27.3 \text{ L}}$$

Robert needs 27.3 liters gas to get to Vancouver.

- State the answer: 30L > 27.3L $\boxed{\text{Yes, Robert can make his trip.}}$

Topic C: Exponents and Order of Operations

Introduction to Exponents

Power: the ***product*** of a number repeatedly multiplied by itself.

Example: $3^2 = 3 \cdot 3 = 9$, the "3^2" is the ***product*** of 3 repeatedly multiplied by itself.

Exponent: the ***number of times*** a number is multiplied by itself.

Example: In $3^{\mathbf{2}}$, the "2" means 3 is multiplied by itself ***two times***.

Base, exponent and power:

$$a^n \begin{cases} a \text{ is the base.} \\ n \text{ is the exponent.} \\ a^n \text{ is the power} \end{cases}$$

Exponential notation (exponential expression): a^n or $\text{Base}^{\text{Exponent}}$

Exponential notation	Example
Power, Exponent, Base: $\boxed{\mathbf{a^n}} = a \cdot a \cdot a \cdot a \ldots a$ Read "*a* to the *n*th" or "the *n*th power of *a*."	$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$ Read "2 to the 4th."

2 is repeatedly multiplied by itself 4 times.

Exponents make it easier to write very long numbers (for multiplications).

Any non-zero number to the zero power equals 1 ($a^0 = 1$).

0^0 is undefined.

Example: $2^0 = \boxed{1}$, $13000^0 = \boxed{1}$

Any number raised to the power of 1 equals the number itself ($a^1 = a$).

Example: $4^1 = \boxed{4}$, $1000^1 = \boxed{1000}$

Anything raised to the first power is itself.

(4 is multiplied by itself one time)

1 raised to any power is still 1 ($1^n = 1$). **Example**: $1^3 = \boxed{1}$, $1^{10000} = \boxed{1}$

$1^3 = 1 \cdot 1 \cdot 1 = 1$

Exponents: basic properties:

Name	Property	Example	
Zero exponent a^0	$a^0 = 1$ (0^0 is undefined)	$(\frac{3}{4})^{\mathbf{0}} = 1$,	$(2xy)^0 = 1$
One exponent a^1	$a^1 = a$	$4.5^1 = 4.5$,	$(3x)^1 = 3x$
	$1^n = 1$	$1^7 = 1$,	$1^{389} = 1$

Read and Write Exponential Expressions

How to read exponent expressions:

Base $^{\text{Exponent}}$	Repeated multiplication	Product	Read	
3^2	$3 \cdot 3$	9	3^2	3 squared
10^3	$10 \cdot 10 \cdot 10$	1000	10^3	10 cubed
$(0.2)^2$	$0.2 \cdot 0.2$	0.04	$(0.2)^2$	0.2 squared
1^{10}	$1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$	1	1^{10}	1 to the tenth
$(\frac{2}{3})^3$	$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$	$\frac{8}{27}$	$(\frac{2}{3})^3$	two thirds cubed
10000^0		1	10000^0	10000 to the zero
y^5	$y \cdot y \cdot y \cdot y \cdot y$	y^5	y^5	y to the fifth

Example: Write the following exponential expressions in expanded form.

Exponential expressions	Expanded form	
1) 6^4	$\boxed{6 \cdot 6 \cdot 6 \cdot 6}$	$a^n = a \cdot a \cdot a \ldots$
2) $(-x)^3$	$\boxed{(-x)\,(-x)\,(-x)}$	
3) $(3x^2y)^2$	$\boxed{(3x^2y)\,(3x^2y)}$	
4) $\left(\frac{3}{4}u\right)^4$	$\boxed{\left(\frac{3}{4}u\right)\left(\frac{3}{4}u\right)\left(\frac{3}{4}u\right)\left(\frac{3}{4}u\right)}$	

Example: Write each of the following in the exponential form.

Expanded form	Exponential notation
1) $(0.2)\,(0.2)\,(0.2)$	$\boxed{(0.2)^3}$
2) $(5a)\,(5a)\,(5a)\,(5a)$	$\boxed{(5a)^4}$
3) $\left(\frac{5}{7}t\right)\left(\frac{5}{7}t\right)$	$\boxed{(\frac{5}{7}t)^2}$

Example: Evaluate $(4^2)\,(3^3)\,(6^0)\,(9^1)$.

$$4^2 \cdot 3^3 \cdot 6^0 \cdot 9^1 = (4 \cdot 4)\,(3 \cdot 3 \cdot 3)\,(1)\,(9)$$

$$= 16 \cdot 27 \cdot 1 \cdot 9 = \boxed{3888}$$

$a^0 = 1$, $a^1 = a$

Example: Write each of the following as a base with an exponent.

1) Six to the power of eight. $\boxed{6^8}$

2) x to the seventh power. $\boxed{x^7}$

3) Eight cubed. $\boxed{8^3}$

Example: Evaluate $\frac{6x^2}{y+3} + 7x - 2$, given $x = 2$ and $y = 9$.

$$\frac{6x^2}{y+3} + 7x - 2 = \frac{6 \cdot 2^2}{9+3} + 7 \cdot 2 - 2$$ Substitute x for 2 and y for 9.

$$= \frac{24}{12} + 14 - 2 = \boxed{14}$$ Calculate.

Order of Operations

Basic operations: addition, subtraction, multiplication, division, exponent, etc.

The order of operations are the rules of which calculation comes first in an expression (when doing expressions with more than one operation).

Order of operations:

Order of operations		
1.	the brackets or parentheses (innermost first)	( ) , [ ] , { }
2.	exponent (power)	a^n
3.	multiplication and division (from left-to-right)	× and ÷
4.	addition and subtraction (from left-to-right)	+ and −

Example:

$$
\begin{aligned}
4 \cdot 3^2 + 5 + \mathbf{(2 + 1)} - 2 &= 4 \cdot \mathbf{3^2} + 5 + 3 - 2 && (\ ),\ a^n \\
&= \mathbf{4 \cdot 9} + 5 + 3 - 2 && \times \\
&= \mathbf{36 + 5} + 3 - 2 && + \\
&= \mathbf{41 + 3} - 2 && + \\
&= 44 - 2 && - \\
&= \boxed{42}
\end{aligned}
$$

Memory aid - BEDMAS

B	E	DM	AS
Brackets	**E**xponents	**D**ivide or **M**ultiply	**A**dd or **S**ubtract

Grouping symbols: if parentheses are inside one another, calculate the inside set first.

- Parentheses ( ) are used in the inner most grouping.
- Square brackets [ ] are used in the second higher level grouping.

Example:

$$
\begin{aligned}
4 \cdot 3 + [5 + \mathbf{(2 + 1)}] - 3^2 &= 4 \cdot 3 + \mathbf{[5 + 3]} - 3^2 && (\ ),[\ ] \\
&= 4 \cdot 3 + 8 - \mathbf{3^2} && a^n \\
&= \mathbf{4 \cdot 3} + 8 - 9 && \times \\
&= \mathbf{12 + 8} - 9 && + \\
&= 20 - 9 && - \\
&= \boxed{11}
\end{aligned}
$$

Unit 2: Summary

Introduction to Algebra

Basic algebraic terms

Algebraic term	Description	Example
Algebraic expression	A mathematical phrase that contains numbers, letters, grouping symbols (parentheses) and arithmetic operations.	$5x + 2$, $3a + (4b - 6)$, $\frac{2}{3} + 4$
Constant	A number.	$x + \boldsymbol{2}$ constant: 2
Variable	A letter that can be assigned different values.	$3 - \boldsymbol{x}$ variable: x
Coefficient	The number in front of a variable.	$\boldsymbol{-6}\,x$ coefficient: -6 x coefficient: 1
Term	A term can be a constant, variable, or the product of a number and variable(s). (Terms are separated by addition or subtraction signs.)	$3x - \frac{2}{5} + 13y^2 + 7xy$ Terms: $3x$, $-\frac{2}{5}$, $13y^2$, $7xy$
Like terms	The terms that have the same variables and exponents.	$2x - y^2 - \frac{2}{5} + 5x - 7 + 13y^2$ Like terms: $2x$ and $5x$ $-y^2$ and $13y^2$, $-\frac{2}{5}$ and -7

Evaluating an algebraic expression: substitute a specific value for a variable and perform the mathematical operations (+, −, ×, ÷, etc.).

To evaluate an expression:

- Replace the variable(s) with number(s).
- Calculate.

Key or clue words in word problems:

Addition (+)	Subtraction (−)	Multiplication (×)	Division (÷)	Equals to (=)
add	subtract	times	divided by	equals
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total (of)	minus	double	split up	are
altogether	less (than)	twice	fit into	were
increased by	decreased by	triple	per	amounts to
gain (of)	loss (of)	of	each	totals
combined	balance	how much (total)	goes into	results in
entire	(amount) left	how many	as much as	the same as
in all	savings		out of	gives
greater than	withdraw		ratio (of)	yields
complete	reduced by		percent	
together	fewer (than)		share	
more (than)	how much more		distribute	
and	how many extra		average	
additional	how long			

Steps for solving word problems:

Steps for solving word problems
- ***Organize*** the ***facts*** given from the problem (create a ***table*** or ***diagram*** if it will make the problem clearer). - Identify and label the unknown quantity (***let x = unknown***). - Convert words into mathematical symbols, and ***determine the operation*** – write an ***equation*** (looking for 'key' or 'clue' words). - ***Estimate*** and **solve** the equation and find the solution(s). - ***Check*** and state the ***answer***. (Check the solution with the equation and check it back into the problem – is it logical?)

Power: the ***product*** of a number repeatedly multiplied by itself.

Exponent: the ***number of times*** a number is multiplied by itself.

Base, exponent and power:

$$a^n \begin{cases} a \text{ is the base.} \\ n \text{ is the exponent.} \\ a^n \text{ is the power} \end{cases}$$

Exponential notation (exponential expression): a^n or $\text{Base}^{\text{Exponent}}$

Exponential notation	Example
Power Exponent $\boxed{a^n} = a \cdot a \cdot a \cdot a \ldots a$ Base Read "*a* to the *n*th" or "the *n*th power of *a*."	$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$ Read "2 to the 4th."

Exponents: basic properties:

Name	Property
Zero Exponent a^0	$a^0 = 1$ (0^0 is undefined)
One Exponent a^1	$a^1 = a$
	$1^n = 1$

Order of operations:

Order of operations		
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Memory aid - BEDMAS

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Brackets	**E**xponents	**D**ivide or **M**ultiply	**A**dd or **S**ubtract

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- Parentheses ( ) are used in the inner most grouping.
- Square brackets [ ] are used in the second higher level grouping.

Unit 2: Self-Test

Introduction to Algebra

Topic A

1. Identify the constant, coefficient and the variable:

a) $2x - 3$

b) $-4t + 13 + \frac{5}{7}t$

2. Identify the terms for each of the following:

a) $5x + 3 - y$

b) $2r + 16r^2 - \frac{3}{14}r + 1$

3. Identify the like terms in the following expressions:

a) $7 + 2y^2 - \frac{5}{9}x + 5x - 1 + 13y^2$

b) $0.6t + 9uv - 7t + 1.67uv$

4. Evaluate the following algebraic expressions.

a) $7x - 4 + 13x$, given $x = 4$.

b) $\frac{3}{a-7} + 9b + 12$, given $a = 10$ and $b = 5$.

Topic B

5. Write an expression/equation for each of the following:

a) The product of ten and *y*.

b) The quotient of *t* and six.

c) The difference between fifteen and a number more than the quotient of three by seven is six.

d) Seven less than six times a number is fifteen.

6. Write an expression for each of the following:

a) Susan has $375 in her checking account. If she makes a deposit of *y* dollars, how much in total will she have in her account?

b) Mark weighs 175 pounds. If he loses y pounds, how much will he weigh?

c) A piece of wire 45 meters long was cut in two pieces and one piece is w meters long. How long is the other piece?

d) Emily made 4 dozen muffins. If it cost her x dollars, what was her cost per dozen muffins? What was her cost per muffin?

Topic C

7. **a)** In x^3, the base is ( ).

b) In y^4, the exponent is ( ).

8. Write the following exponential expressions in expanded form.

a) 9^3

b) $(-y)^4$

c) $(0.5a^3b)^2$

d) $\left(\frac{2}{7}x\right)^1$

9. Write each of the following in the exponential form.

a) (0.06) (0.06) (0.06) (0.06)

b) $(12y)\,(12y)\,(12y)$

c) $\left(\frac{-2}{9}x\right)\left(\frac{-2}{9}x\right)$

10. Evaluate $(3^2)\,(2^4)\,(23^0)(10^1)$.

11. Write each of the following as a base with an exponent.

a) y to the eighth power.

b) Five cubed.

12. Evaluate the following:

a) $\frac{9a^2}{b+6} + 3a + 4$, if $a = 1$ and $b = 3$.

b) $8xy + 7y^4$, if $x = \frac{1}{4}$ and $y = 1$.

13. Calculate the following:

a) $2 \cdot 4^3 + 7 - (4 + 3) + 5$

b) $5 \cdot 7 + [11 + (4 - 3)] + 4^2$

c) $\frac{104 - 4^2}{6 + 5}$

Unit 3

Introduction to Geometry

Topic A: Perimeter, area, and volume

- Perimeter of plane figures
- Circle
- Perimeter
- Perimeters of irregular / composite shapes

Topic B: Area

- Areas of quadrilaterals and circles
- Arears of irregular / composite shapes

Topic C: Volume

- Volume of solids

Topic D: Surface and lateral area

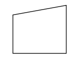
- Surface and lateral area – rectangular solids
- Surface and lateral area – cylinders, cones and spheres

Unit 3 Summary

Unit 3 Self - test

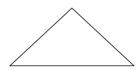
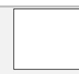
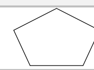
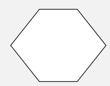
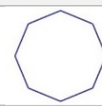
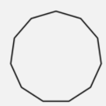
Topic A: Perimeter, Area, and Volume

Perimeter of Plane Figures

Polygon: a closed figure made up of three or more line segments. 

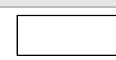
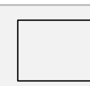
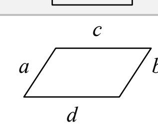
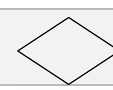
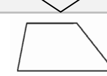
Regular polygon: a polygon that has all angles equal and all sides equal.

Classify regular polygons):

Number of sides	Name of polygon	Figure
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
8	Octagon	
10	Decagon	

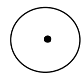
Quadrilateral: a four-sided polygon.

Classify quadrilaterals:

Name of quadrilateral	Definition	Figure
Rectangle	A four-sided figure that has four right angles (90^0).	
Square	A four-sided figure that has four equal sides and four right angles.	
Parallelogram	A four-sided figure that has opposite sides parallel (//) and equal. ($a \mathbin{//} b$, $c \mathbin{//} d$; $a = b$, $c = d$)	
Rhombus (diamond)	A four-sided figure that has four equal sides, but no right angle.	
Trapezoid	A four-sided figure that has one pair of parallel sides.	

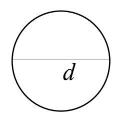
Circle

Circle: a round shape bounded by a curved line that is always the same distance from the center.

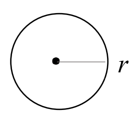


Circumference (*C*): the line bounding the edge of a circle.

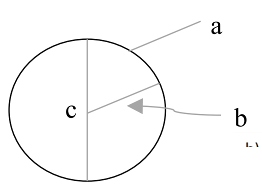
Diameter (*d*): a straight line between any two points on the circle through the center of the circle.



Radius (*r*): a straight line between any point on the circle to the center of the circle (half of the diameter, $r = \frac{1}{2}d$ (or $d = 2r$).



Example: Identify the parts of a circle (what is a, b and c?).



a. Circumference b. Radius c. Diameter

Example:

1) Find the radius of a circle with a diameter of 12 meters.

$d = 12$ m , $r = \frac{1}{2}d = \frac{1}{2} \cdot 12 \text{ m} = \boxed{6 \text{ m}}$

2) If the radius of a circle is 15 meters, what is the diameter of this circle?

$d = 2r = 2 \cdot 15\ m = \boxed{30 \text{ m}}$

Perimeter

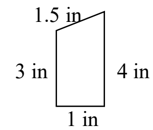
Perimeter (P): the total length of the outer boundary of a figure.

Find the perimeter: add together the length of each side.

Example: To find the perimeter (P) of the following figure, add the lengths of all 4 sides.

$P = 3 \text{ in} + 1 \text{ in} + 4 \text{ in} + 1.5 \text{ in}$

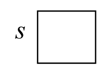
$= \boxed{9.5 \text{ in}}$



The perimeter of any regular (equal sided) polygon: the number of sides (n) times the length of any side (s) of that polygon. $P = ns$

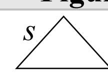
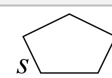
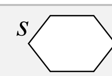
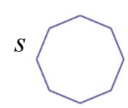
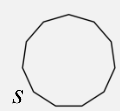
Example: The perimeter (P) of a square is $P = 4\ s$

4 sides



Units of perimeter: the meter (m), centimeter (cm), foot (ft), inch (in), yard (yd), etc.
(The same units as length.)

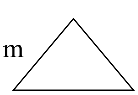
The perimeter of regular polygons: s – the length of the side

Name of the figure	Perimeter ($P = ns$)	Figure
Equilateral triangle (A triangle with three equal sides.)	$P = 3\ s$	
Square	$P = 4\ s$	
Pentagon	$P = 5\ s$	
Hexagon	$P = 6\ s$	
Octagon	$P = 8\ s$	
Decagon	$P = 10\ s$	

Example:

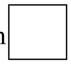
1) What is the perimeter (P) of the following triangle?

$s = 3.5$ m

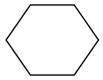


$P = 3\ s = (3)\ (3.5 \text{ m}) = \boxed{10.5 \text{ m}}$

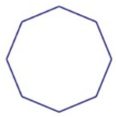
2) What is the perimeter (P) of the following square?

s = 2.3cm  $P = 4\,s = 4\,(2.3 \text{ cm}) = \boxed{9.2 \text{ cm}}$

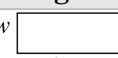
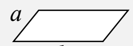
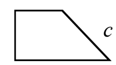
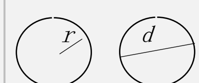
3) What is the perimeter (P) of the following hexagon?

s = 5ft  $P = 6\,s = (6)\,(5 \text{ ft}) = \boxed{30 \text{ ft}}$

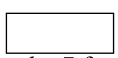
4) What is the perimeter (P) of the following octagon?

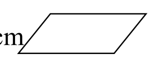
$s = \frac{3}{4}$ yd  $P = 8\,s = 8 \cdot \frac{3}{4} \text{ yd} = \boxed{6 \text{ yd}}$

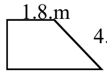
The perimeter of some basic geometric shapes:

Name of the figure	Perimeter formula	Figure
Rectangle	$P = 2\,w + 2\,l$ (w – width, l – length)	w  l
Parallelogram	$P = 2\,a + 2\,b$ (a and b – the length of the sides)	a  b
Trapezoid	$P = a + b + c + d$	b a  c d
Circle (The perimeter of the circle is its circumference C)	$C = \pi d$ or $C = 2\pi r$ ($\pi \approx 3.14$) π (pi) is the ratio of circle's circumference C to its diameter d, that is approximately 3.14. ($\pi = \frac{\text{Circumference}}{\text{diameter}} = \frac{C}{d} \approx 3.14159265359 \ldots$)	

Example: What is the perimeter (P) of the following polygons?

1) w = 5ft  l = 7 ft $P = 2\,w + 2\,l = 2\,(5 \text{ ft}) + 2\,(7 \text{ ft}) = \boxed{24 \text{ ft}}$

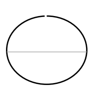
2) a = 3.4cm  b = 5.2cm $P = 2a + 2b = 2\,(3.4 \text{ cm}) + 2\,(5.2 \text{ cm}) = \boxed{17.2 \text{ cm}}$

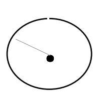
3) 

$P = a + b + c + d$

$= 2.4 \text{ m} + 1.8\text{m} + 4.3 \text{ m} + 5.8\text{m} = \boxed{14.3\text{m}}$

Example: What are the circumferences (C) of the circles shown below?

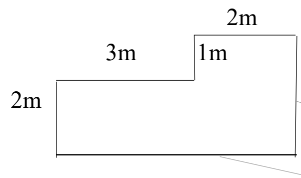
1)  d=5cm $C = \pi d \approx (3.14)(5\text{cm}) = \boxed{15.7 \text{ cm}}$

2)  r =2.8cm $C = 2\pi r \approx 2(3.14)(2.8\text{cm}) \approx \boxed{17.58 \text{ cm}}$

Perimeters of Irregular / Composite Shapes

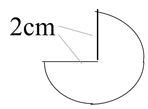
Example: What are the perimeters (P) of the following figures?

1)



$P = 2\text{m} + 3\text{m} + 1\text{m} + 2\text{m} + (3\text{m} + 2\text{m}) + (1\text{m} + 2\text{m}) = \boxed{16\text{ m}}$

2)



P is equal to $\frac{3}{4}$ of the circumference of the circle ($C = 2\pi r$) and two sides with 2m.

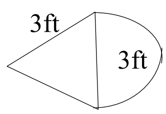
$$P = (2\text{cm} + 2\text{cm}) + \frac{3}{4}\,(2\pi r)$$

$$= 4\text{ cm} + \frac{3}{4}\,(2\pi \cdot 2\text{cm}).$$

$r = 2\text{cm}$

$$\approx \boxed{13.42\text{ cm}}$$

3)



P is equal to $\frac{1}{2}$ of the circumference of the circle and two sides with 3ft.

$$P = (3\,ft + 3ft) + \frac{1}{2}\,(\pi d)$$

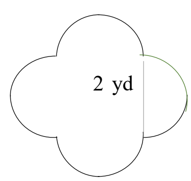
$C = \pi d$

$$= 6\text{ ft} + \frac{1}{2}\,(\pi \cdot 3\text{ ft})$$

$d = 3\text{ft}$ (An equilateral triangle.)

$$\approx \boxed{10.71\text{ ft}}$$

4)



$$P = 4 \cdot \frac{1}{2}\,(\pi d)$$

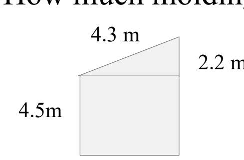
P is the circumference of 4 half circles.

$$= 4 \cdot \frac{1}{2}\,(\pi \cdot 2\text{ yd})$$

$C = \pi d$, $d = 2$ yd

$$\approx \boxed{12.57\text{ yd}}$$

Example: Damon is renovating his living room that is the shape indicated in the diagram below. He wishes to put molding around the base of the walls of the living room. How much molding does he need?



$$P = 3(4.5\text{m}) + 2.2\text{m} + 4.3\text{ m} = \boxed{20\text{ m}}$$

Topic B: Area

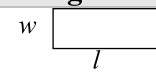
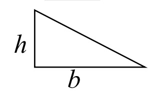
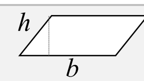
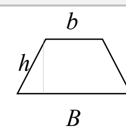
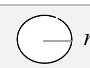
Areas of Quadrilaterals and Circles

Area (A): the size of the outermost surface of a shape (space within its boundaries).

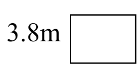
Units of area: the units of measurement of area are always expressed as square units.

Such as square meter (m^2), square centimeter (cm^2), square foot (ft^2), square inch (in^2), square yard (yd^2), etc.

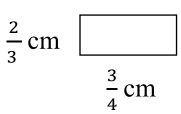
Areas of some basic geometric shapes:

Name of the figure	Area formula (A)	Figure
Rectangle	$A = wl$ (w – width, h – height)	
Square	$A = s^2$ (s – the length of the side)	
Triangle	$A = \frac{1}{2}bh$ (b - base, h – height)	
Parallelogram	$A = bh$ (b - base, h - height)	
Trapezoid	$A = \frac{1}{2}h(b + B)$ (b-upper base, B-lower base, h- height)	
Circle	$A = \pi r^2$ (r - radius, $\pi \approx 3.14$)	

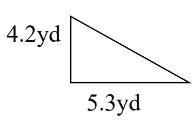
Example: What are the areas (A) of the following figures?

1) 

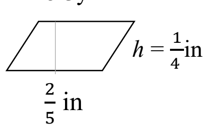
$A = s^2 = (3.8 \text{ m})(3.8 \text{ m}) = \boxed{14.44 \text{ m}^2}$ $\quad \text{m} \cdot \text{m} = \text{m}^2$

2) 

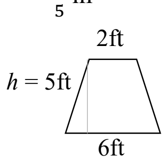
$A = w\,l = (\frac{2}{3}\text{ cm})(\frac{3}{4}\text{ cm}) = \boxed{\frac{1}{2}\text{ cm}^2}$ $\quad \text{cm} \cdot \text{cm} = \text{cm}^2$

3) 

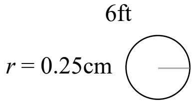
$A = \frac{1}{2}bh = \frac{1}{2}(5.3 \text{ yd})(4.2 \text{ yd}) = \boxed{11.13 \text{ yd}^2}$

4) 

$A = bh = (\frac{2}{5}\text{ in})(\frac{1}{4}\text{ in}) = \boxed{\frac{1}{10}\text{ in}^2}$

5) 

$A = \frac{1}{2}h(b + B) = \frac{1}{2}5ft(2\text{ft} + 6\text{ft}) = \boxed{20 \text{ ft}^2}$

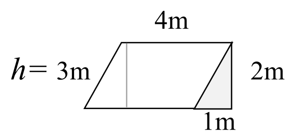
6) 

$A = \pi r^2 \approx (3.14)(0.25\text{cm})^2 \approx \boxed{0.2 \text{ cm}^2}$

Areas of Irregular / Composite Shapes

Example: Find the areas (A) of the following figures.

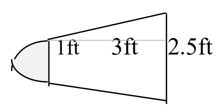
1)



Total area = Area of parallelogram + Area of triangle

$$\boldsymbol{A} = (bh) + (\frac{1}{2}bh) = (3m)(4m) + \frac{1}{2}(1m)(2m) = 12m^2 + 1m^2 = 13m^2$$

2)

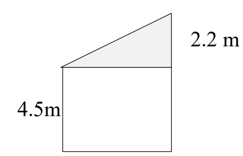


Total area = Area of trapezoid + Area of ($\frac{1}{2}$) circle

(d = 1ft, $r = \frac{1}{2}d = 0.5$ ft)

$$\text{A} = [\frac{1}{2}h(b+B)] + (\frac{1}{2}\pi r^2) = [\frac{1}{2}(3\text{ ft})(1\text{ ft} + 2.5\text{ ft}) + \frac{1}{2}(3.14)(0.5\text{ ft})^2 \approx 5.64\text{ ft}^2$$

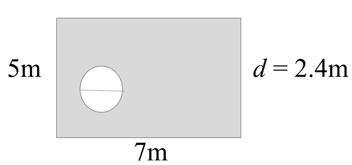
Example: Damon is renovating his living room that is the shape indicated in the diagram below. He wishes to purchase new flooring. How much does he need to order to cover the entire living room floor?

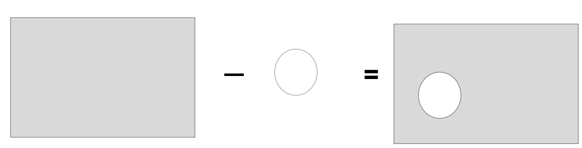


Total area = Area of square + Area of triangle

$$A = S^2 + \frac{1}{2}bh = (4.5\text{ m})^2 + \frac{1}{2}(4.5\text{ m})(2.2\text{m}) = \boxed{25.2\text{ m}^2}$$

Example: William built a wooden deck at the back of his home. It is shown in the following diagram. He decides to insert a circular hot tub that has a diameter of 2.4 m. Calculate the area of the remaining exposed wooded floor of the deck.





Shaded area = Area of rectangle − Area of circle

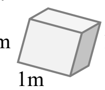
(d = 2.4m, $r = \frac{1}{2}d = 1.2$m)

$$A = (wl) - (\pi r^2) = (5\text{m})(7\text{m}) - (3.14)(1.2\text{m})^2 \approx \boxed{30.48\text{ m}^2}$$

Topic C: Volume

Volume of Solids

Volume (V): the amount of space a solid object (three-dimensional) occupies.

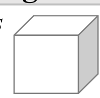
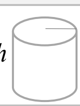
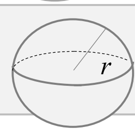
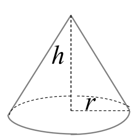
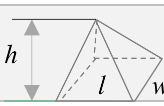


Example: the volume of a can of food is the amount of food inside.

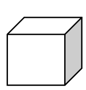
Units of volume: the units of measurement of volume are always expressed as cubic units.

Such as the cubic meter (m^3), cubic centimeter (cm^3), cubic foot (ft^3), cubic inch (in^3), cubic yard (yd^3), etc.

Volumes of basic geometric shapes:

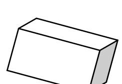
Name	Figure	Volume formula (V)
Cube		$V = s^3$ (s – the length of the side)
Rectangular solid		$V = w\,l\,h$ (w – width, l – length, h - height)
Cylinder		$V = \pi r^2 h$ (r - radius, h – height, $\pi \approx 3.14$)
Sphere		$V = \frac{4}{3}\pi r^3$ (r- radius)
Cone		$V = \frac{1}{3}\pi r^2 h$ (r - radius, h - height)
Pyramid		$V = \frac{1}{3} wlh$ (w – width, l – length, h - height)

Example: Find the volumes (V) of the following figures.

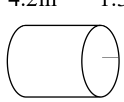
1) 

$V = s^3 = (1.4\text{ m})\,(1.4\text{ m})\,(1.4\text{ m})$

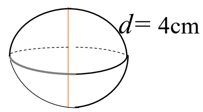
$= (1.4\text{ m})^3 = \boxed{2.744\text{ m}^3}$ $\qquad \text{m} \cdot \text{m} \cdot \text{m} = \text{m}^3$

2) 

$V = w\,l\,h = (4.2\text{in})\,(1.3\text{in})\,(2.4\text{in}) \approx \boxed{13.1\text{ in}^3}$ $\qquad \text{in} \cdot \text{in} \cdot \text{in} = \text{in}^3$

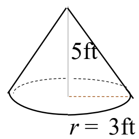
3) 

$V = \pi\, r^2 h = \pi (3m)^2 (8m) \approx \boxed{226.2\text{ m}^3}$

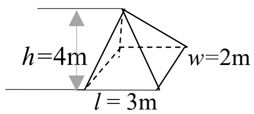
4) 

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2\text{cm})^3 \approx \boxed{33.51\text{ cm}^3}$

($d = 4$ cm, $r = \frac{1}{2}d = 2$ cm)

5) 

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3\text{ft})^2(5\text{ft}) \approx \boxed{47.1\text{ ft}^3}$

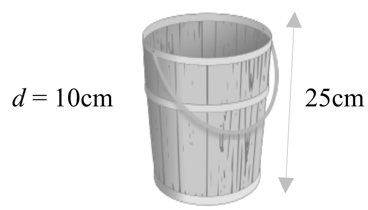
6) 

$V = \frac{1}{3}wlh = \frac{1}{3}(2\text{m})(3\text{m})(4\text{m}) = \boxed{8\text{ m}^3}$

7) Determine the amount of water that will fill the following bucket.

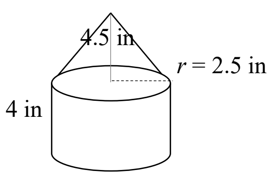
$V = \pi r^2 h = \pi(5)^2(25\text{cm}) \approx \boxed{1963.5\text{ cm}^3}$

($d = 10$ cm, $r = \frac{1}{2}d = 5$ cm)



Volume of composite shapes

Example: Find the volume (V) of the following figure.

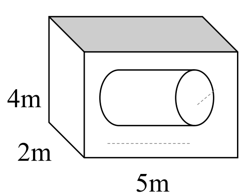


Total volume = <u>Volume of the cylinder</u> + <u>Volume of the cone</u>

$V = (\pi r^2 h) + (\frac{1}{3}\pi r^2 h) = [\pi(2.5\text{ in})^2(4\text{ in})] + [\frac{1}{3}\pi\,(2.5\text{ in})^2(4.5\text{ in})]$

$= \boxed{107.99\text{ in}^3}$

Example: Find the volumes (V) of the following figure (a rectangular solid with a cylinder removed from inside).



(Cylinder: $h = 4$m, $r = 1$m)

Unknown volume = <u>Volume of the rectangular solid</u> − <u>Volume of the cylinder</u>

$V = (wlh) - (\pi r^2 h) = [(2\text{m})(5\text{m})(4\text{m})] - [\pi(1\text{m})^2(4\text{m})] \approx \boxed{27.43\text{ m}^3}$

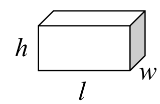
Topic D: Surface and Lateral Area

Surface and Lateral Area
– Rectangular Solids

Surface area (SA): the total area on the surface of a solid object (a three-dimensional object).

Lateral area (LA): the surface area of a solid object excluding its top and bottom.

Lateral area (LA) of a rectangular solid: the sum of the surface areas of the four sides excluding its top and bottom.



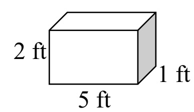
LA of a rectangular solid = front side + back side + 2 sides

$= 2\,(l\,h) + 2\,(wh)$

The front and back sides. The left and right sides.

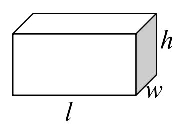
(w – width, l – length, h - height)

Example: Determine the lateral area (LA) of the rectangular solid.



$$\text{LA} = 2(5\text{ft} \cdot 2\text{ft}) + 2(1\text{ft} \cdot 2\text{ft})$$
$$= 20\text{ ft}^2 + 4\text{ ft}^2$$
$$= \boxed{24\text{ ft}^2}$$

Surface area (SA) of a rectangular solid: the sum of the areas of the top, bottom and the four sides.



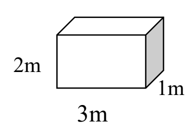
SA of a rectangular solid = top area + bottom area + 4 sides

$= (l\,w) + (l\,w) + 2(lh) + 2(wh)$

$= 2\,(l\,w) + 2(lh) + 2(wh)$ (w – width, l – length, h - height)

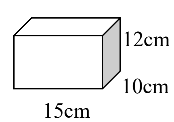
The top & bottom. The front and back sides. The left and right sides.

Example: Determine the SA of the rectangular solid.



$$\text{SA} = 2\,(3\text{m} \cdot 1\text{m}) + 2(3\text{m} \cdot 2\text{m}) + 2(1\text{m} \cdot 2\text{m})$$
$$= 6\text{m}^2 + 12\text{m}^2 + 4\text{m}^2 = \boxed{22\text{m}^2}$$

Example: How many square centimeters of glass are needed to make a fish tank which is 15 cm long by 10 cm wide by 12 cm high if the top is left open?



$$A = 2\,(15\text{cm} \cdot 12\text{m}) + 2(12\text{cm} \cdot 10\text{cm}) + (15\text{cm} \cdot 10\text{cm}) = \boxed{750\text{ cm}^2}$$

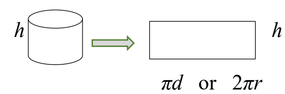
The front and back sides. The left and right sides. The bottom part.

Surface and Lateral Area
– Cylinders, Cones and Spheres

Cylinders

- Lateral area (LA) of a cylinder: the area of the the rectangular side that wraps around the cylinder's side (the rectangular side folded around).

  LA of a cylinder = πdh or $2\pi rh$



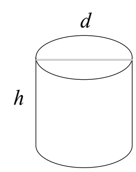
  - Imagine a fruit can that is cut down the side and rolled flat.

  - Recall: the circumference of a circle $C = \pi d$ or $2\pi r$ (r – radius, d – diameter)

- Surface area (SA) of a cylinder: the sum of the surface areas of the top, bottom and the side (the lateral area).

  SA of a cylinder = top area + bottom area + LA of a cylinder

  SA of a cylinder = $2(\pi r^2) + \pi dh$



  Recall: the area of a circle: $A = \pi r^2$

  (r – radius, d – diameter, h - height)

Example: Determine the lateral area and surface area of the following cylinder.

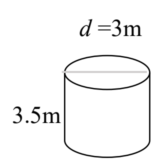
$$LA = \pi dh = \pi\,(3\text{m})(3.5\text{m}) \approx \boxed{32.99\ \text{m}^2}$$

$$SA = 2(\pi r^2) + \pi dh$$

$$= 2[\pi\,(1.5\ \text{m})^2] + 32.99\ \text{m}^2 \qquad d = 3\text{m},\ \ r = \tfrac{1}{2}d = 1.5\text{m}$$

$$\approx 14.137\ \text{m}^2 + 32.99\ \text{m}^2$$

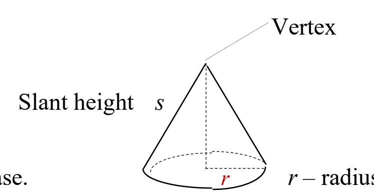
$$\approx \boxed{47.13\ \text{m}^2}$$



Cones

- Lateral area of a cone:

  LA of a cone = (π) (radius) (slant height) = πrs



  Slant height (s): the height from the vertex to a point on the circle base.

  (r – radius, s – slant height)

- Surface area (SA) of a cone:

SA of a cone = LA of a cone + area of the circular base (a circle)

$\boxed{\text{SA of a cone} = \pi rs + \pi r^2}$ s - slant height, r - radius

Example: Determine the lateral area and total area of a cone whose diameter is 2m and slant height is 4m.

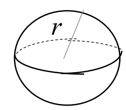
$$\text{LA} = \pi rs = \pi\,(1\text{m})(4\text{m}) \approx \boxed{12.57\ \text{m}^2}$$ $d = 2\text{m}$, $r = \frac{1}{2}d = 1\text{m}$

$$\text{SA} = \pi rs + \pi r^2 = 12.57\ \text{m}^2 + \pi\,(1m)^2 \approx \boxed{15.71\ \text{m}^2}$$

Spheres

Surface area (SA) of a sphere:

$\boxed{\text{SA of a sphere} = 4\pi r^2}$ r - radius



Example: Determine the surface area of a sphere whose radius is 4.5cm.

$$\text{SA} = 4\pi r^2 = 4\pi\,(4.5\text{cm})^2 \approx \boxed{254.47\ \text{cm}^2}$$

Example: Mary wishes to paint 5 balls with green paint. The diameter of each ball is 18 cm. What area should Mary tell the paint store she needs to cover?

$$\text{SA} = 4\pi r^2 = 4\pi\,(9\text{cm})^2 \approx \boxed{1017.88\ \text{cm}^2}$$ (The surface area of one ball)

d = 18 cm, $r = \frac{1}{2}d$ = 9 cm

$$5\,(\text{SA}) = 5\,(1017.88\ \text{cm}^2) = \boxed{5089.4\ \text{cm}^2}$$ (The surface area of 5 balls)

Surface and lateral area summary:

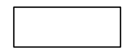
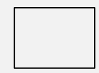
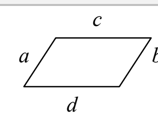
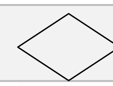
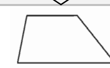
Figure	Lateral area (LA)	Surface area (SA)
Rectangular Solid	Front side + back side + 2 sides $2(l\,h) + 2(wh)$	Top area + bottom area + 4 sides $(l\,w) + (l\,w) + 2(lh) + 2(wh)$
Cylinder	πdh or $2\pi rh$	$2(\pi r^2) + \pi dh$
Cone	πrs	$\pi rs + \pi r^2$
Sphere		$4\pi r^2$

There is no difference between lateral area and surface area in a sphere.

Unit 3: Summary

Introduction to Geometry

Classify quadrilaterals (four-sided shapes):

Name of quadrilateral	Definition	Figure
Rectangle	A four-sided figure that has four right angles (90^0).	
Square	A four-sided figure that has four equal sides and four right angles.	
Parallelogram	A four-sided figure that has opposite sides parallel (//) and equal. ($a \mathbin{//} b$, $c \mathbin{//} d$; $a = b$, $c = d$)	
Rhombus (diamond)	A four-sided figure that has four equal sides, but no right angle.	
Trapezoid	A four-sided figure that has one pair of parallel sides.	

Terms of geometry:

Term	Definition
Perimeter (*P*)	The total length of the outer boundary of a shape.
Circumference (*C*)	The line bounding the edge of a circle.
Diameter (*d*)	A straight line between any two points on the circle through the center of the circle.
Radius (*r*)	A straight line between any point on the circle to the center of the circle (half of the diameter, $r = \frac{1}{2}d$ or $d = 2r$).
Area (*A*)	The size of the outermost surface of a shape.
Volume (*V*)	The amount of space a solid object (3D) occupied.
Surface area (SA)	The total area on the surface of a solid object (a 3D object).
Lateral area (LA)	The surface area of a solid object excluding its top and bottom.

Units of perimeter: the meter (m), centimeter (cm), foot (ft or'), inch (in or"), yard (yd), etc.
The same units as length.

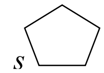
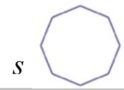
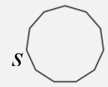
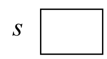
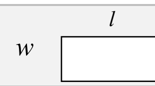
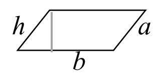
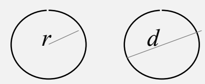
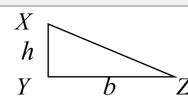
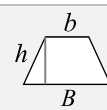
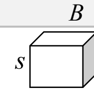
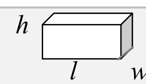
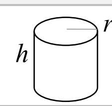
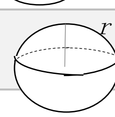
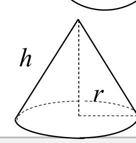
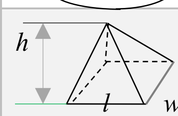
Units of area: the units of measurement of area are always expressed as square units.

Units of volume: the units of measurement of volume are always expressed as cubic units.

Surface and lateral area summary:

Figure	Lateral area (LA)	Surface area (SA)
Rectangular Solid	Front side + back side + 2 sides $2(l\,h) + 2(wh)$	Top area + bottom area + 4 sides $(l\,w) + (l\,w) + 2(lh) + 2(wh)$
Cylinder	πdh or $2\pi rh$	$2(\pi r^2) + \pi dh$
Cone	πrs	$\pi rs + \pi r^2$
Sphere		$4\pi r^2$

Geometry formulas: *s* – side, *P* – perimeter, *C* – Circumference, *A* – area, *V* – volume

Name of the figure	Formula	Figure
Equilateral triangle	$P = 3s$	
Pentagon	$P = 5s$	
Hexagon	$P = 6s$	
Octagon	$P = 8s$	
Decagon	$P = 10s$	
Square	$P = 4s$ $A = s^2$	
Rectangle	$P = 2w + 2l$ $A = wl$	
Parallelogram	$P = 2a + 2b$ $A = bh$	
Circle	$C = \pi d = 2\pi r$ $A = \pi r^2$	
Triangle	$< X + < Y + < Z = 180^0$ $A = \frac{1}{2}bh$	
Trapezoid	$A = \frac{1}{2}h(b + B)$	
Cube	$V = s^3$	
Rectangular solid	$V = w\,l\,h$	
Cylinder	$V = \pi r^2 h$	
Sphere	$V = \frac{4}{3}\pi r^3$	
Cone	$V = \frac{1}{3}\pi r^2 h$	
Pyramid	$V = \frac{1}{3}wlh$	

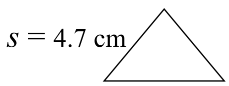
Unit 3: Self - Test

Introduction to Geometry

Topic A

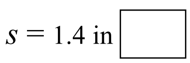
1. Find the radius of a circle with a diameter of 42 centimeters.

2. What is the perimeter (P) of the following triangle?

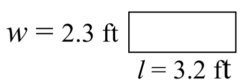


3. What is the perimeter (P) of the following polygons?

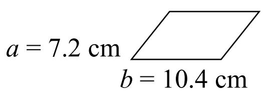
a)



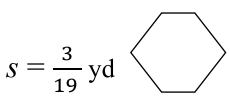
b)



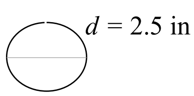
c)



d)

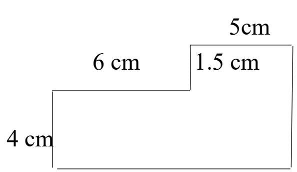


4. What is the circumferences (C) of the circle shown below?

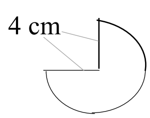


5. What are the perimeters (P) of the following figures?

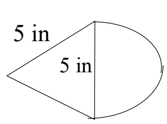
a)

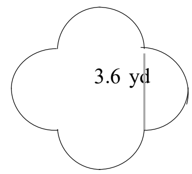


b)



c)



d) 

6. A flower bed in the shape of a parallelogram has sides of 5.5 inches and 3.4 inches. What is its perimeter?

7. The floor of a rectangular room measures 5.2 m by 4.3 m. The doorway is 1 m wide. Baseboard is to be installed around the perimeter of the room, except in the doorway. What length of baseboard needs to be purchased?

8. Tom's rectangular yard is 10 meters wide and 15 meters long.

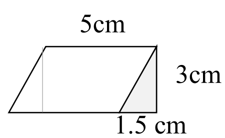
a. If Tom wants to fence the whole lot, how many meters of fencing would Tom has to buy?

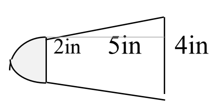
b. If the fencing cost $15 per meter, estimate the cost of fencing the yard.

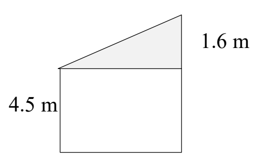
9. A rectangular swimming pool is 8 m long and 4 m wide. It is surrounded by concrete deck 1.5 m wide on all sides. Find the outside perimeter of the deck.

Topic B

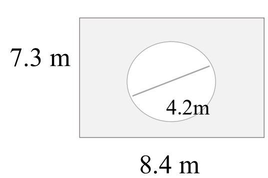
10. Find the areas of the following figures.

a) 

b) 

c) 

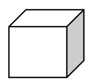
11. Find the area (A) of the shaded area in the following figure.

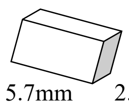


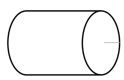
12. A rectangular lawn measuring 24 m by 18 m has 3 circular flowerbeds cut from it. If the circular flowerbeds each have a diameter of 8 m, find the area of the grass remaining.

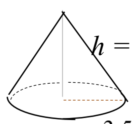
Topic C

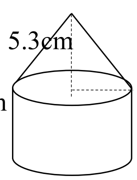
13. Find the volumes (V) of the following figures.

a) 

b) 

c) 

d) 

e) 

14. A snowman is made of three balls of snow. One has a diameter of 28 cm, one of 18 cm, and one of 8 cm. What volume of snow does the snowman contain?

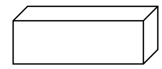
15. A conveyor belt unloading salt from a ship makes a conical pile 18 m high with a base diameter of 8 m. What is the volume of the salt in the pile?

16. A spherical balloon is filled with water and has a diameter of 30 cm. If the water was poured out into an empty tin can measuring 24 cm across and 28 cm high, would the water all fit?

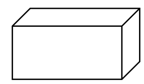
17. The height of a cylindrical pail is 26 cm and the radius of the base is 10 cm. A ball with radius 6 cm is dropped in the pail. Find the volume of the region inside the pail but outside of the ball.

Topic D

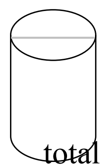
18. Determine the LA of the rectangular solid.



19. Determine the SA of the rectangular solid.



20. Determine the lateral area and surface area of the following cylinder.

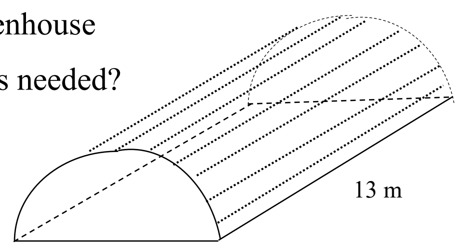


21. Determine the lateral area and total area of a cone whose diameter is 6.4 cm and slant height is 7.3 cm.

22. Determine the SA of a sphere whose diameter is 1.8 m.

23. A toy box measures 0.7 m long by 0.6 m wide and is 0.5 m high. What is the total area of plywood needed to build the box if it has no top?

24. A greenhouse is semi-cylindrical in shape. If a clear vinyl is used to cover the greenhouse (including the ends), how much vinyl is needed?



Unit 4
Measurement

Topic A: Metric system of measurement

- International system of units
- Metric conversion
- The unit factor method

Topic B: Metric units for area and volume

- Convert units of area and volume
- The relationship between mL, g and cm^3

Topic C: Imperial system

- The system of imperial units
- Imperial unit conversion

Topic D: Converting between metric and imperial units

- Imperial and metric conversions

Unit 4 Summary

Unit 4 Self - test

Topic A: Metric System of Measurement

International System of Units

Metric system (SI – international system of units): the most widely used system of measurement in the world. It is based on the basic units of meter, kilogram, second, etc.

SI common units:

Quantity	Unit	Unit symbol
Length	meter	m
Mass (or weight)	gram	kg
Volume	litre	L
Time	second	s
Temperature	degree (Celsius)	^{0}C

Metric prefixes (SI prefixes): large and small numbers are made by adding SI prefixes, which is based on multiples of 10.

Key metric prefix:

Prefix	Symbol (abbreviation)	Power of 10	Multiple value	Example
mega	M	10^6	1,000,000	1 Mm = 1,000,000 m
kilo-	k	10^3	1,000	1 km = 1,000 m
hecto-	h	10^2	100	1 hm = 100 m
deka-	da	10^1	10	1 dam = 10 m
meter/gram/liter		1		
deci-	d	10^{-1}	0.1	1 m = 10 dm
centi-	c	10^{-2}	0.01	1 m = 100 cm
milli-	m	10^{-3}	0.001	1 m = 1,000 mm
micro	µ	10^{-6}	0.000 001	1 m = 1,000,000 µm

Metric prefix for length, weight and volume:

Prefix	Length (m - meter)	Weight (g - gram)	Liquid volume (L - liter)	
mega (M)	Mm (Megameter)	Mg (Megagram)	ML (Megaliter)	Large
kilo (k)	km (Kilometer)	kg (Kilogram)	kL (Kiloliter)	
hecto (h)	hm (hectometer)	hg (hectogram)	hL (hectoliter)	
deka (da)	dam (dekameter)	dag (dekagram)	daL (dekaliter)	
meter/gram/liter	m (meter)	g (gram)	L (liter)	
deci (d)	dm (decimeter)	dg (decigram)	dL (deciliter)	
centi (c)	cm (centimeter)	cg (centigram)	cL (centiliter)	
milli (m)	mm (millimeter)	mg (milligram)	mL (milliliter)	
micro (µ)	µm (micrometer)	µg (microgram)	µL (microliter)	Small

Metric Conversion

Metric conversion table:

Value	1,000	100	10	1	.	0.1	0.01	0.001
Prefix	kilo	hecto	deka	**meter (m)** **gram (g)** **liter (L)**	.	deci	centi	milli
Symbol	k	h	da		.	d	c	m

Larger ← → Small

Steps for metric conversion through decimal movement:

- Identify the number of places to move on the metric conversion table.
- Move the decimal point.
  - Convert a ***smaller*** unit ***to*** a ***larger*** unit: move the decimal point to the ***left***.
  - Convert a ***larger*** unit ***to*** a ***smaller*** unit: move the decimal point to the ***right***.

Example: 326 mm = (?) m

- Identify mm (*millimeters*) and m (*meters*) on the conversion table.

  Count places from mm to m: 3 places

  meter . d c m
  3 2 1

- Move 3 decimal places. (1 m = 1000 mm)

  Convert a smaller unit (mm) to a larger (m) unit: move the decimal point to the left.

  326. mm = $\boxed{0.326\text{ m}}$

  Move the decimal point three places to the left (326 = 326.).

Example: 4.675 hg = (?) g

- Identify hg (*hectograms*) and g (*grams*) on the conversion table.

  Count places from hg to g: 2 places

  h da gram
  1 2

- Move 2 decimal places. (1 hg = 100 g)

  Convert a larger unit (hg) to a smaller (g) unit: move the decimal point to the right.

  4.765 hg = $\boxed{476.5\text{ g}}$

  Move the decimal point two places to the right.

Example: 30.5 mL = (?) kL

- Identify mL (*milliliters*) and kL (*kiloliters*) on the conversion table.

  Count places from mL to kL: 6 places

  k h da liter. d c m
  6 5 4 3 2 1

- Move 6 decimal place. (1 kL = 1,000,000 mL)

  Convert a smaller unit (mL) to a larger (kL) unit: move the decimal point to the left.

  30.5 mL = $\boxed{0.0000305\text{ kL}}$

  Move the decimal point six place to the left (add 0s).

The Unit Factor Method

Convert units using the unit factor method (or the factor-label method)

- Write the original term as a fraction (over 1). Example: 10g can be written as $\frac{10\text{ g}}{1}$
- Write the conversion formula as a fraction $\frac{1}{(\ )}$ or $\frac{(\ )}{1}$.
  Example: 1m = 100 cm can be written as $\frac{1\text{m}}{(100\text{cm})}$ or $\frac{(100\text{cm})}{1\text{m}}$

  (Put the desired or unknown unit on the top.)
- Multiply the original term by $\frac{1}{(\ )}$ or $\frac{(\ )}{1}$. (Cancel out the same units).

Metric conversion using the unit factor method:

Example: 1200 g = (?) kg

- Write the original term (the left side) as a fraction: $1200\text{ g} = \frac{1200\text{ g}}{1}$
- Write the conversion formula as a fraction. 1 kg = 1000 g: $\frac{1\text{ kg}}{(1000\text{ g})}$ "kg" is the desired unit.
- Multiply: $1200\text{ g} = \frac{1200\ \cancel{\text{g}}}{1} \cdot \frac{1\text{ kg}}{(1000\ \cancel{\text{g}})}$ The units "g" cancel out.

$$= \frac{1200\text{ kg}}{1000}$$

$$= \boxed{1.2\text{ kg}}$$

Example: 30 cm = (?) mm

- Write the original term (the left side) as a fraction: $30\text{ cm} = \frac{30\text{ cm}}{1}$
- Write the conversion formula as a fraction. 1 cm = 10 mm: $\frac{(10\text{ mm})}{1\text{cm}}$

  "mm" is the desired unit.
- Multiply: $30\text{ cm} = \frac{30\ \cancel{\text{cm}}}{1} \cdot \frac{(10\text{ mm})}{1\ \cancel{\text{cm}}}$ The units "cm" cancel out.

$$= \frac{(30)(10)\text{ mm}}{1}$$

$$= \boxed{300\text{ mm}}$$

Adding and subtracting SI measurements:

Example: 3 m − 2000 mm ⟹ 3000 mm − 2000 mm = $\boxed{1000\text{ mm}}$ 1 m = 1,000 mm

Combine after converting to the same unit.

Example: 25 kg + 4 g ⟹ 25000 g + 4 g = $\boxed{25004\text{ g}}$ 1 kg = 1000 g

Topic B: Metric Units for Area and Volume

Convert Units of Area and Volume

Area unit conversion

- Area unit conversion: convert the length or distance ***twice***.

  Since the units of area are always expressed as square units (in m^2, cm^2, ft^2, yd^2, etc.)

  Example: The area of a square is side squared ($A = s^2$).
  (Convert the unit of the side twice.)

  s 

- Steps for area unit conversion:

Steps

- Determine the number of decimal places it would move with ordinary units of length.
- ***Double*** this number, and move that number of decimal places for units of area.

  (Since area is in m^2, cm^2, ft^2, yd^2, etc.)

Example: 3200 cm^2 = (?) m^2

Convert cm to m: move ***2*** decimal places left.

1m = 100cm

2 × 2 = 4, move 4 places left for area.

3200. cm^2 = 0.3200 m^2 = $\boxed{0.32\ m^2}$

Example: Convert.

0.03 km^2 = (?) m^2

0.03 km^2 = 0030000. m^2 = $\boxed{30000\ m^2}$

km to m: move **3** decimal places right (1km = 1,000cm)

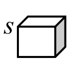
2 × 3 = 6, move 6 places right for area.

Volume unit conversion

- Volume unit conversion: convert the length or distance ***three times***.

  Since the units of volume are always expressed as cubic units (in m^3, cm^3, ft^3, yd^3, etc.)

  Example: The volume of a cube is side cubed ($V = s^3$).
  (Convert the unit of the side three times.)

  s 

- Steps for volume unit conversion:

Steps

- Determine the number of decimal places it would move with ordinary units of length.
- ***Triple*** this number, and move that number of decimal places for units of volume.

  (Since volume is in m^3, cm^3, ft^3, yd^3, etc.)

Example: 3m^3 = (?) cm^3

m to cm: move ***2*** decimal places right.

1m = 100cm

3 × 2 = 6, move 6 places right for volume.

3m^3 = $\boxed{3000000\ cm^3}$ 3 = 3.

Example: Convert.

5300 mm^3 = (?) cm^3

5300 mm^3 = $\boxed{5.3\ cm^3}$

(5300 = 5300.)

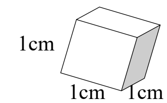
mm to cm: move **1** place left. 1cm = 10 mm

3 × 1 = 3, move 3 places left for volume.

The Relationship between *mL*, *g* and cm^3

How are mL, g, and cm^3 related?

- Recall: millimeter = mL, gram = g, cubic centimeter = cm^3
- A cube takes up 1 cm^3 of space (1 cm × 1 cm × 1 cm = 1cm^3).
  (cm^3 = cc (cubic centimeter) in chemistry and medicine)
- A cube holds 1 mL of water and has a mass of 1 gram at 4^0 C.



The relationship between mL, g and cm^3 – formulas:

$\boxed{1\text{ cm}^3 = 1\text{ mL} = 1\text{ g}}$

Or $\boxed{1\text{ cm}^3 = 1\text{ mL}}$ $\boxed{1\text{ mL} = 1\text{ g}}$ $\boxed{1\text{ cm}^3 = 1\text{ g}}$

Example: Convert.

1) 16cm^3 = (?) g

16cm^3 = $\boxed{16\text{ g}}$ 1 cm^3 = 1 g

2) 9 L = (?) cm^3

9 L = 9000 mL 1 L = 1,000 mL

= $\boxed{9000\text{ cm}^3}$ 1 mL = 1 cm^3

3) 35 cm^3 = (?) cL

35cm^3 = 35 mL 1 cm^3 = 1 mL

= $\boxed{3.5\text{ cL}}$ move 1 decimal place left. c m

450 kg = (?) L

4) 450 kg = 450,000 g 1 kg = 1,000 g

= 450,000 mL 1 g = 1 mL

= $\boxed{450\text{ L}}$ 1 L = 1,000 mL

Example: A swimming pool that measures 10 m by 8 m by 2 m. How many ***kiloliters*** of water will it hold?

$V = w\,l\,h$ = (8m) (10m) (2m) = 160 m^3 160 m^3 = (?) kL

160m^3 = 160,000,000 cm^3 1 m = 100 cm, **3** × 2 = 6, move 6 places right for volume.

160,000,000 cm^3 = 160,000,000 mL 1 mL = 1 cm^3

160,000,000 mL = 160 kL 1 kL = 1,000,000 mL

160 m^3 = $\boxed{160\text{ kL}}$ The swimming pool will hold 160 kL of water.

Topic C: Imperial System

The System of Imperial Units

Imperial system units: a system of measurement units originally defined in England, including the foot, pound, quart, ounce, gallon, mile, yard, etc.

Length, weight, liquid volume and time:

Quantity	Units
Length	inch, foot, yard, mile, etc.
Weight	pound, ounce, ton, etc.
Liquid volume	fluid ounce, pint, quart, gallon, cup, teaspoon, etc.
Time	year, week, day, hour, minute, second, etc.
Temperature	degree / Fahrenheit (^{0}F)

Imperial equivalents:

Unit name	Symbol (abbreviation)	Relationship
Length		
inch	in. or ”	
foot	ft. or ’	1ft = 12 in
yard	yd.	1yd = 3 ft
mile	mi.	1 mi = 5280 ft
Weight		
ounce	oz.	
pound	lb.	1 lb = 16 oz
ton	ton	1 ton = 2000 lb
Liquid volume		
fluid ounce	fl oz.	
pint	pt.	1 pt = 16 fl oz
quart	qt.	1 qt = 2 pt
gallon	gal.	1 gal = 4 qt
cup	c.	1 c = 8 fl oz
teaspoon	tsp.	3 tsp = 1 tbsp
tablespoon	tbsp.	16 tbsp = 1 c
Time		
second	s.	1 min. = 60 s
minute	min.	1 hr = 60 min = 3600 s
hour	hr.	1 d = 24 hr
day	d.	1 wk = 7 d
week	wk.	1 yr = 52 wk
year	yr.	1 yr = 365 d

Imperial Unit Conversion

Imperial conversion using the unit factor method:

- Write the original term as a fraction (over 1). Example: 10g can be written as $\frac{10\text{ g}}{1}$
- Write the conversion formula as a fraction $\frac{1}{(\ )}$ or $\frac{(\ )}{1}$.
  Example: 1 ft = 12 in can be written as $\frac{1\text{ ft}}{(12\text{ in})}$ or $\frac{(12\text{ in})}{1\text{ ft}}$

  (Put the unknown or desired unit on the top.)
- Multiply the original term by $\frac{1}{(\ )}$ or $\frac{(\ )}{1}$. (Cancel out the same units).

Example: 4 ft = (?) in

- Write the original term (the left side) as a fraction: $4\text{ ft} = \frac{4\text{ ft}}{1}$
- Write the conversion formula as a fraction. 1 ft = 12 in: $\frac{(12\text{ in})}{1\text{ ft}}$ "in" is the desired unit.
- Multiply: $4\text{ ft} = \frac{4\cancel{\text{ ft}}}{1} \cdot \frac{(12\text{ in})}{1\cancel{\text{ ft}}} = \frac{(4)(12\text{ in})}{1} = \boxed{48\text{ in}}$ The units "ft" cancel out.

Example: 20 qt = (?) pt

- Write the original term as a fraction: $20\text{ qt} = \frac{20\text{ qt}}{1}$
- Write the conversion formula as a fraction. 1 qt = 2pt: $\frac{(2\text{ pt})}{1\text{qt}}$ "pt" is the desired unit.
- Multiply: $20\text{ qt} = \frac{20\cancel{\text{ qt}}}{1} \cdot \frac{(2\text{ pt})}{1\cancel{\text{qt}}} = \boxed{40\text{ pt}}$ The units "qt" cancel out.

Example: 8 mi = (?) yd mi" to ft **to** yd

- Write the original term as a fraction: $8\text{ mi} = \frac{8\text{mi}}{1}$
- Write the conversion formula as a fraction.

  1 mi = 5280 ft: $\frac{(5280\text{ ft})}{1\text{mi}}$ "ft" is the desired unit.

  1 yd = 3 ft: $\frac{1\text{yd}}{(3\text{ ft})}$ "yd" is the desired unit.
- Multiply: $8\text{ mi} = \frac{8\cancel{\text{ mi}}}{1} \cdot \frac{(5280\cancel{\text{ ft}})}{1\cancel{\text{mi}}} \cdot \frac{1\text{ yd}}{(3\cancel{\text{ ft}})} = \frac{(8)(5280)(1\text{yd})}{3} = \boxed{14080\text{ yd}}$

Topic D: Converting between Metric and Imperial Units

Imperial and Metric Conversion

Key imperial and metric unit conversions:

Quantity	Metric to imperial	Imperial to metric	Abbreviation
	1 m ≈ 39 in	1 in = 2.54 cm	inch: in. or ”
Length	1 m ≈ 3.28 ft	1 ft ≈ 30.48 cm	foot: ft. or ’
	1 m ≈ 1.09 yd	1 mi ≈ 1.61 km	yard: yd.
	1 km ≈ 0.6214 mi	1 yd ≈ 0.914 m	mile: mi.
	1kg ≈ 2.2 1b	1 oz ≈ 28.35 g	pound: lb.
Weight	1 g ≈ 0.035 oz	1 lb ≈ 454 g	ounce: oz.
	1 ton ≈ 910 kg		
	1 L ≈ 0.264 gal	1 qt ≈ 0.946 L	gallon: gal.
Volume	1 L ≈ 2.1 pt	1 gal ≈ 3.79 L	pint: pt.
	1 L ≈ 1.06 qt	1 pt ≈ 470 mL	quart: qt.
	1 mL = 0.2 tsp	1 tsp = 5 mL	teaspoon: tsp.

Imperial - metric unit conversion (the unit factor method):

- Write the original term as a fraction (over 1). Example: 10 gal can be written as $\frac{10\text{ gal}}{1}$
- Write the conversion formula as a fraction $\frac{1}{(\ )}$ or $\frac{(\ )}{1}$.

Example: 1mL = 0.2 tsp can be written as $\frac{1\text{ mL}}{(0.2\text{ tsp})}$ or $\frac{(0.2\text{ tsp})}{1\text{ mL}}$

(Put the desired or unknown unit on the top.)

- Multiply the original term by $\frac{1}{(\ )}$ or $\frac{(\ )}{1}$. (Cancel out the same units).

Example: 2 ft = (?) m

- Write the original term (the left side) as a fraction: $2\text{ ft} = \frac{2\text{ ft}}{1}$
- Write the conversion formula as a fraction. 1 m ≈ 3.28 ft: $\frac{1\text{m}}{(3.28\text{ ft})}$ "m" is the desired unit.
- Multiply: $2\text{ ft} = \frac{2\text{ ft}}{1} \cdot \frac{1\text{m}}{(3.28\text{ ft})} \approx \boxed{0.61\text{m}}$

Example: 120 oz = (?) kg "oz" to "g" to "kg"

- Write the original term (the left side) as a fraction: $120\,oz = \dfrac{120\,oz}{1}$
- Write the conversion formula as a fraction. 1 oz ≈ 28.35 g: $\dfrac{(28.35\,g)}{1\,oz}$ "g" is the desired unit.
- Multiply: $120\,oz = \dfrac{120\,\cancel{oz}}{1}\cdot\dfrac{(28.35\,g)}{1\,\cancel{oz}} = 3402\,g = \boxed{3.402\text{ kg}}$ 1 kg = 1000 g

Example: 250 mL = (?) tsp

- Original term to fraction: $250\text{ mL} = \dfrac{250\text{ mL}}{1}$
- Conversion formula: 1 tsp = 5 mL: $\dfrac{1\text{ tsp}}{(5\text{ mL})}$ "tsp" is the desired unit.
- Multiply: $250\text{ mL} = \dfrac{250\text{ }\cancel{\text{mL}}}{1}\cdot\dfrac{1\text{ tsp}}{(5\text{ }\cancel{\text{mL}})}$

$$= \boxed{50\text{ tsp}}$$

Example: 10560 yd = (?) mi "yd" to "ft" to "mi"

- Original term to fraction: $10560\text{ yd} = \dfrac{10560\text{ yd}}{1}$
- Conversion formula: 3ft = 1yd: $\dfrac{(3\text{ ft})}{1\text{ yd}}$ "ft" is the desired unit.

  1 mi = 5280 ft: $\dfrac{1\text{ mi}}{(5280\text{ ft})}$ "mi" is the desired unit.
- Multiply: $10560\text{ yd} = \dfrac{10560\,\cancel{\text{yd}}}{1}\cdot\dfrac{(3\,\cancel{\text{ft}})}{1\,\cancel{\text{yd}}}\cdot\dfrac{1\text{ mi}}{(5280\,\cancel{\text{ft}})}$

$$= \frac{10560}{1}\cdot\frac{3}{1}\cdot\frac{1\text{ mi}}{5280}$$

$$= \frac{(10560)(3)\text{ mi}}{5280} = \boxed{6\text{ mi}}$$

Example: Two towns are 600 miles apart. How many kilometers separate them?

- 600 miles = (?) km
- Original term to fraction: $600\text{ mi} = \dfrac{600\text{ mi}}{1}$
- Conversion formula: 1 km ≈ 0.6214 mi: $\dfrac{1\text{ km}}{(0.6214\text{ mi})}$ "km" is the desired unit.
- Multiply: $600\text{ miles} = \dfrac{600\,\cancel{\text{mi}}}{1}\cdot\dfrac{1\text{ km}}{(0.6214\,\cancel{\text{mi}})}$

$$\approx \boxed{965.6\text{ km}}$$

The distance between two towns is 965.6 km.

Unit 4: Summary

Measurement

Metric system (SI – international system of units): the most widely used system of measurement in the world. It is based on the basic units of meter, kilogram, second, etc.

Imperial system units: a system of measurement units originally defined in England, including the foot, pound, quart, ounce, gallon, mile, yard, etc.

Metric prefixes (SI prefixes): large and small numbers are made by adding SI prefixes, which is based on multiples of 10.

Steps for metric conversion through decimal movement:

- Identify the number of places to move on the metric conversion table.
- Move the decimal point.
  - Convert a ***smaller*** unit ***to*** a ***larger*** unit: move the decimal point to the ***left***.
  - Convert a ***larger*** unit ***to*** a ***smaller*** unit: move the decimal point to the ***right***.

Convert units using the unit factor method (or the factor-label method):

- Write the original term as a fraction (over 1). Example: 10g can be written as $\frac{10\text{ g}}{1}$
- Write the conversion formula as a fraction $\frac{1}{(\ )}$ or $\frac{(\ )}{1}$.

  Example: 1m = 100 cm can be written as $\frac{1\text{m}}{(100\text{cm})}$ or $\frac{(100\text{cm})}{1\text{m}}$

  (Put the desired or unknown unit on the top.)
- Multiply the original term by $\frac{1}{(\ )}$ or $\frac{(\ )}{1}$. (Cancel out the same units).

Key metric prefix:

Prefix	Symbol (abbreviation)	Power of 10	Example
mega	M	10^6	1 Mm = 1,000,000 m
kilo-	k	10^3	1 km = 1,000 m
hecto-	h	10^2	1 hm = 100 m
deka-	da	10^1	1 dam = 10 m
meter/gram/liter		1	
deci-	d	10^{-1}	1 m = 10 dm
centi-	c	10^{-2}	1 m = 100 cm
milli-	m	10^{-3}	1 m = 1,000 mm
micro	μ	10^{-6}	1 m = 1,000,000 μm

Metric conversion table:

Value	1,000,000	1,000	100	10	1	.	0.1	.01	0.001	0.000 001
Prefix	Mega	kilo	hecto	deka	**meter (m)** **gram (g)** **liter (L)**	.	dec	centi	milli	micro
Symbol	M	k	h	da		.	d	c	m	µ

Larger — Small

Steps for area unit conversion:

- Determine the number of decimal places it would move with ordinary units of length.
- ***Double*** this number, and move that number of decimal places for units of area.

Steps for volume unit conversion:

- Determine the number of decimal places it would move with ordinary units of length.
- ***Triple*** this number, and move that number of decimal placed for units of volume.

The relationship between mL, g and cm^3 − formulas:

- A cube holds 1 mL of water and has a mass of 1 gram at 40 C.
- $\boxed{1\ cm^3 = 1\ mL = 1\ g}$

Or $\boxed{1\ cm^3 = 1\ mL}$ $\boxed{1\ mL = 1\ g}$ $\boxed{1\ cm^3 = 1\ g}$

Unit 4: Self - Test

Measurement

Topic A

1. Convert each measurement using the metric conversion table**.**

a) 439 mm = (?) m

b) 2.236 hg = (?) g

c) 48.3 mL = (?) kL

d) 2.5 kg = (?) hg

2. Convert each measurement using the unit factor method.

a) 7230 g = (?) kg

b) 52 cm = (?) mm

c) 3.4 dL = (?) L

d) 52 daL = (?) cL

3. Combine.

a) 7 m – 3000 mm = (?) mm

b) 63 kg + 6 g = (?) g

c) 0.72 L + 4.58 L – 10mL = (?) mL

d) 25.3 km + 357 dam = (?) km

Topic B

4. Convert.

a) 7400 cm^2 = (?) m^2

b) 0.09 km^2 = (?) m^2

c) 5m^3 = (?) cm^3

d) 567 mm^3 = (?) cm^3

5. Complete.

a) A cube holds 1 mL of water and has a mass of 1 gram at (   ) ^{0}C.

b) $38\ cm^3$ = (   ) g

c) 5 L = (   ) cm^3

d) $27\ cm^3$ = (?) cL

e) $76\ cm^3$ of water at 4°C has a mass of (   ) g.

f) 18 L of water has a volume of ___________ cm^3 .

g) 257 kg = (?) L

h) A fish box that measures 45 cm by 35 cm by 25 cm. How many kiloliters of water will it hold?

Topic C

6. Convert the following imperial system units.

a) 9 ft to inches

b) 47 qt to pints

c) 4 mi to yards

d) 9276 pounds to tons

Topic D

7. Convert.

a) 8 ft. to meters

b) 268 oz. to kilograms

c) 465 mL to tsp

d) 15840 yd. to miles

e) Two towns are 450 miles apart. How many kilometers separate them?

Unit 5

The Real Number System

Topic A: Rational and irrational numbers

- Real numbers

Topic B: Properties of addition and multiplication

- Properties of addition
- Properties of multiplication
- Properties of addition & multiplication

Topic C: Signed numbers and absolute value

- Signed numbers
- Absolute value

Topic D: Operations with signed numbers

- Adding and subtracting signed numbers
- Multiplying signed numbers
- Dividing signed numbers

Unit 5 Summary

Unit 5: Self - test

Topic A: Rational and Irrational Numbers

Real Numbers

Natural numbers: the numbers used for counting. 1, 2, 3, 4, 5, 6 …

Whole numbers: the natural numbers plus 0. ***0***, 1, 2, 3, 4, 5, 6 …

Integers: all the whole numbers and their negatives. ***… -4, -3, -2, -1***, 0, 1, 2, 3, 4 …

Rational number: a number that can be expressed as a fraction of two integers ($\frac{a}{b}$).

Examples of rational numbers:

$\frac{3}{4}$, $4\frac{2}{3}$ $(=\frac{14}{3})$, $11\ (=\frac{11}{1})$, $0\ (=\frac{0}{7})$, 0.52 $(=\frac{52}{100})$, -4.5 $(=\frac{-9}{2})$, $\sqrt{4}$ (=2)

Rational numbers can be expressed as terminating decimals or repeating decimals.

Example: $\frac{3}{4} = 0.75$ A terminating decimal.

$\frac{2}{3} = 0.66666\ldots = 0.\overline{6}$ A repeating decimal.

$0.232323\ldots = 0.\overline{23}$ A repeating decimal.

Irrational number: a number that ***cannot*** be represented by the fractions of two integers.

Examples of irrational numbers: π, $\sqrt{3}$, $\sqrt{19}$, $5\sqrt{13}$

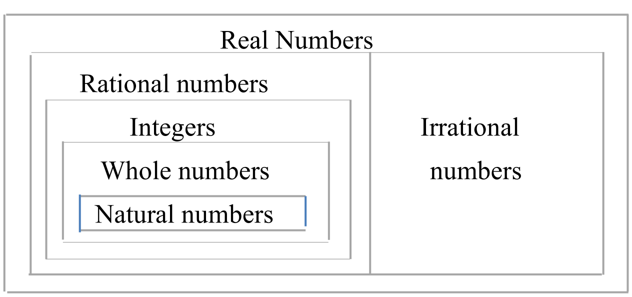
Irrational numbers ***cannot*** be expressed as terminating decimals or repeating decimals.

$\pi \approx 3.14159265358979323\ldots$ A non-terminating and non-repeating decimal.

$\sqrt{3} \approx 1.73205\ldots$ A non-terminating and non-repeating decimal.

Real numbers (*R*): rational numbers plus irrational numbers.

The real number system:



Topic B: Properties of Addition and Multiplication

Properties of Addition

Commutative property: changing the order of the numbers does not change the sum (order does not matter).

$\boldsymbol{a} + b = b + \boldsymbol{a}$ **Example**: $\mathbf{2} + 3 = 3 + \mathbf{2}$ $5 = 5$

Associative property: regrouping the numbers does not change the sum (it does not matter where you put the parenthesis).

$(a + b) + c = a + (b + c)$ **Example**: $(2 + 1) + 3 = 2 + (1 + 3)$ $5 = 5$

Additive identity property: the sum of any number and zero leaves that number unchanged.

$\boldsymbol{a} + 0 = \boldsymbol{a}$ **Example**: $\mathbf{100} + 0 = \mathbf{100}$

Closure property of addition: the sum of any two real numbers equals another real number.

Example: If **3** and **8** are real numbers, then 3 + 8 = **11** is another real number.

Additive inverse property: the sum of any real number and its negative is always a zero.

$-a + a = \mathbf{0}$ **Example**: $7 + (-7) = \mathbf{0}$

A summary of properties of addition:

Additive Properties		Example
Commutative property (switch order)	$a + b = b + a$	$2 + 3 = 3 + 2$
Associative property (switch parentheses)	$(a + b) + c = a + (b + c)$	$(2 + 1) + 3 = 2 + (1 + 3)$
Identity property	$a + 0 = a$	$100 + 0 = 100$
Closure property	If a and b are real numbers, then $a + b$ is a real number.	2 and 5 are real numbers, so $2 + 5 = 7$ is a real number.
Inverse property	$-a + a = 0$	$-2 + 2 = 0$

Example: Name the properties.

		Answer
1)	$7x + 0 = 7x$	Identity property
2)	$(97 + 22) + 3 = (97 + 3) + 22$	Commutative property (switch order)
3)	$(3 + 11x) + 7x = 3 + (11x + 7x)$	Associative property (switch parentheses)
4)	$(4y + 3) + [-(4y + 3)] = 0$	Inverse property of addition

Properties of Multiplication

Commutative property: changing the order of the numbers does not change the product (order does not matter). $\boxed{\boldsymbol{a}\, b = b\, \boldsymbol{a}}$

Example: $\mathbf{2} \cdot 6 = 6 \cdot \mathbf{2}$ $12 = 12$

Associative property: regrouping the numbers does not change the product (it does not matter where you put the parenthesis). $\boxed{\mathbf{(}a\, b\mathbf{)}\, c = a\, \mathbf{(}b\, c\mathbf{)}}$

Example: $\mathbf{(}2 \cdot 4\mathbf{)} \cdot 3 = 2 \cdot \mathbf{(}4 \cdot 3\mathbf{)}$ $24 = 24$

Multiplicative identity property: a number does not change when it is multiplied by 1.

Example: $9 \cdot \mathbf{1} = 9$ $\boxed{\boldsymbol{a} \cdot 1 = \boldsymbol{a}}$

Distributive property: multiply the number outside the parenthesis by each of the numbers inside the parenthesis. $\boxed{\boldsymbol{a}\, (b + c) = \boldsymbol{a}b + \boldsymbol{a}c}$ or $\boxed{\boldsymbol{a}\, (b - c) = \boldsymbol{a}b - \boldsymbol{a}c}$

Example: $\mathbf{2}\, (3 + 4) = \mathbf{2} \cdot 3 + \mathbf{2} \cdot 4$ $14 = 14$

$\mathbf{5}\, (6 - 3) = \mathbf{5} \cdot 6 - \mathbf{5} \cdot 3$ $15 = 15$

Multiplicative property of zero: any number multiplied by zero always equals zero.

Example: $100 \cdot \mathbf{0} = \mathbf{0}$ $\boxed{\boldsymbol{a} \cdot 0 = \mathbf{0}}$

Closure property of multiplication: the product of any two real numbers equals another real number.

Example: If **5** and **4** are real numbers, then $5 \cdot 4 = \mathbf{20}$ is another real number.

Multiplicative inverse property: the product of any nonzero real number and its reciprocal is always one. $\boxed{\mathrm{a} \cdot \frac{1}{\mathrm{a}} = 1}$

Example: **1)** $9 \cdot \frac{1}{9} = 1$

2) $(12x)\,(\frac{1}{12\,x}) = 1$

Recall reciprocal: Reciprocal = $\frac{1}{\text{number}}$

For example, the reciprocal of 4 is $\frac{1}{4}$

number its reciprocal

A summary of properties of multiplication:

Multiplicative properties		Example
Commutative property (Switch order)	$ab = ba$	$2 \cdot 3 = 3 \cdot 2$
Associative property (Switch parentheses)	$(ab)c = a(bc)$	$(2 \cdot 1)\,3 = 2\,(1 \cdot 3)$
Identity property of 1	$a \cdot 1 = a$	$100 \cdot 1 = 100$
Closure property	If a and b are real numbers, then ab is a real number.	**3** and **4** are real numbers, so $3\,(4) = 12$ is a real number
Distributive property	$a(b + c) = ab + ac$ $a(b - c) = ab - ac$	$2\,(3 + 4) = 2 \cdot 3 + 2 \cdot 4$ $3\,(4 - 2) = 3 \cdot 4 - 3 \cdot 2$
Property of zero	$a \cdot 0 = 0$	$35 \cdot 0 = 0$
Inverse property	$a \cdot \frac{1}{a} = 1$	$5 \cdot \frac{1}{5} = 1$

Example: Name the properties

Answer

1) $(3y)\,(5y) = (5 \cdot 3)\,(y \cdot y)$

$= 15\,y^2$

Commutative property of multiplication

2) $(9\,x)\,x^2 = 9\,(x \cdot x^2)$

$= 9\,x^3$

Associative property of multiplication

3) $\frac{1}{5}(10\,x - 15) = \frac{1}{5} \cdot 10\,x - \frac{1}{5} \cdot 15$

$= 2\,x - 3$

Distributive property of multiplication

4) $-(7 + 3x) \cdot \frac{1}{-(7+3x)} = 1$

Inverse property of multiplication

5) $(2x - 3y)\,x = 2x^2 - 3xy$

Distributive property

6) $\frac{1}{4x} \cdot 0 = 0$

Multiplicative property of zero

7) $(1000 \cdot 8) \cdot 9 = 1000\,(8 \cdot 9)$

$= 1000\,(72) = 72000$

Associative property of multiplication

Properties of Addition & Multiplication

Properties of addition and multiplication:

Name	Additive properties	Multiplicative properties
Commutative property	$\boldsymbol{a} + b = b + \boldsymbol{a}$	$\boldsymbol{a}\, b = b\, \boldsymbol{a}$
Associative property	$(a + b) + c = a + (b + c)$	$(a\, b)\, c = a\, (b\, c)$
Identity property	$\boldsymbol{a} + 0 = \boldsymbol{a}$	$\boldsymbol{a} \cdot 1 = \boldsymbol{a}$
Closure property	If a and b are real numbers, then $a + b$ is a real number.	If a and b are real numbers, then $a \cdot b$ is a real number.
Inverse property	$-a + a = 0$	$a \cdot \frac{1}{a} = 1$
Distributive property		$\boldsymbol{a}\,(b + c) = \boldsymbol{a}b + \boldsymbol{a}c$
Property of zero		$\boldsymbol{a} \cdot 0 = \mathbf{0}$

Example: Regroup and simplify the calculations using properties.

1) $(43 + 1998) + 2 = ?$

$43 + (1998 + 2) = \boxed{2043}$ Associative property of addition

2) $(7 \cdot 1000) \cdot 9 = ?$

$(7 \cdot 9) \cdot 1000 = \boxed{63{,}000}$ Commutative property of multiplication

Example: Solving the problems in two ways.

1) $3\,(4 + 2) = ?$

a) $3 \cdot 6 = \boxed{18}$

b) $3 \cdot 4 + 3 \cdot 2 = \boxed{18}$ Distributive property

2) $\frac{1}{2}\left(\frac{1}{2} + 1\frac{2}{3}\right) = ?$

a) $\frac{1}{2}\left(\frac{1}{2} + \frac{5}{3}\right) = \frac{1}{2}\left(\frac{3}{6} + \frac{10}{6}\right)$ $1\frac{2}{3} = \frac{5}{3}$

$= \frac{1}{2}\left(\frac{13}{6}\right) = \frac{13}{12} = \boxed{1\frac{1}{12}}$

b) $\frac{1}{2}\left(\frac{1}{2} + \frac{5}{3}\right) = \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{5}{3}\right)$ Distributive property

$= \frac{1}{4} + \frac{5}{6} = \frac{3}{12} + \frac{10}{12} = \frac{13}{12} = \boxed{1\frac{1}{12}}$

Topic C: Signed Numbers and Absolute Value

Signed Numbers

Signed number: a positive number is written with a plus sign (or without sign) in front and a negative number is written with a minus sign in front.

Example: Positive number: +5 (or 5) , $7x$, $4y^2$
Negative number: -3, -2 , $-9x$

Positive and negative numbers in real life:

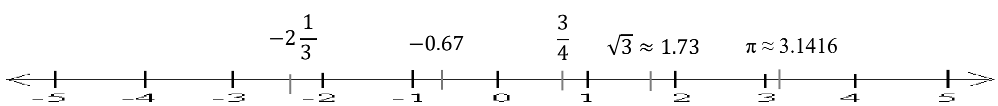
	Meaning	Example
Temperature	+ ^{0}C: above 0 degree - ^{0}C: below 0 degree	+20^{0}C -5^{0}C
Money	+ \$: gain or own - \$: loss or owe	Own: +\$10000 Owe: -\$500
Sports	+ points: gain - points: loss	Gain 3 points: +3 Lost 2 points: -2

Positive and negative numbers: positive numbers are greater than zero; negative numbers are less than zero.

The real number line: a straight line on which every point corresponds to a real number.

Example: Put the following numbers on the real number line.

$\frac{3}{4}$, $-2\frac{1}{3}$, -0.67, $\sqrt{3} \approx 1.732$, $\pi \approx 3.1416$



The number on the right is greater than the number on the left on the number line.

Example: $-5 < -3$, $-1 < 4$, $0 > -2$, $2 > \frac{1}{3}$, $-\frac{4}{5} < -\frac{2}{5}$

big > small, small < big

Example: Arrange the following numbers from the smallest to the largest number.

a) -17, 3, -3, -6, 11, 0

$\boxed{-17 < -6 < -3 < 0 < 3 < 11}$

b) $-\frac{1}{2}$, $\frac{2}{3}$, $-\frac{1}{4}$, $2\frac{2}{3}$

$-\frac{1}{2} = -0.5$, $\frac{2}{3} \approx 0.67$, $-\frac{1}{4} = -0.25$, $2\frac{2}{3} = \frac{8}{3} \approx 2.67$

$-0.5 < -0.25 < 0.67 < 2.67$

$\boxed{-\frac{1}{2} < -\frac{1}{4} < \frac{2}{3} < 2\frac{2}{3}}$

Absolute Value

Absolute value: geometrically, it is the distance of a number *x* from zero on the number line. It is symbolized "$|x|$".

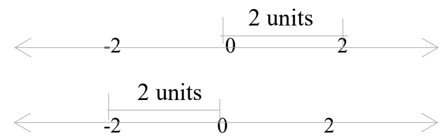
Example: $|5|$ is 5 units away from 0.

$|18|$ is 18 units away from 0.

No negatives for absolute value: the distance is always positive, and absolute value is distance, so the absolute value is never negative.

Example: $|2|$ is 2 units away from 0.

$|-2|$ is also 2 units away from 0.



Example:

a) $|-8| = \boxed{8}$

b) $|12 - 2| = \boxed{10}$

c) $|0.8 - 0.6| = \boxed{0.2}$

d) $-|-5| = -(5) = \boxed{-5}$

e) $-|-6^2| = -(36) = \boxed{-36}$

Order of operations:

Order of operations	
Clear the brackets or parentheses and absolute values (innermost first).	( ) , [ ] , { } and | |
Calculator exponents (power) and radicals.	a^n and $\sqrt{\ }$
Perform multiplication or division (from left-to-right).	$\times$ and $\div$
Perform addition or subtraction (from left-to-right).	+ and -

Example: **1)**

$$3\,[7 - 4 + (10 - 2)] = 3\,[7 - 4 + 8]$$ Parentheses

$$= 3\,[3 + 8]$$ Brackets / subtraction

$$= 3 \cdot 11$$ Brackets /addition

$$= \boxed{33}$$ Multiplication

2)

$$\frac{|-8|}{2^2} - (4 - 3) = \frac{8}{2^2} - 1$$ Parentheses and absolute value

$$= \frac{8}{4} - 1$$ Exponent

$$= 2 - 1$$ Division

$$= \boxed{1}$$ Subtraction

Topic D: Operations with Signed Numbers

Adding and Subtracting Signed Numbers

Adding signed numbers

- Add two numbers with the same sign: add their values and keep their common sign.

Example: **1)** $5 + 4 = \boxed{9}$ Add and keep the (+) sign.

2) $(-6) + (-2) = \boxed{-8}$ Add and keep the (−) sign.

3) $-\frac{1}{2} + (-1\frac{1}{2}) = -\frac{1}{2} + (-\frac{3}{2}) = -\frac{4}{2} = \boxed{-2}$ Add and keep the (−) sign.

- Add two numbers with different signs: subtract their values and keep the sign of the larger absolute value.

Example: **1)** $2 + (-5) = \boxed{-3}$ Subtract and keep the sign of -5, since $|-5| > |2|$.

2) $(-3) + 7 = \boxed{4}$ Subtract and keep the sign of 7, since $|7| > |-3|$.

3) $3.2 + (-0.2) = \boxed{3}$ Subtract and keep the sign of 3.2, since $|3.2| > |-0.2|$.

Subtracting signed numbers

- Subtract a number by adding its opposite (additive inverse), i.e. $a - b = a + (-b)$

(Change the sign of *b* and then follow the rules for adding signed numbers.)

Example: **1)** $(-3) - (-4) = (-3) + (4) = \boxed{1}$ Change the sign of the (-4), then add (-3) and 4.

2) $(-7) \underbrace{- 2}_{-(+2)} = (-7) + (-2) = \boxed{-9}$ Change the sign of the 2, then add (-7) and (-2).

3) $-\frac{1}{3} \underbrace{- \frac{2}{3}}_{-(+\frac{2}{3})} = -\frac{1}{3} + (-\frac{2}{3}) = -\frac{3}{3} = \boxed{-1}$

4) $\left|\frac{3}{5} - 1\frac{\mathbf{1}}{\mathbf{2}}\right| = \left|\frac{3}{5} - \frac{\mathbf{3}}{\mathbf{2}}\right| = \left|\frac{6}{10} - \frac{15}{10}\right| = \left|-\frac{9}{10}\right| = \boxed{\frac{9}{10}}$

- Opposite (or additive inverse): the opposite of a number (two numbers whose sum is 0).

Example: **1)** The additive inverse of 7 is $\boxed{-7}$ $7 + (-7) = 0$

2) The additive inverse of $-\frac{2}{5}$ is $\boxed{\frac{2}{5}}$ $-\frac{2}{5} + \frac{2}{5} = 0$

Multiplying Signed Numbers

Multiplying two numbers with the same sign: the product is positive.

$a \cdot b = c$

Example: $4 \cdot 5 = \boxed{20}$

$(-3)\ (-5) = \boxed{15}$

Multiplying two numbers with different signs: the product is negative.

Example: $(-5)\ (6) = \boxed{-30}$

$(0.3)\ (-3) = \boxed{-\ 0.9}$

$(-4)^2 = (-4)\ (-4) = \boxed{16}$

Multiplying by -1: $-1 \cdot a = \boxed{-\ a}$

Example: $-\ 1\ (6\ x) = \boxed{-\ 6x}$

$-4^2 = -\ 1 \cdot 4^2 = \boxed{-16}$

Signs of multiplication:

Multiplication		Example
Positive × Positive = Positive	(+) (+) = (+)	4 · 3 = 12
Negative × Positive = Negative	(–) (+) = (–)	(-4) (3) = –12
Positive × Negative = Negative	(+) (–) = (–)	(4) (–3) = –12
Negative × Negative = Positive	(–) (–) = (+)	(–4) (–3) = 12

Multiplying two or more numbers:

Multiplying	Example
▪ If the two signs are the same, the result is positive.	$(-3)\ (-4) = \boxed{12}$
▪ If the two signs are different, the result is negative.	$(-0.5)\ (0.6) = \boxed{-0.3}$
▪ The product of an ***even*** number of negative numbers is always ***positive***.	$(-4)\ (-2)\ (-5)\ (-1) = \boxed{40}$ $(-1)^4 = \boxed{1}$
▪ The product of an ***odd*** number of negative numbers is always ***negative***.	$(-\frac{2}{3})(-\frac{1}{2})(-\frac{3}{4}) = \boxed{-\ \frac{1}{4}}$ $(-1)^7 = \boxed{-1}$

Evaluating expressions:

Example: Evaluate $a^4 - b + c$ if $a = -1$, $b = -2$, $c = 4$.

$a^4 - b + c = (-1)^4 - (-2) + 4$ Substitute a for -1, b for -2 (add parentheses), and c for 4.

$= 1 + 2 + 4 = \boxed{7}$

Dividing Signed Numbers

Dividing signed numbers

- Dividing two numbers with the same sign: the quotient is positive.

Example: **1)** $-9 \div (-3) = \boxed{3}$ $a \div b = c$

2) $\frac{1.8}{2} = \boxed{0.9}$

3) $\frac{-8}{4} \div \left(\frac{-1}{4}\right) = \frac{-8}{4} \times \left(\frac{4}{-1}\right) = \boxed{8}$

- Dividing two numbers with different signs: the quotient is negative.

Example: **1)** $8 \div (-2) = \boxed{-4}$

2) $\frac{-49}{7} = \boxed{-7}$

3) $\frac{3}{9} \div \left(-\frac{6}{3}\right) = \frac{\cancel{3}^{1}}{\cancel{9}_{3}} \times \left(-\frac{\cancel{3}^{1}}{\cancel{6}_{2}}\right) = \boxed{\frac{-1}{6}}$

Signs of division:

Division	Sign	Example
Positive ÷ Positive = Positive	$\frac{+}{+} = +$	$\frac{28}{7} = 4$
Negative ÷ Positive = Negative	$\frac{-}{+} = -$	$\frac{-9}{3} = -3$
Positive ÷ Negative = Negative	$\frac{+}{-} = -$	$\frac{4.9}{-0.7} = -7$
Negative ÷ Negative = Positive	$\frac{-}{-} = +$	$\frac{-72}{-8} = 9$

Properties of zero:

Property	Example
▪ The number 0 divided by any nonzero number is zero.	$\frac{0}{6} = 0$
▪ A number divided by 0 is undefined (not allowed).	$\frac{4}{0}$ is undefined.

Evaluating expressions:

Example: Evaluate $a^2 - \frac{a}{abc}$ if $a = -2$, $b = 1$, $c = (-1)$, and $d = 0$.

$$a^2 - \frac{a}{abc} + \frac{d}{c} = (-2)^2 - \frac{-2}{(-2)\,(1)\,(-1)} + \frac{0}{-1}$$ Substitute a for -2, b for 1, c for -1 and d for 0.

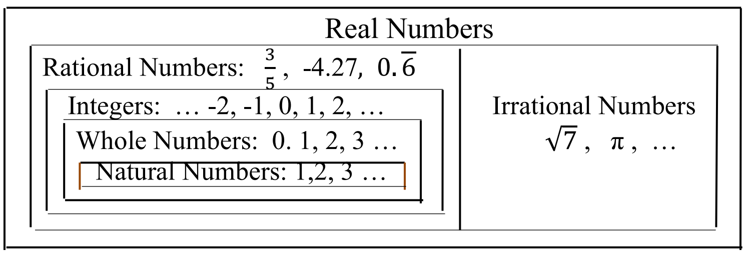
$$= 4 - \frac{-2}{2} + 0$$

$$= \boxed{5}$$

Unit 5: Summary

The Real Number System

The real number system:



Properties of addition and multiplication:

Name	Additive properties	Multiplicative properties
Commutative property	$\boldsymbol{a} + b = b + \boldsymbol{a}$	$\boldsymbol{a}\, b = b\, \boldsymbol{a}$
Associative property	$(a + b) + c = a + (b + c)$	$(a\, b)\, c = a\, (b\, c)$
Identity property	$\boldsymbol{a} + 0 = \boldsymbol{a}$	$\boldsymbol{a} \cdot 1 = \boldsymbol{a}$
Closure property	If a and b are real numbers, then $a + b$ is a real number.	If a and b are real numbers, then $a \cdot b$ is a real number.
Inverse property	$-a + a = 0$	$a \cdot \frac{1}{a} = 1$
Distributive property		$\boldsymbol{a}\,(b + c) = \boldsymbol{a}b + \boldsymbol{a}c$ $\boldsymbol{a}\,(b - c) = \boldsymbol{a}b - \boldsymbol{a}c$
Property of zero		$\boldsymbol{a} \cdot 0 = \mathbf{0}$

Signed number: a positive number is written with a plus sign (or without sign) in front and a negative number is written with a minus sign in front.

Positive and negative numbers: positive numbers are greater than zero; negative numbers are less than zero.

The real number line: a straight line on which every point corresponds to a real number.

The number on the right is greater than the number on the left on the number line.

Absolute value: geometrically, it is the distance of a number x from zero on the number line. It is symbolized "$|x|$".

No negatives for absolute value: the distance is always positive, and absolute value is distance, so the absolute value is never negative.

Order of operations with absolute value:

Order of operations	
Clear the brackets or parentheses and absolute values (innermost first).	( ), [ ], { } or | |
Calculator exponents (power) and absolute value.	a^n and $\sqrt{\ }$
Perform multiplication or division (from left-to-right).	× and ÷
Perform addition or subtraction (from left-to-right).	+ and −

Signed numbers summary:

Operation	Method
Adding signed numbers	▪ Add two numbers with the *same* sign: Add their values, and keep their common sign. ▪ Add two numbers with *different* signs: Subtract their values, and keep the sign of the larger number.
Subtracting signed numbers	Subtract a number by adding its opposite.
Multiplying signed numbers	(+)(+) = (+), (−)(−) = (+), (−)(+) = (−), (+)(−) = (−)
Dividing signed numbers	$\frac{+}{+} = +$, $\frac{-}{-} = +$, $\frac{+}{-} = -$, $\frac{-}{+} = -$ Note: $\frac{0}{A} = 0$, $\frac{A}{0}$ is undefined

Multiplying two or more numbers:

- If the two signs are the same, the result is positive.
- If the two signs are different, the result is negative.
- The product of an ***even*** number of negative numbers is always ***positive***.
- The product of an ***odd*** number of negative numbers is always ***negative***.

Opposite (or additive inverse): the opposite of a number.

Properties of zero

- The number 0 divided by any nonzero number is zero. $\frac{0}{A} = 0$
- A number divided by 0 is undefined (not allowed). $\frac{A}{0}$ is undefined.

Unit 5: Self-Test

The Real Number System

Topic A

1. Give two examples of rational numbers that are not integers.

2. Given the set of numbers:

-3, 4.7, 0, 8, $\frac{3}{5}$, $2.\overline{56}$, 5.4259…., π, $\sqrt{5}$

Determine which of the numbers above are

a) natural numbers?

b) integers?

c) rational numbers?

d) irrational numbers?

Topic B

3. Name the properties.

a) $12a + 0 = 12a$

b) $(3x + 11y) + 7 = 7 + (3x + 11y)$

c) $(4 + x) + 11 = 4 + (x + 11)$

d) $(6a + 5) + [-(6a + 5)] = 0$

e) $7(3y + 4) = 7 \cdot 3y + 7 \cdot 4$
$= 21y + 28$

f) $(0.5a)b = 0.5(ab)$

g) $(4x)(7y) = (4 \cdot 7)(xy)$

h) $-(8y) \cdot \frac{1}{-(8y)} = 1$

i) $(4 - 7y)3 = 12 - 21y$

j) $\frac{1}{23+7x} \cdot 0 = 0$

k) $(199 + 36) + 1 = (199 + 1) + 36$

l) $(1000 \cdot 8) \cdot 9 = 1000(8 \cdot 9)$

4. Regroup and simplify the calculations using properties.

a) $12 + (45 + 88)$

b) $9\,(1000 \cdot 8)$

c) $3 + (2997 + 56)$

5. Use the distributive property to write an equivalent expression without parentheses.

a) $4y\,(y + 0.3)$

b) $(2 - 3y^2)\,5$

c) $\frac{1}{3}\left(\frac{2}{3} - \frac{1}{2}x\right)$

Topic C

6. Compare these numbers using either $<$ or $>$.

a) 6 8

b) 0 -6

c) -4 - 2

d) $-\frac{3}{7}$ $\frac{1}{7}$

e) -0.6 -0.8

f) $1\frac{1}{2}$ $\frac{3}{8}$

7. Arrange the following numbers from the smallest to the largest number (using < to order them).

a) 8, -9, -4, 23, 0, -17

b) 0.05 , -8 , $\frac{2}{5}$, $\frac{3}{5}$, -3.24

c) $-\frac{1}{3}$, $\frac{2}{5}$, $-\frac{1}{7}$, $1\frac{3}{4}$

8. Preform the indicated operation.

a) $|-67|$

b) $|35 - 14|$

c) $|-0.45 + 0|$

d) $-|-7^2|$

e) $\left|-\frac{1}{8}\right|$

9. Preform the indicated operation.

a) $4\,[7 - 3 + (30 - 5)]$

b) $\frac{|-9|}{3^2} + (27 - 3)$

Topic D

10. Preform the indicated operation.

a) 13 + 24

b) (-7) + (-8)

c) $-\frac{1}{5} + (-2\frac{2}{5})$

d) 9 + (-4)

e) (-25) + 12

f) 8.4 + (-0.9)

g) (-7) – (6)

h) (-5) – (-7)

i) $-\frac{3}{7} - \frac{2}{7}$

j) $|\frac{1}{7} - 1\frac{3}{4}|$

k) -45 ÷ (-9)

l) $\frac{-3.6}{6}$

m) $\frac{-9}{5} \div \left(\frac{-1}{15}\right)$

n) -72 ÷ 9

o) $\frac{0}{1789}$

p) $\frac{3.78}{0}$

11. Write the additive inverse (opposite) of each number.

a) – 45

b) $\frac{5}{8}$

c) -1

12. If $x = -2$, $y = 5$, $z = 4$ and $w = 0$, evaluate each of the following.

a) $zy + x^3$

b) $x^2 - 2xy + y^2 + \frac{w}{3xyz}$

c) $(x + y)(x - y) - 5z$

d) $4\left(\frac{2xy}{3w}\right)$

Unit 6

Polynomials

Topic A: Introduction to polynomials

- Polynomials
- Degree of a polynomial
- Combine like terms
- Removing parentheses

Topic B: Multiplying and dividing polynomials

- Multiplying and dividing monomials
- Multiplying / dividing polynomials by monomials
- FOIL method to multiply binomials

Unit 6 Summary

Unit R Self-test

Topic A: Introduction to Polynomials

Polynomials

Basic algebraic terms:

Algebraic term	Description	Example
Algebraic expression	A mathematical phrase that contains numbers, variables (letters), and arithmetic operations (+, – , ×, ÷, etc.).	$3x - 4$ $5a^2 - b + 3$ $12y^3 + 7y^2 - 5y + \frac{2}{3}$
Constant	A number on its own.	$2y + 5$ constant: 5
Coefficient	The number in front of a variable.	$-9x^2$ coefficient: -9 x coefficient: 1 $(x = 1 \cdot x)$
Term	A term can be a constant, a variable, or the product of a number and variable. (Terms are separated by a plus or minus sign.)	$2x^3 + 7x^2 - 9y - 8$ Terms: $2x^3$, $7x^2$, $-9y$, -8
Like terms	The terms that have the same variables and exponents (differ only in their coefficients).	$2x$ and $-7x$ $-4y^2$ and $9y^2$ $0.5pq^2$ and $\frac{2}{3}pq^2$

Polynomial: an algebraic expression that contains one or more terms.

The prefix "poly-" means many.

Example: $7x$, $5ax - 9b$, $6x^2 - 5x + \frac{2}{3}$, $7a^2 + 8b + ab - 5$

There are special names for polynomials that have one, two, or three terms:

- **Mono**mial: an algebraic expression that contains only one term.

  Example: $9x$, $4xy^2$, $0.8mn^2$, $\frac{1}{3}a^2b$ — The prefix "mono" means one.

- **Bino**mial: an algebraic expression that contains two terms. — The prefix "bino-" means two.

  Example: $7x + 9$, $9t^2 - 2t$, $0.3y + \frac{1}{3}$

- **Tri**nomial: an algebraic expression that contains three terms.

  Example: $ax^2 + bx + c$, $-4qp^2 + 3q + 5$ — The prefix "tri-" means three.

Polynomials in ascending or descending order: a polynomial can be arranged in ascending or descending order.

- Descending order: the exponents of variables are arranged from largest to smallest number.

  Example: $5a^3 - 3a^2 + a + 1$ — The exponents of *a* decrease from left to right.

  $19y^4 + 31y^3 - y^2 + 2y - \frac{2}{3}$ — The exponents of *y* decrease from left to right.

- Ascending order: the exponents of variables are arranged from smallest to largest number.

  Example: $2 - 0.3x + 4.5x^2 - 7x^3$ — The exponents of *x* increase from left to right.

  $7 + \frac{3}{7}w + 4w^2 - 8w^3 + w^4$ — The exponents of *w* increase from left to right.

Degree of a Polynomial

Classification of polynomial: polynomials are classified according to their number of terms and degrees.

Degree of a term:

- The degree of a term with one variable: the exponent of its variable.

  Example: $9x^3$ The degree of the term: $\boxed{3}$

  $-7y^5$ The degree of the term: $\boxed{5}$

- The degree of a term with more variables: the sum of the exponents of its variables.

  Example: $-8a^2 b^3 c^6$ The degree of the term: $\boxed{11}$ $(2 + 3 + 6 = 11)$

- **More examples:**

Monomial	Degree	Reason	
$4x$	1	$x = x^1$	(x has an exponent of 1.)
$7xy^3$	4	$1 + 3 = 4$	
$-\frac{3}{5}x^2y^4z$	7	$2 + 4 + 1 = 7$	$(z = z^1)$
13	0	$13 = 13 \cdot 1 = 13 \cdot x^0 = 13$	$(x^0 = 1)$

Degree of a polynomial: the highest degree of any individual term in it.

Examples:

Polynomial	Degree	Reason
$7x^8 + 5x^5 + 8$	8	The highest exponent of the term is $7x^8$.
$3a^2 + 4a^2b^3 + 7a^4b^5c^2$ $2 \quad 2+3=5 \quad 4+5+2=11$	11	The highest degree of the term is $7a^4b^5c^2$.

Example: Arrange polynomials in descending order and identify the degrees and coefficients.

a) $5 + 2a - 4a^2 + a^3$

Descending order: $\boxed{a^3 - 4a^2 + 2a + 5}$

Coefficients: $\boxed{1}$ $\boxed{-4}$ $\boxed{2}$

Degree of the polynomial: $\boxed{3}$

b) $-2xy + 9x^3 + 5x^5y + \frac{3}{4} + 7x^2 - \frac{1}{2}x^4$

Descending order: $5x^5y - \frac{1}{2}x^4 + 9x^3 + 7x^2 - 2xy + \frac{3}{4}$

Coefficients: $\boxed{5}$ $\boxed{-\frac{1}{2}}$ $\boxed{9}$ $\boxed{7}$ $\boxed{-2}$

Degree of the polynomial: $\boxed{6}$ $y = y^1$

Combine Like Terms

Like terms: terms that have the same variables and exponents (the coefficients can be different).

Examples:

Example	Like or unlike terms
$7y$ and $-9y$	Like terms
$6a^2$, $-32a^2$, and $-a^2$	Like terms
$0.3\,x^2y$ and $-4.8\,x^2y$	Like terms
$\frac{-2}{7}u^2v^3$ and $\frac{3}{5}u^2v^3$	Like terms
$-8y$ and $78x$	Unlike terms
$6m^3$ and $-9m^2$	Unlike terms
$-9u^3w^2$ and $-9w^3u^2$	Unlike terms

Combine like terms: add or subtract their coefficients and keep the same variables and exponents.

Note: unlike terms cannot be combined.

Example: Combine like terms.

a) $3a + 7b - 9a + 15b = (3a - 9a) + (7b + 15b)$ Regroup like terms.

$= \boxed{-6a + 22b}$ Combine like terms.

b) $2y^2 - 4x + 3x - 5y^2 = (2y^2 - 5y^2) + (-4x + 3x)$ Regroup like terms.

$= -3y^2 - 1x$ Combine like terms.

$= \boxed{-3y^2 - x}$

c) $8xy^2 - x^2y + 4x^2y - 6xy^2$

$= \underline{8xy^2} \boldsymbol{- x^2y + 4x^2y} \underline{- 6xy^2}$ Or underline like terms and without regrouping.

$= \boxed{2xy^2 + 3x^2y}$ Combine like terms.

d) $2(2m + 3n) + 3(m - 4n) = \underline{4m} + \mathbf{6n} + \underline{3m} - \mathbf{12n}$ Distributive property.

$= \boxed{7m - 6n}$ Combine like terms.

e) $8v + 4(2v - u^2) + 3(u^2 + v) = \underline{8v} + \underline{8v} - \mathbf{4u^2} + \mathbf{3u^2} + \underline{3v}$ Distributive property.

$= \boxed{-u^2 + 19v}$ Combine like terms.

Removing Parentheses

If the sign preceding the parentheses is positive (+), do not change the sign of terms inside the parentheses, just remove the parentheses.

Example: $(x - 5) = x - 5$

If the sign preceding the parentheses is negative (-), remove the parentheses and the negative sign (in front of parentheses), and change the sign of each term inside the parentheses.

Example: $-(x - 7) = -x + 7$

Remove parentheses:

Algebraic expression	Remove parentheses	Example
$(ax + b)$	$ax + b$	$(5x + 2) = 5x + 2$
$(ax - b)$	$ax - b$	$(9y - 4) = 9y - 4$
$-(ax + b)$	$-ax - b$	$-(\frac{3}{4}x + 7) = -\frac{3}{4}x - 7$
$-(ax - b)$	$-ax + b$	$-(0.5b - 2.4) = -0.5b + 2.4$

Example: Simplify.

a) $9x^2 + 7 - (2x^2 - 2) = \underline{9x^2} + \mathbf{7} - \underline{2x^2} + \mathbf{2}$ Remove parentheses.

$= \boxed{7x^2 + 9}$ Combine like terms.

b) $(-8y + 5z) - 4(y - 7z) = \underline{-8y} + \mathbf{5z} \underline{- 4y} + \mathbf{28z}$ Remove parentheses.

$= \boxed{-12y + 33z}$ Combine like terms.

c) $-(3a^2 + 4a - 4) + 3(4a^2 - 6a + 7)$ Remove parentheses.

$= \mathbf{-3a^2} \underline{- 4a} + 4 + \mathbf{12a^2} \underline{- 18a} + 21$ Distributive property.

$= \boxed{9a^2 - 22a + 25}$ Combine like terms.

d) $-5(u^2 - 3u) + 3(2u - 4) - (5 - 3u + 4u^2)$ Distributive property.

$= \mathbf{-5u^2} + 1\underline{5u} + \underline{6u} - 12 - 5 \underline{+ 3u} - \mathbf{4u^2}$ Remove parentheses.

$= \boxed{-9u^2 + 24u - 17}$ Combine like terms.

e) $8(pq - 4cd) - 3(-pq + 5cd) = \underline{8pq} - \mathbf{32cd} \underline{+ 3pq} - \mathbf{15cd}$ Distributive property.

$= \boxed{11pq - 47cd}$ Combine like terms.

Topic B: Multiplying and Dividing Polynomials

Multiplying and Dividing Monomials

Basic rules of exponents:

Name	Rule	Example
Product of like bases (The same base)	$a^m a^n = a^{m+n}$	$2^3\, 2^2 = 2^{3+2} = 2^5$ Since $2^3\, 2^2 = (2 \cdot 2 \cdot 2)\,(2 \cdot 2) = 2^5$
Quotient of like bases (The same base)	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^5}{x^3} = x^{5-3} = x^2$ Since $\frac{x^5}{x^3} = \frac{x\,x\,x\,x\,x}{x\,x\,x} = x^2$
Negative exponent a^{-n}	$a^{-n} = \frac{1}{a^n}$	$3^{-2} = \frac{1}{3^2} = \frac{1}{9} = 0.11$ Since $3^{-1} = \frac{1}{3} = 1 \div 3$, $3^{-2} = \frac{1}{3 \cdot 3} = \frac{1}{9} \approx 0.11$

Example: Simplify the following.

a) $x^4\, x^3 = x^{4+3} = \boxed{x^7}$ $\qquad a^m\, a^n = a^{m+n}$

b) $\frac{y^{-6}}{y^3} = y^{-6-3} = y^{-9} = \boxed{\frac{1}{y^9}}$ $\qquad \frac{a^m}{a^n} = a^{m-n}$, $a^{-n} = \frac{1}{a^n}$

Multiplying monomials (one term):

- Regroup coefficients and variables.
- Multiply coefficients (the numbers in front of the variable).
- Multiply variables (add exponents with the same base, apply $a^m\, a^n = a^{m+n}$).

Example: **1)** $(-4x^4\, y^3)\,(7x^3\, y^2) = (-4 \cdot 7)\,(x^4 \cdot x^3)\,(y^3 \cdot y^2)$ Regroup the coefficients & the variables.

$= -28\, x^{4+3}\, y^{3+2} = \boxed{-28\, x^7 y^5}$ Multiply the coefficients & add the exponents. $a^m\, a^n = a^{m+n}$

2) $\left(\frac{3}{4}a^2b^3c^2\right)\left(\frac{4}{6}ab^2c^2\right) = \left(\frac{3}{4} \cdot \frac{4}{6}\right)(a^2a)\,(b^3b^2)(c^2c^2)$ Regroup.

$= \boxed{\frac{1}{2}a^3b^5c^4}$ $\qquad a = a^1$, $\quad a^m\, a^n = a^{m+n}$

Dividing monomials:

- Divide coefficients.
- Divide variables (subtract exponents with the same base, apply $\frac{a^m}{a^n} = a^{m-n}$).

Example: **1)** $\frac{4a^5}{16a^2} = \left(\frac{4}{16}\right)\left(\frac{a^5}{a^2}\right)$ Regroup the coefficients & the variables.

$= \frac{1}{4}\, a^{5-2} = \boxed{\frac{1}{4}a^3}$ Divide the coefficients & subtract the exponents.

2) $\frac{t^2}{t^7} = t^{2-7} = t^{-5} = \boxed{\frac{1}{t^5}}$ $\qquad \frac{a^m}{a^n} = a^{m-n}$, $a^{-n} = \frac{1}{a^n}$

3) $\frac{-12x^2y^5}{4x^3y^5} = \left(\frac{-12}{4}\right)\left(\frac{x^2}{x^3}\right)\left(\frac{y^5}{y^5}\right)$ Regroup.

$= -3x^{2-3}y^{5-5}$ $\qquad \frac{a^m}{a^n} = a^{m-n}$

$= -3x^{-1}y^0 = \boxed{\frac{-3}{x}}$ $\qquad x^{-1} = \frac{1}{x^1} = \frac{1}{x}$, $\quad y^0 = 1$

Multiplying / Dividing Polynomials by Monomials

Multiplying a monomial and a polynomial:

- Use the distributive property: $a(b + c) = ab + ac$
- Multiply coefficients and add exponents with the same base. Apply $a^m a^n = a^{m+n}$

Examples:

1) $\mathbf{3x^3 (5x^2 - 2x)} = (3x^3)(5x^2) - (3x^3)(2x)$ Distributive property: $a(b + c) = ab + ac$

$= (3 \cdot 5)(x^3 x^2) - (3 \cdot 2)(x^3 x^1)$ Regroup $x = x^1$

$= 15(x^{3+2}) - 6(x^{3+1})$ Multiply the coefficients & add the exponents.

$= \boxed{15x^5 - 6x^4}$ $a^m \cdot a^n = a^{m+n}$

2) $\mathbf{5ab^2 (2a^2b + ab^2 - a)}$ Distribute.

$= (5ab^2)(2a^2b) + (5ab^2)(ab^2) + (5ab^2)(-a)$ Multiply the coefficients and add exponents.

$= (5 \cdot 2)(a^{1+2} b^{2+1}) + (5a^{1+1} b^{2+2}) - (5a^{1+1}b^2)$ $b = b^1$, $a = a^1$

$= \boxed{10a^3b^3 + 5a^2b^4 - 5a^2b^2}$ $a^m \cdot a^n = a^{m+n}$

Dividing a polynomial by a monomial

- Split the polynomial into several parts.
- Divide a monomial by a monomial. Apply $\frac{a^m}{a^n} = a^{m-n}$.

Example: $\frac{12x^2+4x-2}{4x}$

Steps	Solution
Split the polynomial into three parts:	$\frac{12x^2+4x-2}{4x} = \frac{12x^2}{4x} + \frac{4x}{4x} - \frac{2}{4x}$
Divide a monomial by a monomial:	$= \boxed{3x + 1 - \frac{1}{2x}}$ $\frac{a^m}{a^n} = a^{m-n}$

FOIL Method to Multiply Binomials

The FOIL method: an easy way to find the product of two binomials (two terms).

$(a + b)\ (c + d) = ac + ad + bc + bd$ F O I L		Example
F - First terms	first term × first term $(a + b)\ (c + d)$	$(x + 5)\ (x + 4)$
O - Outer terms	outside term × outside term $(a + b)\ (c + d)$	$(x + 5)\ (x + 4)$
I - Inner terms	inside term × inside term $(a + b)\ (c + d)$	$(x + 5)\ (x + 4)$
L - Last terms	last term × last term $(a + b)\ (c + d)$	$(x + 5)\ (x + 4)$

FOIL method	Example
$(a + b)\ (c + d) = ac + ad + bc + bd$ F O I L	$(x + 5)\ (x + 4) = x{\cdot}x + x{\cdot}4 + 5x + 5{\cdot}4 = x^2 + 9x + 20$ F O I L

Multiplying binomials (2 terms × 2 terms)

Example: Multiply.

1) $(2x + 3)\ (5x - 6) = \underset{F}{2x \cdot 5x} + \underset{O}{2x(-6)} + \underset{I}{3 \cdot 5x} + \underset{L}{3(-6)}$ The FOIL method.

$= 10x^2 - 12x + 15x - 18$ $a^n a^m = a^{n+m}$

$= \boxed{10x^2 + 3x - 18}$ Combine like terms.

2) $(3r - t)\ (5r + t^2) = 3r \cdot 5r + 3r \cdot t^2 - t \cdot 5r - t \cdot t^2$ FOIL

$= \boxed{15r^2 + 3rt^2 - 5rt - t^3}$ $a^n a^m = a^{n+m}$

3) $(xy^2 + y)\ (2x^2y + x) = xy^2 \cdot 2x^2y + xy^2 \cdot x + y \cdot 2x^2\, y + y\, x$ FOIL

$= 2x^3\, y^3 + x^2y^2 + 2x^2\, y^2 + x\, y$ $a^n a^m = a^{n+m}$

$= \boxed{2x^3\, y^3 + 3x^2y^2 + x\, y}$ Combine like terms.

4) $(a - \frac{1}{3})\ (a - \frac{1}{3}) = a^2 - \frac{1}{3}a - \frac{1}{3}a + (-\frac{1}{3})(-\frac{1}{3})$ FOIL

$= \boxed{a^2 - \frac{2}{3}a + \frac{1}{9}}$. Combine like terms.

Unit 6: Summary

Polynomials

Basic algebraic terms:

Algebraic term	Description	Example
Algebraic expression	A mathematical phrase that contains numbers, variables (letters), and arithmetic operations (+, −, ×, ÷, etc.).	$3x - 4$, $5a^2 - b + 3$
Constant	A number on its own.	$2y + 5$ constant: 5
Coefficient	The number in front of a variable.	$-9x^2$ coefficient: -9 x coefficient: 1
Term	A term can be a constant, a variable, or the product of a number and variable. (Terms are separated by a plus or minus sign.)	$7a^2 - 6b + 8$ Terms: $7a^2$, $-6b$, 8
Like terms	The terms that have the same variables and exponents (differ only in their coefficients).	$2x$ and $-7x$ $-4y^2$ and $9y^2$

Polynomial	Example
Monomial **(one term)**	$0.67x$
Binomial **(two terms)**	$4x - \frac{2}{3}$
Trinomial **(three terms)**	$2a^2 - ab + 5$
Polynomial (one or more terms)	$2xy$, $4x^3 + 11$, $-\frac{2x}{3} + x - 5y + 4$

Descending order: the exponents of variables are arranged from largest to smallest number.

Ascending order: the exponents of variables are arranged from smallest to largest number.

Degree of a term/polynomial:

- The degree of a term with one variable: the exponent of its variable.
- The degree of a term with more variables: the sum of the exponents of its variables.
- Degree of a polynomial: the highest degree of any individual term in it.

Like terms: terms that have the same variables and exponents (the coefficients can be different.)

Combine like terms: add or subtract their numerical coefficients and keep the same variables and exponents.

Remove parentheses:

- If the sign preceding the parentheses is positive (+), do not change the sign of terms inside the parentheses, just remove the parentheses.

- If the sign preceding the parentheses is negative (-), remove the parentheses and the negative sign (in front of parentheses), and change the sign of terms inside the parentheses.

Basic rules of exponents:

Name	Rule		Example
Product of like bases (The same base)	$a^m a^n = a^{m+n}$	$(a \neq 0)$	$2^3 2^2 = 2^{3+2} = 2^5 = 32$
Quotient of like bases (The same base)	$\frac{a^m}{a^n} = a^{m-n}$	$(a \neq 0)$	$\frac{y^3}{y^2} = y^{3-2} = y^1 = y$
Negative exponent a^{-n}	$a^{-n} = \frac{1}{a^n}$	$(a \neq 0)$	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

Multiply monomials (one term):

- Multiply coefficients.
- Multiply variables (add exponents with the same base, apply $a^m a^n = a^{m+n}$).

Dividing monomials:

- Divide coefficients.
- Divide variables (subtract exponents with the same base, apply $\frac{a^m}{a^n} = a^{m-n}$).

Multiplying a monomial and a polynomial:

- Use the distributive property: $a(b + c) = ab + ac$
- Multiply coefficients and add exponents with the same base. Apply $a^m a^n = a^{m+n}$

Dividing a polynomial by a monomial

- Split the polynomial into several parts.
- Divide a monomial by a monomial. Apply $\frac{a^m}{a^n} = a^{m-n}$

The FOIL method:

$\mathbf{(a + b)(c + d) = ac + ad + bc + bd}$ **F O I L**		
F - First terms	first term × first term	$(a + b)(c + d)$
O - Outer terms	outside term × outside term	$(a + b)(c + d)$
I - Inner terms	inside term × inside term	$(a + b)(c + d)$
L - Last terms	last term × last term	$(a + b)(c + d)$

Unit 6: Self-Test

Polynomials

Topic A

1. Identify the terms of each polynomial.

a) $5x^3 - 8x^2 + 2x$

b) $-\frac{2}{3}y^4 + 9a^2 + a - 1$

2. Identify the coefficients and the degree of the polynomials.

a) $2a^3 - 7a^2b^3 + 9b + 11$

b) $-8xy^5 - \frac{2}{3}y^4 + 11x^2y^3 + 4y^2 - 23y + \frac{5}{6}$

3. Identify each polynomial as a monomial, binomial, or trinomial.

a) $3x^2 - 7x$

b) $-29xy^3$

c) $8mn^2 + 7m - 45$

4. Arrange polynomials in descending order.

a) $3 + 8x - 23x^2 + 15x^3$

b) $-3y^3 - 45y^2 + 4y + \frac{2}{3}y^4$

5. Combine like terms.

a) $7x + 10y - 8x + 9y$

b) $12a^2 - 33b + 2b - 6a^2$

c) $12uv^2 - 5u^2v + 15u^2v - 8uv^2$

d) $5(4t - 6r) + 3(t + 7r)$

e) $13n + 5(6n - m^2) + 7(2m^2 + 3n)$

6. Simplify.

a) $15a^2 + 9 - (5a^2 - 4)$

b) $(-13x + 9y) - 6(x - 5y)$

c) $-(7z^2 + 6z - 15) + 2(7z^2 - 5z + 8)$

d) $-11(y^2 - 3y) + 4(2y - 5) - (13 - 6y + 9y^2)$

e) $5(ab - 2xy) - 6(-2ab + 3xy)$

Topic B

7. Simplify the following.

a) $a^3\, a^6$

b) $\frac{x^{-4}}{x^7}$

c) $\frac{t^3}{t^9}$

d) $(-6a^3\, b^5)\,(7a^4\, b^6)$

e) $\left(\frac{5}{6}x^3y^4z^5\right)\left(\frac{3}{10}xy^3z^4\right)$

f) $\frac{6y^8}{36y^3}$

g) $\frac{-81m^3n^9}{9m^4n^9}$

8. Perform the indicated operation.

a) $-4x^3\,(3x^4 - 7x)$

b) $9a^3b\,(3ab^2 + 2a^2\,b^2 - a)$

c) $\frac{35a^2 + 5a - 4}{5a}$

d) $(5y - 7)\,(8y + 9)$

e) $(7r - 2t)\,(3r + 4t^2)$

f) $(2ab^2 + 3b)\,(5a^2b + 3a)$

g) $(x - \frac{1}{3})\,(x - \frac{2}{3})$

Unit 7

Equations

Topic A: Properties of equations

- Introduction to equations
- Solving one-step equations
- Properties of equality

Topic B: Solving equations

- Solving multi-step equations
- Equation solving strategy
- Equations involving decimals / fractions

Topic C: One solution, no solutions, infinite solutions

- Types of equations

Topic D: Writing and solving equations

- Number problems
- Consecutive integers:
- Mixed problems

Unit 7 Summary

Unit 7 Self-test

Topic A: Properties of Equations

Introduction to Equations

Equation: a mathematical sentence that contains two expressions and separated by an equal sign (both sides of the equation have the same value).

Example: $4 + 3 = 7$, $9x - 4 = 5$, $2y - \frac{1}{3} = y$

To solve an equation is the process of finding a particular value for the variable in the equation that makes the equation true (left side = right side or LS = RS).

Example: For the equation $x + 4 = 5$

only $x = 1$ can make it true, since $1 + 4 = 5$ (LS = RS)

Solution of an equation: the value of the variable in the equation that makes the equation true.

Example: For the equation $x + 4 = 5$, $x = 1$ is the solution.

Examples: Indicate whether each of the given number is a solution to the given equation.

1)	**2:**	$4x - 3 = 5$	$4 \cdot 2 - 3 \stackrel{?}{=} 5$	$5 \stackrel{\surd}{=} 5$	$\boxed{\text{Yes}}$	Replace x with 2.
2)	**15:**	$\frac{-3}{15}y = -3$	$\frac{-3}{15}(15) \stackrel{?}{=} -3$	$-3 \stackrel{\surd}{=} -3$	$\boxed{\text{Yes}}$	Replace y with 15.
3)	$\mathbf{\frac{1}{2}}$**:**	$8t = 3$	$8\left(\frac{1}{2}\right) \stackrel{?}{=} 3$	$4 \neq 3$	$\boxed{\text{No}}$	Replace t with $\frac{1}{2}$.

An equation behaves like a pair of balanced scales. The scales remain balanced when the same weight is put on to or taken away from each side. Always do the same thing on both sides to keep an equation true.

Left side = Right side (LS = RS)

Left side ≠ Right side (LS ≠ RS)

Solving One-Step Equations

To solve a one-step addition equation: $x + a = b$

Isolate the variable "***x***" by **subtracting** the same number ***a*** from each side of the equation (to get rid of the constant ***a*** on the left side of the equal sign so that the letter x is on its own).

Example: Solve $x + 7 = 9$ $\qquad a = 7$

$x + \cancel{7} - \cancel{7} = 9 - 7$ $\qquad$ Subtract 7 from both sides.

$x = 2$

or $\quad x + \cancel{7} = 9$

$\quad\;\; -\cancel{7} \quad -7$

Solution: $x = \boxed{2}$

Check: substitute the solution into the equation to verify that is true.

(Left side = Right side).

$x + 7 = 9$ $\qquad$ Original equation

$2 + 7 \overset{?}{=} 9, \quad 9 \overset{\surd}{=} 9$ $\quad$ LS = RS (correct) $\qquad$ Replace x with 2.

Example: Solve $u + \frac{2}{5} = \frac{3}{5}$

$u + \cancel{\frac{2}{5}} - \cancel{\frac{2}{5}} = \frac{3}{5} - \frac{2}{5}$ $\qquad$ Subtract $\frac{2}{5}$ from both sides.

$u = \frac{1}{5}$

or $\quad u + \frac{2}{5} = \frac{3}{5}$

$\quad\;\; -\frac{2}{5} \quad -\frac{2}{5}$

Solution: $u = \boxed{\frac{1}{5}}$

Check: $u + \frac{2}{5} = \frac{3}{5}$ $\qquad$ Replace u with $\frac{1}{5}$.

$\frac{1}{5} + \frac{2}{5} \overset{?}{=} \frac{3}{5}, \qquad \frac{3}{5} \overset{\surd}{=} \frac{3}{5}$ $\qquad$ LS = RS (correct)

To solve a one-step subtraction equation: $\boxed{x - a = b}$

Isolate the variable by **adding** the same number ***a*** to each side of the equation.

Example: Solve $x - 5 = 8$ $a = 5$

$$x \cancel{- 5} \cancel{+ \mathbf{5}} = 8 + \mathbf{5}$$ Add 5 to both sides.

Solution: $x = \boxed{13}$

To solve a one-step multiplication equation: $\boxed{a\,x = b}$

Isolate the variable "***x***" by **dividing** the same number ***a*** from each side of the equation.

Example: Solve $6x = 42$ $a = 6$

$$\frac{6x}{\mathbf{6}} = \frac{42}{\mathbf{6}}$$ Divide both sides by 6.

Solution: $x = \boxed{7}$

Example: Solve $\frac{4y}{5} = \frac{4}{15}$ $a = \frac{4}{5}$, $\frac{4y}{5} = \frac{4}{5}y$

$$\frac{4y}{5} \div \frac{\mathbf{4}}{\mathbf{5}} = \frac{4}{15} \div \frac{\mathbf{4}}{\mathbf{5}}$$ Divide both sides by $\frac{4}{5}$.

$$\frac{\cancel{4}y}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{4}} = \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{15}}} \cdot \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{4}}}$$

Solution: $y = \boxed{\frac{1}{3}}$

To solve a one-step division equation: $\boxed{\frac{x}{a} = b}$

Isolate the variable by **multiplying** the same number ***a*** to each side of the equation.

Example: Solve $\frac{x}{7} = 6$ $a = 7$

$$\frac{x}{\cancel{7}} \cdot \cancel{7} = 6 \cdot 7$$ Multiply both sides by 7.

Solution: $x = \boxed{42}$

Example: Solve $-\frac{1}{5}y = 8$ $a = -5$

$$\cancel{-\frac{1}{5}}\,\mathbf{(-5)}\,y = 8\,(\mathbf{-5})$$ Multiply both sides by -5.

Solution: $y = \boxed{-40}$

Properties of Equality

Basic rules for solving one-step equations:

- Add, subtract, multiply or divide the same quantity to both sides of an equation can result in a valid equation.
- Remember to always do the same thing to both sides of the equation (balance).

Properties for solving equations:

Properties	Equality	Example
Addition property of equality	$A = B \quad A + \boldsymbol{C} = B + \boldsymbol{C}$	Solve $x - 6 = 3$ $x \cancel{-6} + \cancel{\boldsymbol{6}} = 3 + \boldsymbol{6} \qquad x = \mathbf{9}$
Subtraction property of equality	$A = B \quad A - \boldsymbol{C} = B - \boldsymbol{C}$	Solve $y + 5 = -8$ $y \cancel{+ 5} \cancel{- \mathbf{5}} = -8 - \mathbf{5} \qquad y = \mathbf{-13}$
Multiplication property of equality	$A = B \quad A \cdot \boldsymbol{C} = B \cdot \boldsymbol{C}$	Solve $\frac{m}{9} = 2$ $\cancel{\mathbf{9}} \cdot \frac{m}{\cancel{9}} = 2 \cdot \mathbf{9} \qquad m = \mathbf{18}$
Division property of equality	$A = B \quad \frac{A}{\boldsymbol{C}} = \frac{B}{\boldsymbol{C}} \quad (C \neq 0)$	Solve $3n = -15$ $\frac{\cancel{3}n}{\cancel{\mathbf{3}}} = \frac{-15}{\mathbf{3}} \qquad n = \mathbf{-5}$

Example: Solve the following equations.

1) $-9 + x = 5$ $\quad \cancel{-9} + x + \cancel{\boldsymbol{9}} = 5 + \boldsymbol{9}$ — Property of addition.

$x = \boxed{14}$

Check: $-9 + \boldsymbol{14} \overset{?}{=} 5 \qquad 5 \overset{\surd}{=} 5$ — Replace x with 14.

2) $t + \frac{2}{5} = -\frac{1}{5}$ $\quad y + \frac{2}{5} - \frac{\mathbf{2}}{\mathbf{5}} = -\frac{1}{5} - \frac{\mathbf{2}}{\mathbf{5}}$ — Property of subtraction.

$y = \boxed{-\frac{3}{5}}$

3) $\frac{-1}{6}x = 7$ $\quad \mathbf{-6} \cdot \frac{-1}{6}x = 7(\mathbf{-6})$ — Property of multiplication.

$x = \boxed{-42}$

4) $-5x = 30$ $\quad \frac{-5x}{\mathbf{-5}} = \frac{30}{\mathbf{-5}}$ — Property of division.

$x = \boxed{-9}$

5) $0.7y = -0.63$ $\quad \frac{0.7y}{\mathbf{0.7}} = \frac{-0.63}{\mathbf{0.7}}$ — Property of division.

$y = \boxed{-0.9}$

6) $y - 3\frac{2}{5} = 2\frac{3}{10}$ $\quad y - 3\frac{2}{5} + \mathbf{3\frac{2}{5}} = 2\frac{3}{10} + \mathbf{3\frac{2}{5}}$ — Property of addition.

$y = 2\frac{3}{10} + 3\frac{4}{10}$ — The LCD = 10

$y = \boxed{5\frac{7}{10}}$

Topic B: Solving Equations

Solving Multi-Step Equations

Multi-step equation: an equation that requires more than one step to solve it.

Steps for solving multi-step equations:

- Simplify the equation and remove parentheses if necessary.
- Combine like terms on each side of the equation.
- Collect the variable (letter) terms on one side of the equation and the numerical terms (numbers) on the other side.
- Isolate the variable and find the solution: make the coefficient of the variable (number in front of the variable) equal to one.
- Check: substitute the solution back into the equation to verify that it is true (LS = RS).

Example: Solve **$9x + 6 = 12$**

- Simplify: $3x + 2 = 4$ — Divide each term by 3.
- Combine like terms: $3x + \cancel{2} - \cancel{2} = 4 - 2$ — Subtract 2 from both sides.

$3x = 2$

Variable term Constant term

- Isolate the variable $\frac{3x}{3} = \frac{2}{3}$ — Divide both sides by 3.

Solution: $x = \boxed{\frac{2}{3}}$

- Check: $9x + 6 = 12$ — Original equation.

$9 \cdot \frac{2}{3} + 6 \overset{?}{=} 12$ $\quad 12 \overset{\surd}{=} 12$ — Replace x with $\frac{2}{3}$.

Example: Solve **$13t - 10 = 3$**

$13t - \cancel{10} + \cancel{10} = 3 + 10$ — Add 10 to both sides.

$13t = 13$

$\frac{13t}{13} = \frac{13}{13}$ — Divide both sides by 13.

$t = \boxed{1}$ — Solution.

Example: Solve **$2(x - 4) + 5x + 3 = 3(2 - 3x)$**. — Remove parentheses.

$\underline{2x} - \mathbf{8} + \underline{5x} + \mathbf{3} = 6 - 9x$ — Combine like terms.

$7x - 5 = 6 - 9x$

$7x - \cancel{5} + \cancel{5} = 6 - 9x + 5$ — Add 5 to both sides.

$7x = 11 - 9x$

$7x + 9x = 11 - \cancel{9x} + \cancel{9x}$ — Add $9x$ to both sides.

$16x = 11$ $\quad \boxed{x = \frac{11}{16}}$ — Divide both sides by 16.

Equations Solving Strategy

Procedure for solving equations

Equation solving strategy
▪ Clear the fractions or decimals if necessary. ▪ Simplify and remove parentheses if necessary. ▪ Combine like terms on each side of the equation. ▪ Collect the variable terms on one side of the equation and the constants on the other side. ▪ Isolate the variable (to get the variable alone on one side of the equation). ▪ Check the solution with the original equation.

Steps for solving equations:

Steps	**Example**
	Solve $\frac{1}{5}(y+10) = 3y - \frac{9}{5}y$
▪ Eliminate the denominators if the equation has fractions.	$\mathbf{5} \cdot \frac{1}{5}(y+10) = \mathbf{5}(3y) - \mathbf{5}\left(\frac{9}{5}y\right)$ Multiply each term by 5.
▪ Remove parentheses.	$y + 10 = 15y - 9y$
▪ Combine like terms.	$y + 10 = 6y$
▪ Collect variable terms on one side and the constants on the other side.	$y + \cancel{10} - \cancel{\mathbf{10}} = 6y - \mathbf{10}$
	$y = 6y - \mathbf{10}$ Subtract 10 from both sides.
	$y - \mathbf{6y} = \cancel{6y} - 10 - \cancel{\mathbf{6y}}$ Subtract $6y$ from both sides.
▪ Isolate the variable.	$-5y = -10$ Divide both sides by -5.
	$y = \frac{-10}{-5}$
	$\boxed{y = 2}$
▪ Check with the original equation.	$\frac{1}{5}(2+10) \overset{?}{=} 3 \cdot 2 - \frac{9}{5} \cdot 2$ Replace y with 2.
	$5 \cdot \frac{1}{5}(2+10) \overset{?}{=} 5 \cdot 3 \cdot 2 - 5 \cdot \frac{9}{5} \cdot 2$ Multiply each term by 5.
	$(2+10) \overset{?}{=} 30 - 18$
	$12 \overset{\surd}{=} 12$ LS = RS (correct)

Equations Involving Decimals / Fractions

Equations involving decimals

Tip: Multiply every term of both sides of the equation by a multiple of 10 (10, 100, 1000, etc.) to clear the decimals (based on the number with the largest number of decimal places in the equation).

Steps	Example
	Solve **$0.34x - 0.12 = -4.26x$**.
▪ Multiply each term by 100 to clear the decimal.	$\mathbf{100}(0.34x) - \mathbf{100}(0.12) = \mathbf{100}(-4.26x)$ The largest number of decimal place is two.
▪ Collect the variable terms on one side of the equation and the constants on the other side.	$34x - 12 = -426x$ Add 12 to both sides. $34x + 426x = 12$ Add 426x to both sides. $460x = 12$
▪ Isolate the variable.	$\boxed{x \approx 0.026}$

Example: Solve **$0.4y + 0.08 = 0.016$** The largest number of decimal place is three.

$\mathbf{1000}(0.4y) + \mathbf{1000}(0.08) = \mathbf{1000}(0.016)$ Multiply each term by 1000.

$400y + 80 = 16$ Combine like terms.

$400y = -64$ Divide both sides by 400.

$\boxed{y = -0.16}$

Equations involving fractions

Steps	Example
	Solve $\frac{t}{3} + \frac{3}{4} = -\frac{t}{2} - \frac{1}{3}$.
▪ Multiply each term by the LCD.	$\mathbf{12} \cdot \frac{t}{3} + \mathbf{12} \cdot \frac{3}{4} = \mathbf{12}(-\frac{t}{2}) - \mathbf{12} \cdot \frac{1}{3}$ $\begin{array}{r\|rrrr} 2 & 3 & 4 & 2 & 3 \\ \hline 3 & 3 & 2 & 1 & 3 \\ \hline & 1 & 2 & 1 & 1 \end{array}$ LCD = 2 × 3 × 2 = 12
▪ Collect the variable terms on one side of the equation and the constants on the other side.	$4t + 9 = -6t - 4$ Add 6t to both sides. $10t = -13$ Subtract 9 from both sides.
▪ Isolate the variable.	$t = \frac{-13}{10} = \boxed{-1\frac{3}{10}}$ Divide both sides by 10.

Topic C: One Solution, No Solutions, Infinite Solutions

Types of Equations

Types of equations: a mathematical equation can be a contradiction, an identity, or a conditional equation.

Contradiction equation: an equation which is never true, regardless of the value of the variable, and thus has no solution.

Example: $3(x + 1) - 3x = -7$ Distribute property.

$\cancel{3x} + 3 - \cancel{3x} = -7$ Combine like terms.

$3 = -7$ False, $3 \neq -7$

No solution There are no real numbers that can make this equation true.

Note: If the resulting equation is a **false** statement with **no variables**, it is a contradiction equation.

Identity equation: an equation which is always true for every value of the variable and thus has an infinite number of solutions (the solution is all real numbers).

Example: $12x - 3(2 + 4x) = -6$ Distribute property.

$\cancel{12x} - 6 - \cancel{12x} = -6$ Combine like terms.

$-6 = -6$

$\boxed{\text{The solution is all real numbers}}$.

The equation is always true no matter what value is substituted for the variable.

Note: If the resulting equation is a **true** statement and with **no variables**, it is an identity equation.

Conditional equation: an equation is true only for the certain value of the variable (one solution).

Example: $2x - 3 = -7x$ Add $7x$ to both sides.

$9x - 3 = 0$ Add 3 to both sides.

$9x = 3$

$x = \frac{1}{3}$ Divide both sides by 9.

$\boxed{\text{If } x = \frac{1}{3}\text{, the equation is true, otherwise, the equation is false.}}$

Summary: types of equations

Types of equations	Characteristic	Solution
Contradiction equation	Always false	No solution
Identity equation	Always true	All real numbers
Conditional equation	It is true only for the certain value.	One solution

Example: Determine each equation as a contradiction, an identity, or a conditional equation.

1) $4x - (3 - x) = 5(x - 1)$ Remove parentheses.

$4x - 3 + x = 5x - 5$ Combine like terms.

$5x - 3 = 5x - 5$

$\cancel{5x} - 3 - \cancel{5x} = \cancel{5x} - 5 - \cancel{5x}$ Subtract $5x$ from both sides.

$-3 = -5$

$\boxed{\text{No solution – contradiction equation}}$

The resulting equation is a false statement with no variables.

2) $\frac{y}{2} + 2(y - 3) = 2 - 3y$ Multiply each term by 2.

$\cancel{2} \cdot \frac{y}{\cancel{2}} + 2 \cdot 2(y - 3) = 2 \cdot 2 - 2(3y)$ Remove parentheses.

$\underline{y} + \underline{4y} - 12 = 4 - 6y$ Combine like terms.

$5y - \cancel{12} + \mathbf{\cancel{12}} = 4 - 6y + \mathbf{12}$ Add 12 to both sides.

$5y = 16 - 6y$

$5y + \mathbf{6y} = 16 - \cancel{6y} + \mathbf{\cancel{6y}}$ Add 6y to both sides.

$11y = 16$ Divide both sides by 11.

$\boldsymbol{y \approx 1.455}$

$\boxed{\text{One solution – conditional equation}}$

3) $4t - 3(t + 4) = t - 12$ Distribute property.

$\underline{4t} - \underline{3t} - 12 = t - 12$ Combine like terms.

$t - 12 = t - 12$ Add 12 to both sides.

$\quad + 12 \qquad + 12$

$t = t$ Subtract t from both sides.

$-t \quad -t$

$0 = 0$

$\boxed{\text{All real numbers – identity equation}}$

The resulting equation is a true statement with no variables.

Topic D: Writing and Solving Equations

Number Problems

Number problems - examples

English phrase	Algebraic expression / equation
Seven more than the difference of a number and four.	$(x - 4) + 7$
The quotient of five and the product of six and a number.	$\frac{5}{6x}$
The product of nine and a number, decreased by five.	$9x - 5$
Ten less than three times two numbers is seven more than their sum.	$3xy - 10 = x + y + 7$
The sum of the squares of two numbers is nine less than their product.	$x^2 + y^2 = xy - 9$
Two more than the quotient of $11x$ by 5 is seven times that number.	$2 + \frac{11x}{5} = 7x$

Let x = a number, y = a number

Steps for solving word problems:

Procedure for solving word problems
▪ **Organize** the **facts** given from the problem.
▪ Identify and **label** the **unknown** quantity (**let *x* = unknown**).
▪ Draw a **diagram** if it will make the problem clearer.
▪ Convert words into a mathematical **equation**.
▪ **Solve** the equation and find the **solution**(s).
▪ **Check** the solution with the original equation (check it back into the problem – is it logical? if necessary).

Example: The product of nine and a number is twenty-seven. Determine the value of this number.

- Organize the facts and assign the unknown quantity:

Facts	The **product** of **9** and ***x*** is **27**
Unknown	Let ***x*** = number

- Write an equation: $9 \cdot x = 27$ or $9x = 27$
- Solve the equation: $\frac{9x}{9} = \frac{27}{9}$ Divide both sides by 9.

  $\boxed{x = 3}$

- Check: $9 \cdot 3 \stackrel{?}{=} 27$ Replace x with 3.

  $27 \stackrel{\surd}{=} 27$ LS = RS (correct)

Answer: The value of the number is 3.

Example: Eight less than two times a number is five less than the number divided by two. Find the number.

Let x = number

- Organize the facts: $-8 \quad 2x \quad = \quad -5 \quad \frac{x}{2}$
- Equation: $\mathbf{2x - 8 = \frac{x}{2} - 5}$ — Multiply each term by 2.

  $2(2x) - 2 \cdot 8 = 2\left(\frac{x}{2}\right) - 2 \cdot 5$ — Remove parentheses.

  $4x - 16 = x - 10$ — Combine like terms.

  $3x = 6$ — Divide both sides by 3.

- Solution: $\boxed{x = 2}$
- Check: $2(\mathbf{2}) - 8 \overset{?}{=} \frac{2}{2} - 5$ — Replace x with 2.

  $4 - 8 \overset{?}{=} 1 - 5$

  $-4 \overset{\surd}{=} -4$ — LS = RS (correct)

Answer: The number is 2.

Example: There are **three** numbers, the **first** is **four less** than **three times** the **second**, and the **third** is **two more** than the **first**. The **sum** of these **three** numbers **is fifteen**.
Find each number.

- Organize the facts:

Number	Words	Algebraic expression
2nd number	Let 2nd number = x	x
1st number	4 less than 3 times the 2nd number	$3x - 4$
3rd number	2 more than the 1st number	$(3x - 4) + 2$
Sum	The sum of three numbers is 15	1st # + 2nd # + 3rd # = 15

- Equation: $\mathbf{(3x - 4) + x + [(3x - 4) + 2)] = 15}$ — Remove parentheses.

  $3x - 4 + x + 3x - 2 = 15$ — Combine like terms.

  $7x - 6 = 15$ — Add 6 to both sides.

  $7x = 21$ — Divide both sides by 7.

- Solution: $\boxed{x = 3}$

1st Number	$3x - 4 = 3 \cdot 3 - 4 = 5$
2nd Number	$x = 3$
3rd Number	$(3x - 4) + 2 = (3 \cdot 3 - 4) + 2 = 7$

- Check: $5 + 3 + 7 \overset{?}{=} 15$ Yes!

Consecutive Integers

Consecutive integers:

English phrase	Algebraic expression	Example
Two consecutive integers	x , $x + 1$	If $x = \mathbf{1}$, $x + 1 = \mathbf{2}$
Three consecutive integers	x , $x + 1$, $x + 2$	If $x = \mathbf{1}$, $x + 1 = \mathbf{2}$, $x + 2 = \mathbf{3}$
Two consecutive **odd** integers	x , $x + 2$	If $x = \mathbf{1}$, $x + 2 = \mathbf{3}$
Three consecutive **odd** integers	x , $x + 2$, $x + 4$	If $x = \mathbf{1}$, $x + 2 = \mathbf{3}$, $x + 4 = \mathbf{5}$
Two consecutive **even** integers	x , $x + 2$	If $x = \mathbf{2}$, $x + 2 = \mathbf{4}$
Three consecutive **even** integers	x , $x + 2$, $x + 4$	If $x = \mathbf{2}$, $x + 2 = \mathbf{4}$, $x + 4 = \mathbf{6}$

Examples:

English phrase	Equation
The difference of two consecutive integers is one.	$(x + 1) - x = 1$
The sum of three consecutive odd integers is nine.	$x + (x + 2) + (x + 4) = 9$
The product of two consecutive even integers is eight.	$x\,(x + 2) = 8$
Three consecutive even integers whose sum is twelve.	$x + (x + 2) + (x + 4) = 12$

Example: The **sum** of **three consecutive odd** integers is **twenty-one**, find each number.

- Organize the facts:

1st consecutive odd number	x
2nd consecutive odd number	$x + 2$
3rd consecutive odd number	$x + 4$

- Write an equation: $x + (x + 2) + (x + 4) = 21$ — Combine like terms.
- Solve the unknown: $3x + 6 = 21$ — Subtract 6 from both sides.

  $3x = 15$ — Divide both sides by 3.

  $\boxed{x = 5}$

1st consecutive even number	$x = 5$
2nd consecutive even number	$x + 2 = 5 + 2 = 7$
3rd consecutive even number	$x + 4 = 5 + 4 = 9$

- Check: $5,\ 7,\ 9 \overset{?}{=}$ consecutive odd integers — Yes!

  $5 + (5 + 2) + (5 + 4) \overset{?}{=} 21$ — Replace x with 5.

  or $5 + 7 + 9 \overset{?}{=} 21$

  $21 \overset{\surd}{=} 12$ — LS = RS (correct)

- State the answer: $\boxed{x = 5,\quad x + 2 = 7,\quad x + 4 = 9}$

Mixed Problems

Example: The second angle of a triangle is twelve times as large as the first. The third angle is five degrees more than the second angle. Find the measure of each angle.

1st angle	x
2nd angle	$12x$
3rd angle	$12x + 5^0$

- Equation $x + 12x + (12x + 5^0) = 180^0$ The sum of three angles of a triangle is 180^0.

  $25x + 5^0 = 180^0$ Remove parentheses and combine like terms.

  $25x = 175^0$ Subtract 5^0 from both sides.

- Solve: $x = \frac{175}{25} = \boxed{7^0}$ Divide both sides by 25.
- The answer:

1st angle	$x = 7^0$
2nd angle	$12x = 12\ (7) = 84^0$
3rd angle	$12x + 5^0 = 12\ (7^0) + 5^0 = 89^0$

- Check: $7^0 + 84^0 + 89^0 \stackrel{?}{=} 180^0$

  $180^0 \stackrel{\surd}{=} 180^0$ Yes!

Example: The perimeter of a rectangle is 164 meters. The width is 13 meters shorter than the length. Find the dimensions (width and length).

- List the facts and sign the unknown quantity:

Facts	Perimeter $P = 164$ m
Unknown	Let l = length, width = $l - 13$

- Equation: $2\,l + 2(l - 13) = 164$ The perimeter of a rectangle: $P = 2l + 2w$

  $4\,l - 26 = 164$ Remove parentheses and combine like terms.

  $4\,l = 190$ Divide both sides by 4.

  Length: $\boxed{l = 47.5 \text{ m}}$

- Find the width: $w = l - 13$

  $w = 47.5 - 13$ Substitute 47.5m for l in the equation.

  $= 34.5$ m

  Width: $\boxed{w = 34.5 \text{ m}}$

	Formulas
Original price	Original price = Sale price + Discount
Discount	Discount = Discount rate $\times$ Original price
Sale price	Sale price = Original price – Discount

Example: After a 35% reduction, a women's jacket is on sale for \$30.55. What is the discount? What was the original price?

- Organize the facts:

Sale price	\$30.55
Discount rate	35 %
Unknown	Let x = original price

- Discount: Discount = Discount rate $\times$ Original price

  $= (35\%)\ x$

- Equation: Original price = Sale price + Discount

  $x = 30.55 + 35\%\ x$

  or $x = 30.55 + 0.35\ x$ Convert percent to decimal.

- Solve: $x - 0.35\ x = 30.55$ Subtract $0.35x$ from both sides.

  $0.65\ x = 30.55$ Combine like terms. $x = 1 \cdot x$

  $x = \frac{30.55}{0.65} = 47$ Divide both sides by 0.65.

  $\boxed{x = \$47}$

- State the answer: The original price was \$47.

Example: A \$159.99 instant pot is labeled "30% off". What is the sale price?

Original price	\$159.99
Discount rate	30 %
Unknown	Let $\boldsymbol{x}$ = sale price

- Equation: Sale price = Original price – Discount

  $x = 159.99 - (30\%)\ (159.99)$ Discount = Discount rate $\times$ Original price

  $x = 159.99 - (0.3)\ (159.99)$ Convert percent to decimal.

  $x \approx 111.99$

- State the answer: The sale price is \$111.99.

Unit 7: Summary

Equations

Equation: a mathematical sentence that contains two expressions and separated by an equal sign.

To solve an equation is the process of finding a particular value for the variable in the equation that makes the equation true.

Solution of an equation: the value of the variable in the equation that makes the equation true.

An equation behaves like a pair of balanced scales. The scales remain balanced when the same weight is put on to or taken away from each side. Always do the same thing on both sides to keep an equation true (LS = RS).

Basic rules for solving one-step equations:

- Add, subtract, multiply or divide the same quantity to both sides of an equation can result in a valid equation.
- Remember to always do the same thing to both sides of the equation (balance).

Properties for solving equations:

Properties	Equality		Example
Addition property of equality	$A = B$	$A + \boldsymbol{C} = B + \boldsymbol{C}$	Solve $x - 6 = 3$ $x - 6 + \boldsymbol{6} = 3 + \boldsymbol{6}$ $\quad x = \mathbf{9}$
Subtraction property of equality	$A = B$	$A - \boldsymbol{C} = B - \boldsymbol{C}$	Solve $y + 5 = -8$ $y + 5 - \mathbf{5} = -8 - \mathbf{5}$ $\quad y = \mathbf{-13}$
Multiplication property of equality	$A = B$	$A \cdot \boldsymbol{C} = B \cdot \boldsymbol{C}$	Solve $\frac{m}{9} = 2$ $\mathbf{9} \cdot \frac{m}{9} = 2 \cdot \mathbf{9}$, $\quad m = \mathbf{18}$
Division property of equality	$A = B$	$\frac{A}{\boldsymbol{C}} = \frac{B}{\boldsymbol{C}}$ $\quad (C \neq 0)$	Solve $3n = -15$ $\frac{3n}{\mathbf{3}} = \frac{-15}{\mathbf{3}}$ $\quad n = \mathbf{-5}$

Steps for solving equations:

Equation solving strategy
▪ Clear the fractions or decimals if necessary. ▪ Simplify and remove parentheses if necessary. ▪ Combine like terms on each side of the equation. ▪ Collect the variable terms on one side of the equation and the constants on the other side. ▪ Isolate the variable. ▪ Check the solution with the original equation.

Types of equations:

Types of equations	Characteristic	Solution
Contradiction equation	Always false	No solution
Identity equation	Always true	All real numbers
Conditional equation	It is true only for the certain value.	One solution

- If the resulting equation is a false statement with no variables, it is a contradiction equation.
- If the resulting equation is a true statement and with no variables, it is an identity equation.

Steps for solving word problems:

Procedure for solving word problems
▪ **Organize** the **facts** given from the problem.
▪ Identify and **label** the **unknown** quantity (**let** x **= unknown**).
▪ Draw a **diagram** if it will make the problem clearer.
▪ Convert words into a mathematical **equation**.
▪ **Solve** the equation and find the **solution**(s).
▪ **Check** the solution with the original equation (check it back into the problem – is it logical? if necessary).

Consecutive integers:

English phrase	Algebraic expression	Example
Three consecutive integers	x, $x+1$, $x+2$	If $x = \mathbf{1}$, $x+1=\mathbf{2}$, $x+2=\mathbf{3}$
Three consecutive **odd** integers	x, $x+2$, $x+4$	If $x=\mathbf{1}$, $x+2=\mathbf{3}$, $x+4=\mathbf{5}$
Three consecutive **even** integers	x, $x+2$, $x+4$	If $x=\mathbf{2}$, $x+2=\mathbf{4}$, $x+4=\mathbf{6}$

	Formulas
Original price	Original price = Sale price + Discount
Discount	Discount = Discount rate $\times$ Original price
Sale price	Sale price = Original price – Discount

Unit 7: Self-Test

Equations

Topic A

1. Indicate whether each of the given number is a solution to the given equation.

a) 2: $9x - 7 = 11$

b) 17: $\frac{-5}{17}y = -9$

c) $\frac{2}{3}$: $9m = 6$

2. Solve the following equations.

a) $x - 7 = 12$

b) $y + \frac{3}{8} = \frac{5}{8}$

c) $m - 6 = 17$

d) $9t = 72$

e) $\frac{3x}{2} = \frac{9}{16}$

f) $\frac{y}{13} = -4$

g) $-21 + x = 7$

h) $y + \frac{4}{9} = -\frac{3}{9}$

i) $\frac{-4}{14}x = -2$

j) $-19t = 38$

k) $0.8y = -0.64$

l) $x - 4\frac{2}{3} = 3\frac{2}{9}$

Topic B

3. Solve the following equations.

a) $14t + 5 = 8$

b) $7m - 23 = 40$

c) $7(x - 3) + 3x - 5 = 2(5 - 4x)$

d) $\frac{1}{7}(y + 12) = 4y - \frac{3}{7}y$

e) $0.63x - 0.29 = -3.56x$

f) $0.5t + 0.05 = 0.025$

g) $\frac{x}{4} + \frac{2}{5} = -\frac{x}{2} - \frac{1}{5}$

Topic C

4. Determine each equation as a contradiction, an identity, or a conditional equation.

a) $5(y + 2) - 5y = -8$

b) $8x - 4(3 + 2x) = -12$

c) $7t - 9 = -3t$

d) $5y - (4 - y) = 6(y - 2)$

e) $\frac{x}{3} + 3(x - 4) = 5 - 8x$

f) $7m - 5(m + 3) = 2m - 15$

Topic D

5. Write each of the following as an algebraic expression.

a) Nine more than the difference of a number and seven.

b) The quotient seven and the product of nine and a number.

c) The product of eleven and a number, decreased by eight.

6. Write each of the following as an algebraic expression or equation.

a) Thirteen less than four times two numbers is six more than their sum.

b) The sum of the squares of two numbers is twenty-six less than their product.

c) Five more than the quotient of $5x$ by 23 is eleven times that number.

d) The difference of two consecutive integers is one.

e) The sum of three consecutive odd integers is fifteen.

f) The product of two consecutive even integers is forty-eight.

g) Three consecutive odd integers whose sum is twenty-one.

7. Solve each problem by writing and solving an equation.

a) The product of seven and a number is forty-two. Determine the value of this number.

b) Three less than four times a number is nine less than the number divided by four. Find the number.

c) There are three numbers, the first is three less than five times the second, and the third is four more than the first. The sum of these three numbers is twenty. Find each number.

d) The sum of three consecutive odd integers is twenty-seven, find each number.

e) The second angle of a triangle is seven times as large as the first. The third angle is thirty degrees more than the second angle. Find the measure of each angle.

f) The perimeter of a rectangle is 128 meters. The width is 8 meters shorter than the length. Find the dimensions (width and length).

g) After a 20% reduction, a TV is on sale for $199.99. What is the discount? What was the original price?

h) A $379.99 laptop is labeled "10% off". What is the sale price?

Unit 8

Formulas

Topic A: Substitution into formulas

- Geometry formulas
- Substituting into formulas

Topic B: Solving formulas

- Solving for a specific variable
- More examples for solving formulas

Topic C: Pythagorean theorem

- Pythagorean theorem
- Applications of the Pythagorean theorem

Unit 8 Summary

Unit 8 Self-test

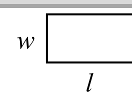
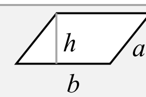
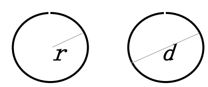
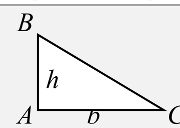
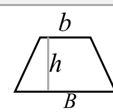
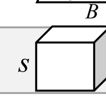
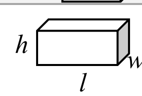
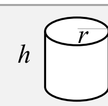
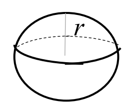
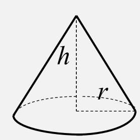
Topic A: Substitution into Formulas

Geometry Formulas

Formula: an equation that contains more than one variable and is used to solve practical problems in everyday life.

Geometry formulas review:

s – side, P – perimeter, C – circumference, A – area, V – volume

Name of the figure	Formula	Figure
Rectangle	$P = 2w + 2l$ $A = wl$ (w = width, l = length)	
Parallelogram	$P = 2a + 2b$ $A = bh$ (a and b = sides, h = height)	
Circle	$C = \pi d = 2\pi r$ $A = \pi r^2$ (r = radius, d = diameter)	
Triangle	$< A + < B + < C = 180^{0}$ $A = \frac{1}{2}bh$ (b = base, h = height)	
Trapezoid	$A = \frac{1}{2}h(b + B)$ (b = top base, B = bottom base, h = height)	
Cube	$V = s^3$ (s = side)	
Rectangular solid	$V = wlh$ (w = width, l = length, h = height)	
Cylinder	$V = \pi r^2 h$ (r = radius, h = height)	
Sphere	$V = \frac{4}{3}\pi r^3$ (r = radius)	
Cone	$V = \frac{1}{3}\pi r^2 h$ (r = radius, h = height)	

Substituting into Formulas

- **Examples of formula:**

Application	Formula	Components
Distance	$d = v\,t$	d – distance v – velocity t – time
Simple interest	$I = P\,r\,t$	I – interest P – principle r – interest rate (%) t – time (years)
Compound interest	$B = P\,(100\% + r)^{\,t}$	B – *balance* P – principle r – interest rate (%) t – time (years)
Percent increase	$\frac{N - O}{O}$	N – new value O – original value
Percent decrease	$\frac{O - N}{O}$	N – new value O – original value
Sale price and discount	$S = P - d\,P$ $D = d\,P$	S – sale price P – price (original or regular price) d – discount rate D – discount
Original price and markup	$P = C + m\,C$ $M = m\,C$	P – price (original or selling price) C – cost m – markup rate M – markup
Intelligence quotient (I.Q.)	$I = \frac{100m}{c}$	I – I.Q. m – mental age c – chronological age
Cost of running electrical appliances	$C = \frac{Wtr}{1000}$	C – Cost (in cents) W – power in watts (watts used) t – time (hours) r – rate (per kilowatt-hour)

Substitution into formula: "substitution" means replacing numbers with variables (letters).

Example: Find the IQ of a 10-year-old girl with a mental age of 12.

- Formula: $I = \frac{100m}{c}$
- Facts: m = 12 years, c = 10 years
- Substituting: $I = \frac{100m}{c}$

$= \frac{100\ (12\ y)}{10\ \cancel{y}}$ Substitute m for 12 y and c for 10 y.

$I = \boxed{120}$

The 10-year-old girl has an IQ of 120.

Example: Find the distance travelled by a train which has a velocity of 83 km per hour for 3 hours.

- Formula: $d = v\,t$
- Facts: $v = 83$ km/h, $t = 3$ h
- Substituting: $d = v\,t = (83 \text{ km/h})\,(3 \text{ h})$ — Substitute *v* for 83 km/h and *t* for 3h.

  $d = \boxed{249 \text{ km}}$ — $\left(\frac{83 \text{ km}}{\cancel{\text{h}}}\right)(3\cancel{\text{h}}) = 249$ km

  The distance is 249 km.

Example: Find the volume of a cylinder with a radius of 2.3 cm and a height of 4.2 cm.

- Formula: $V = \pi\, r^2\, h$
- Facts: $r = 2.3$ cm, $h = 4.2$ cm
- Substituting: $V = \pi r^2 h = \pi\,(2.3 \text{ cm})^2\,(4.2 \text{ cm})$ — Substitute *r* for 2.3cm and *h* for 4.2 cm.

  $V \approx \boxed{69.8 \text{ cm}^3}$ — $(\text{cm}^2)\,(\text{cm}) = \text{cm}^3$

  The volume of the cylinder is 69.8 cm^3.

Example: Find the area of a triangle with a base of 12 ft and a height of 34 ft.

- Formula: $A = \frac{1}{2}bh$
- Facts: $b = 12$ ft , $h = 34$ ft
- Substituting: $A = \frac{1}{2}bh = \frac{1}{2}\,(12 \text{ ft})\,(34 \text{ ft})$ — Substitute *b* for 12 ft and *h* for 34 ft.

  $A = \boxed{204 \text{ ft}^2}$ — $(\text{ft})\,(\text{ft}) = \text{ft}^2$

  The area of the triangle is 204 ft^2.

Example: An electric stove top burner runs for 2 hours and uses 750 watts of electricity at a cost of 10 cents per kilowatt-hour. What is the total cost of running the stove top burner?

- Formula: $C = \frac{Wtr}{1000}$
- Facts: $t = 2$ h , $W = 750$ w , $r = 10$¢ / kwh
- Substituting: $C = \frac{Wtr}{1000} = \frac{(750\text{w})(2\text{h})(10¢/kwh)}{1000}$ — Substitute *W*, *t* and *r*.

  $= \boxed{15\ ¢}$

  The cost of running the stove top burner is 15 cents.

Topic B: Solving Formulas

Solving for a Specific Variable

To solve for a variable in a formula: isolate the unknown or desired variable so that it is by itself on one side of the equals sign and all the other terms are on the other side.

- Use the same process as you would for regular linear equations, the only difference is that you will be working with more variables.
- Remember to always do the same thing to both sides of the formula (add, subtract, multiply or divide the same variable or number to both sides of a formula).

Rearrange the formula so that the unknown or desired variable is by itself on one side of the equals sign. You can reverse the sides of the formula if you want.

Example: Solve each formula for the given variable.

1) Solve $\boldsymbol{d = r\,t}$ **for** $\boldsymbol{t}$. — Isolate t (t is the desired variable).

$\frac{d}{r} = \frac{\cancel{r}t}{\cancel{r}}$ — Divide both sides by r.

$\frac{d}{r} = t$ or $\boxed{t = \frac{d}{r}}$ — Reverse the sides of the formula.

Tip: solve a formula for a given letter by isolating the given letter on one side of the formula.

2) Solve $\boldsymbol{I = P\,r\,t}$ **for** $\boldsymbol{r}$ **and** $\boldsymbol{P}$. — Isolate r (r is the desired variable).

$\boldsymbol{r}$: $\frac{I}{P\,t} = \frac{\cancel{P}\,r\,\cancel{t}}{\cancel{P\,t}}$ — Divide both sides by Pt.

$\frac{I}{Pt} = r$ or $\boxed{r = \frac{I}{P\,t}}$ — Reverse the sides of the formula.

$\boldsymbol{P}$: $\frac{I}{r\,t} = \frac{P\,\cancel{r\,t}}{\cancel{r\,t}}$ — Divide both sides by $r\,t$.

$\frac{I}{r\,t} = P$ or $\boxed{P = \frac{I}{r\,t}}$ — Reverse the sides of the formula.

3) Solve $\boldsymbol{P = 2\,w + 2\,l}$ **for** $\boldsymbol{w}$. — Isolate $2w$ (w is the desired variable).

$P - \mathbf{2\,l} = 2\,w + \cancel{2\,l} - \cancel{\mathbf{2\,l}}$ — Subtract $2l$ from both sides.

$P - 2\,l = 2w$ — Divide both sides by 2.

$\frac{P-2l}{\mathbf{2}} = \frac{\cancel{2}w}{\cancel{\mathbf{2}}}$

$\frac{p-2l}{2} = w$ or $\boxed{w = \frac{p-2l}{2}}$ — Reverse the sides of the formula.

More Examples for Solving Formulas

Example: Solve each formula for the given variable.

1) **a)** Solve $\boldsymbol{F = \frac{9}{5}C + 32}$ **for** $\boldsymbol{C}$. **b)** If $\boldsymbol{F = 68}$, $\boldsymbol{C = ?}$

Solution:

a) $F - \mathbf{32} = \frac{9}{5}C + \cancel{32} \ \cancel{-\mathbf{32}}$ — Subtract 32 from both sides.

$F - \mathbf{32} = \frac{9}{5}C$

$\frac{\mathbf{5}}{\mathbf{9}}(F - 32) = \cancel{\frac{\mathbf{5}}{\mathbf{9}}} \cdot \cancel{\frac{9}{5}}C$ — Multiply both sides by $\frac{5}{9}$.

$\frac{5}{9}(F - 32) = C$ or $\boxed{C = \frac{5}{9}(F - 32)}$ — Reverse the sides of the formula.

b) If $F = 68$, $C = \frac{5}{9}(68 - 32)$ — Substitute 68 for F in the formula.

$\boldsymbol{C} = \frac{5}{9}(36)$

$\boxed{C = 20}$

2) Solve $\boldsymbol{P = C + m\,C}$ **for** $\boldsymbol{C}$.

$P = C\,(1 + m)$ — Factor out C.

$\frac{P}{1+m} = \frac{C\,\cancel{(1+m)}}{\cancel{1+m}}$ — Divide both sides by $(1 + m)$.

$\frac{P}{1+m} = C$ or $\boxed{C = \frac{P}{1+m}}$ — Reverse the sides.

3) Solve $\boldsymbol{p = 35\,q^2 + s\,q}$ *for* $\boldsymbol{s}$.

$p - 35\,q^2 = \cancel{35\,q^2} + s\,q - \cancel{35\,q^2}$ — Subtract $35q^2$ from both sides.

$p - 35\,q^2 = s\,q$

$\frac{P - 35q^2}{q} = \frac{s\cancel{q}}{\cancel{q}}$ — Divide both sides by q.

$\boxed{s = \frac{P - 35q^2}{q}}$ — Reverse the sides.

4) Solve $\boldsymbol{x = \frac{y - z}{t}}$ **for** $\boldsymbol{y}$. — Multiply both sides by t.

$x\,t = \frac{y - z}{\cancel{t}} \cdot \cancel{t}$

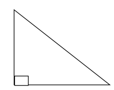
$x\,t + z = y - \cancel{z} + \cancel{z}$ — Add z to both sides.

$\boxed{y = x\,t + z}$ — Reverse the sides.

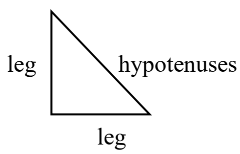
Topic C: Pythagorean Theorem

Pythagorean Theorem

Right triangle: a triangle containing a 90° angle.



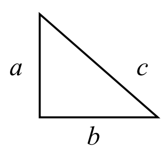
Pythagorean theorem: a relation among the three sides of a right triangle which states that the square of the hypotenuse is equal to the sum of the squares of the other two sides (legs).



$$\boxed{\text{hypotenuse}^2 = \text{leg}^2 + \text{leg}^2}$$

$$\boxed{\text{hypotenuse} = \sqrt{\text{leg}^2 + \text{leg}^2}}\ , \qquad \boxed{\text{leg} = \sqrt{\text{hypotenuse}^2 - \text{leg}^2}}$$

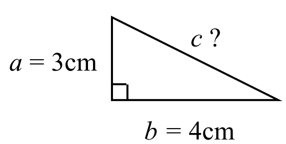
Using the Pythagorean theorem can find the length of the missing side in a right triangle.



$$\boxed{c = \sqrt{a^2 + b^2}} \qquad \boxed{b = \sqrt{c^2 - a^2}} \qquad \boxed{a = \sqrt{c^2 - b^2}}$$

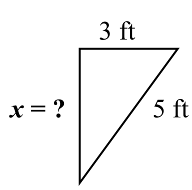
- c is the longest side of the triangle (hypotenuses).
- Other two sides (legs) of the triangle a and b can be exchanged.

Example: Find the missing side of the following triangles.



$$c = \sqrt{a^2 + b^2} = = \sqrt{3^2 + 4^2} = \boxed{5\ \text{cm}}$$

$\text{hypotenuse} = \sqrt{\text{leg}^2 + \text{leg}^2}$

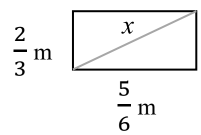


$$x = \sqrt{5^2 - 3^2} = \boxed{4\ \text{ft}}$$

$\text{arm} = \sqrt{\text{hypotenus}^2 - \text{arm}^2}$

Applications of the Pythagorean Theorem

Example: Find the distance of the diagonal across the rectangle.



$x = \sqrt{(\frac{2}{3})^2 + (\frac{5}{6})^2} \approx 1.067$ m $\qquad c = \sqrt{a^2 + b^2}$

$x \approx \boxed{1.067 \text{ m}}$

The distance of the diagonal is 1.067 m.

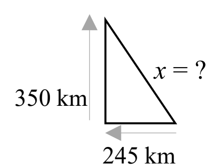
Example: What is the length of one leg of a right triangle whose hypotenuse measures 5.36 cm and the other leg measures 3.24 cm?

$x = \sqrt{5.36^2 - 3.24^2} \approx 4.27$ cm $\qquad a = \sqrt{c^2 - b^2}$

$x \approx \boxed{4.27 \text{ cm}}$

The length of one leg is 4.27 cm.

Example: A plane leaves the Vancouver airport and flies 245 km west, then 350 km north. How far is the plane from the airport?

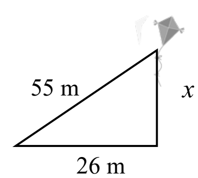


$x = \sqrt{245^2 + 350^2} \approx 427.23$ km

$x \approx \boxed{427.23 \text{ km}}$

The distance of the plane from the airport is 427.23 km.

Example: A kite at the end of a 55 m line is 26 m behind the runner. How high is the kite?



$x = \sqrt{55^2 - 26^2} \approx 48.47$ m

$x \approx \boxed{48.47 \text{ m}}$

The height of the kite is 48.47 m.

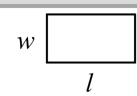
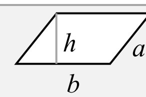
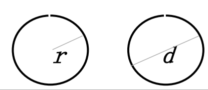
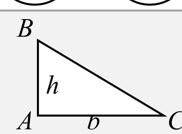
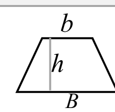
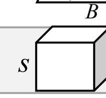
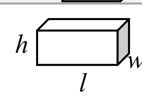
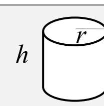
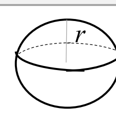
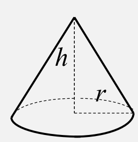
Unit 8: Summary

Formulas

Formula: an equation that contains more than one variable and is used to solve practical problems in everyday life.

Geometry formulas review:

s – side, P – perimeter, C – circumference, A – area, V – volume

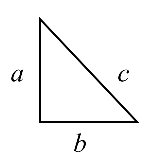
Name of the figure	Formula	Figure
Rectangle	$P = 2w + 2l$ $A = wl$ (w = width, l = length)	
Parallelogram	$P = 2a + 2b$ $A = bh$ (a and b = sides, h = height)	
Circle	$C = \pi d = 2\pi r$ $A = \pi r^2$ (r = radius, d = diameter)	
Triangle	$< A + < B + < C = 180^0$ $A = \frac{1}{2}bh$ (b = base, h = height)	
Trapezoid	$A = \frac{1}{2}h(b + B)$ (b = top base, B = bottom base, h = height)	
Cube	$V = s^3$ (s = side)	
Rectangular solid	$V = wlh$ (w = width, l = length, h = height)	
Cylinder	$V = \pi r^2 h$ (r = radius, h = height)	
Sphere	$V = \frac{4}{3}\pi r^3$ (r = radius)	
Cone	$V = \frac{1}{3}\pi r^2 h$ (r = radius, h = height)	

Substitution into formula: "substitution" means replacing numbers with variables (letters).

Examples of formula:

Application	Formula	Components
Distance	$d = v\,t$	d – distance v – velocity t – time
Simple interest	$I = P\,r\,t$	I – interest P – principle r – interest rate (%) t – time (years)
Compound interest	$B = P\,(100\% + r)^{t}$	B – *balance* P – principle r – interest rate (%) t – time (years)
Percent increase	$\frac{N - O}{O}$	N – new value O – original value
Percent decrease	$\frac{O - N}{O}$	N – new value O – original value
Sale price and Discount	$S = P - d\,P$ $D = d\,P$	S – sale price P – price (original or regular price) d – discount rate D – discount
Original price and Markup	$P = C + m\,C$ $M = m\,C$	P – price (original or selling price) C – cost m – markup rate M – markup
Intelligence quotient (I.Q.)	$I = \frac{100m}{c}$	I – I.Q. m – mental age c – chronological age
Cost of running electrical appliances	$C = \frac{Wtr}{1000}$	C – Cost (in cents) W – power in watts (watts used) t – time (hours) r – rate (per kilowatt-hour)

Pythagorean theorem: a relation among the three sides of a right triangle which states that the square of the hypotenuse is equal to the sum of the squares of the other two sides (legs).



$$c = \sqrt{a^2 + b^2}$$

Using the Pythagorean theorem can find the length of the missing side in a right triangle.

- c is the longest side of the triangle (hypotenuses).
- Other two sides (legs) of the triangle a and b can be exchanged.

Unit 8: Self-Test

Formulas

Topic A

1. Find the IQ of a 70-year-old man with a mental age of 85.

2. Find the distance travelled by a train which has a velocity of 78 km per hour for 2.5 hours.

3. Steve rides his bicycle at a speed of 11 miles per hour. He goes on a 22-mile bike ride. How many hours does this ride take?

4. Find the volume of a cone with a radius of 4.6 cm and a height of 8.4 cm.

5. Find the area and perimeter of a rectangle with a width of 11 cm and a length of 35 cm.

6. Find the area of a triangle with a base of 24 ft and a height of 58 ft.

7. The diameter of a circle is 4.8 ft. Find the circumference and area of the circle.

8. Ann invests $15,000 at an annual interest rate of 0.75%. How much simple interest will she earn by the end of 3 years?

9. An electric stove top burner runs for 2.5 hours and uses 800 watts of electricity per hour at a cost of 9 cents per kilowatt-hour. What is the total cost of running the stove top burner?

Topic B

10. Solve each formula for the given variable.

a) Solve $d = r\,t$ for r.

b) Solve $I = P\,r\,t$ for t.

c) Solve $P = 2\,w + 2\,l$ for l.

d) Solve $C = \frac{5}{9}(F - 32)$ for F.

If $C = 24$, $F = ?$

e) Solve $P = C + m\,C$ for m.

f) Solve $x = 35\,y^2 + z\,y$ for z.

g) Solve $A = \frac{1}{2}\,bh^2$ for b.

h) Solve $x = \frac{y - z}{t}$ for z.

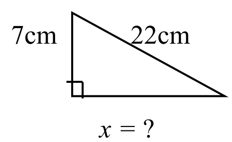
i) Solve $w = \frac{\pi r^2 h}{35}$ for h.

j) Solve $x = y - (2z + 3)\,w$ for w,

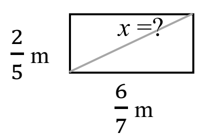
if $x = 2$, $y = 3$, $z = 4$

Topic C

11. Find the missing side of the following triangles.



12. Find the distance of the diagonal across the rectangle.



13. What is the length of one leg of a right triangle whose hypotenuse measures 21.34 ft and the other leg measures 15.27 ft?

14. A plane leaves the Calgary airport and flies 134 km east, then 250 km south. How far is the plane from the airport?

15. A kite at the end of a 89 ft line is 57 ft behind the runner. How high is the kite?

Unit 9

Ratio, Proportion, and Percent

Topic A: Ratio and Rate

Ratio

Ratio

- Ratio: a relationship between two numbers, expressed as the quotient with the ***same unit*** in the denominator and the numerator.
- Express a ratio: there are three ways to write a ratio.

  The ratio of a and b is: a to b or $a:b$ or $\frac{a}{b}$

  Example: Write the ratio of 5 cents to 9 cents.

  5 to 9 or 5 : 9 or $\frac{5}{9}$

- Write a ratio in lowest terms (simplify):
  - Write the ratio in a fractional form.
  - Simplify and drop the units if given (as they cancel each other out).

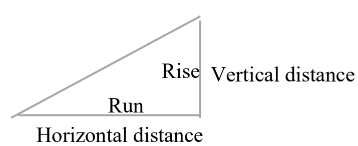
  Example: $4:28=\frac{\overset{\div 4}{4}}{\underset{\div 4}{28}}=\frac{1}{7}$

  Example: 0.75 meters to 0.25 meters $\frac{\overset{\times 100}{0.75m}}{\underset{\times 100}{0.25m}}=\frac{\overset{\div 25}{75}}{\underset{\div 25}{25}}=\frac{3}{1}=3$

Grade and pitch

- Grade (or slope, pitch, incline etc.): the slope of a straight line is the rate of change in height over a distance. It is a measure of the "steepness" "or incline" of a line.
- **The grade or slope formula:**

Formula
Grade or slope $=\frac{\text{vertical distance}}{\text{horizontal distance}}=\frac{\text{rise}}{\text{run}}$



Example: Determine the grade (%) of a road that has a length of 75 m and a vertical height of 3 m.

$$\textbf{Grade}=\frac{\text{vertical distance}}{\text{horizontal distance}}=\frac{3\text{ m}}{75\text{ m}}=0.04=\boxed{4\%}$$

Rate

Rate

- Rate: a ratio of two quantities with different units.

  Example: teachers to students; money to time; distance to time, etc.

  $$\frac{2\text{ teachers}}{83\text{ students}}, \quad \frac{24\text{ dollars}}{3\text{ hours}}, \quad \frac{85\text{ miles}}{2\text{ hours}}$$

- Write a rate in lowest terms (simplify the rate):

  Example: 80 kilometres per 320 minutes: $\frac{80\text{ km}}{320\text{ min}} \overset{\div 80}{\underset{\div 80}{=}} \frac{1\text{ km}}{4\text{ min}}$

Unit rate: a rate in which the number in the ***second term (denominator***) is 1.

Example: 15 dollars per hours: $\frac{\$15}{1\text{ h}} = \boxed{\$\ 15\text{ per h}}$

- Some unit rates:
  - Miles (or kilometres) per hour (or minute).
  - Cost (dollars/cents) per item or quantity.
  - Earnings (dollars) per hour (or week).

- Unit price and the best buy.

  Example: Find the best buy.

  12 eggs for \$ 3.19; 18 eggs for \$4.91; 30 eggs for \$7.13.

  $$\frac{\$3.19}{12\text{ eggs}} \approx \boxed{\$\ 0.266\text{ per egg}}$$

  $$\frac{\$4.91}{18\text{ eggs}} \approx \boxed{\$\ 0.273\text{ per egg}}$$

  $$\frac{\$7.13}{30\text{ eggs}} \approx \boxed{\$\ 0.238\text{ per egg}}$$

  So the best buy is 30 eggs for \$7.13 (the lowest price). $0.238 < 0.266 < 0.273$

Topic B: Proportion

Solving Proportion

Proportion: an equation with a ratio (or rate) on two sides ($\frac{a}{b} = \frac{c}{d}$), in which the two ratios are equal.

Example: Write the following sentence as a proportion.

3 printers is to 18 computers as 2 printers is to 12 computers. $\boxed{\frac{3 \text{ printers}}{18 \text{ computers}} = \frac{2 \text{ printers}}{12 \text{ computers}}}$

Review ratio, rate and proportion:

	Representation		Example
Ratio	a to b or a:b or $\frac{a}{b}$	with the same unit.	5 to 9 or 5:9 or $\frac{5 \text{ km}}{9 \text{ km}}$
Rate	a to b or a:b or $\frac{a}{b}$	with different units.	3 to 7 or 3:7 or $\frac{3 \text{ cm}}{7 \text{ km}}$
Proportion	$\frac{a}{b} = \frac{c}{d}$	an equation with a ratio/rate on each side.	$\frac{x \text{ m}}{5 \text{ km}} = \frac{3\text{m}}{8 \text{ km}}$, $\frac{x \text{ m}}{7 \text{ m}} = \frac{2\text{m}}{5\text{m}}$
Note: the units for both numerators must match and the units for both denominators must match. **Example**: $\frac{\text{in}}{\text{ft}} = \frac{\text{in}}{\text{ft}}$, $\frac{\text{minutes}}{\text{hours}} = \frac{\text{minutes}}{\text{hours}}$			

Solving a proportion:

- Cross multiply: multiply along two diagonals. $\frac{a}{b} = \frac{c}{d}$
- Solve for the unknown.

Example

$$\frac{x}{9} = \frac{2}{6}$$

$$6 \cdot x = 2 \cdot 9$$

$$x = \frac{2 \cdot 9}{6} = \frac{18}{6} = \boxed{3}$$

Application

Example: 4 liters of milk cost \$4.38, how much does a 2 liters of milk cost?

- Facts and unknown:

4 L milk	2 L milk
\$4.38	\$ x = ?

- Proportion: $\frac{4 \text{ L}}{\$4.38} = \frac{2 \text{ L}}{\$x}$

The same units.

The same units.

- Cross multiply: $\frac{4 \text{ L}}{\$4.38} = \frac{2 \text{ L}}{\$x}$

$(4)\,(x) = (2)\,(4.38)$

- Solve for x: $\frac{\cancel{4}x}{\cancel{4}} = \frac{2(4.38)}{4}$ Divide both sides by 4.

  $x = \frac{(2)(4.38)}{4} = 2.19$

  2 liters of milk cost $\boxed{\$2.19}$.

- Check: $\frac{4\text{L}}{\$4.38} = \frac{2\text{L}}{\$2.19}$ Replace x with 2.19.

  $(4)\ (2.19) \overset{?}{=} (2)\ (4.38)$

  $8.76 \overset{\surd}{=} 8.76$ Correct! (LS = RS)

Example: Tom's height is 1.75 meters, and his shadow is 1.09 meters long. A building's shadow is 10 meters long at the same time. How high is the building?

- Facts and unknown:

Tom's height = 1.75 m	Building's height (x) = ?
Tom's shadow = 1.09 m	Building's shadow = 10m

- Proportion: $\frac{1.75\,\text{m}}{1.09\,\text{m}} = \frac{x\,\text{m}}{10\,\text{m}}$ $\frac{\text{Tom's height}}{\text{Tom's shadow}} = \frac{\text{Building's height}}{\text{Building's shadow}}$

- Cross multiply: $\frac{1.75\,\text{m}}{1.09\,\text{m}} = \frac{x\,\text{m}}{10\,\text{m}}$

  $(1.75)\ (10) = (1.09)\ (x)$

- Solve for x: $\frac{(1.75)(10)}{1.09} = \frac{(\cancel{1.09})x}{\cancel{1.09}}$ Divide both sides by 1.09.

  $x = \frac{(1.75)(10)}{1.09} \approx 16.055$ The building's height is $\boxed{16.055\text{m}}$.

- Check: $\frac{1.75\,\text{m}}{1.09\,\text{m}} = \frac{16.055\,\text{m}}{10\,\text{m}}$ Replace x with 16.055.

  $(1.75)\ (10) \overset{?}{=} (16.055)\ (1.09)$

  $17.5 \overset{\surd}{=} 17.5$ Correct! (LS = RS)

Example: If 15 mL of medicine must be mixed with 180 mL of water, how many mL of medicine must be mixed in 230 mL of water?

- Proportion: $\frac{15\text{ mL}}{180\text{ mL}} = \frac{x\text{ mL}}{230\text{ mL}}$ $\frac{15\text{ mL medicine}}{180\text{ mL water}} = \frac{x\text{ mL medicine}}{230\text{ mL water}}$

- Cross multiply: $\frac{15\text{ mL}}{180\text{ mL}} = \frac{x\text{ mL}}{230\text{ mL}}$

- Solve for x: $x = \frac{(15\text{ mL})(230\text{ mL})}{180\text{ mL}} \approx 19.17\text{ mL}$

  $\boxed{19.17\text{ mL}}$ of medicine must be mixed in 230 mL of water.

Topic C: Percent

Percent Review

Percent (%): one part per hundred, or per one hundred.

Review - converting between percent, decimal and fraction:

Conversion	Steps	Example
Percent $\Longrightarrow$ Decimal	Move the decimal point two places to the left, then remove %.	31% = 31.% = 0.31
Decimal $\Longrightarrow$ Percent	Move the decimal point two places to the right, then insert %.	0.317 = 0. 317 = 31.7 %
Percent $\Longrightarrow$ Fraction	Remove %, divide by 100, then simplify.	$15\% = \frac{15}{100} = \frac{3}{20}$
Fraction $\Longrightarrow$ percent	Divide, move the decimal point two places to the right, then insert %.	$\frac{1}{4} = 1 \div 4 = 0.25 = 25\%$
Decimal $\Longrightarrow$ Fraction	Convert the decimal to a percent, then convert the percent to a fraction.	$0.35 = 35\% = \frac{35}{100} = \frac{7}{20}$ % = per one hundred

Two methods to solve percent problems

- Percent proportion method
- Translation (translate the words into mathematical symbols.)

Percent proportion method:

With the word "***is***"

$$\boxed{\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}} \qquad \text{or} \qquad \boxed{\frac{\text{"is" number}}{\text{"of" number}} = \frac{\%}{100}}$$

With the word "***of***"

Step	Example
	8 percent ***of*** what number **is** ***4*** ?
Identify the part, whole, and percent.	Percent Whole (x) Part
Set up the proportion equation.	$\frac{4}{x} = \frac{8}{100}$ $\qquad \frac{\text{Part}}{\text{Whole}} = \frac{\%}{100}$
Solve for unknown (x).	$x = \frac{(4)(100)}{8} = 50$, $\boxed{x = 50}$

Solving Percent Problems

Translation method (translate the words into mathematical symbols):

Translation:

- What ______ x : the word "what" represents an unknown quantity x.
- Is ______ = : the word "is" represents an equal sign.
- of ______ $\times$: the word "of" represents a multiplication sign.
- % to decimal: always change the percent to a decimal.

Example:

1) What is 15% of 80?

$x = 0.15 \bullet 80$ $\qquad x = (0.15)(80) = \boxed{12}$

2) What percent of 90 is 45?

$x\% \bullet 90 = 45$ $\qquad x\% = \frac{45}{90} = 0.5 = \boxed{50\%}$ Divide both sides by 90.

3) 12 is 8% of what number?

$12 = 0.08 \bullet x$ $\qquad x = \frac{12}{0.08} = \boxed{150}$ Divide both sides by 0.08.

- **Percent increase or decrease:**

Application	Formula
Percent increase	Percent increase = $\frac{\text{New value} - \text{Original value}}{\text{Original value}}$, $x = \frac{\text{N} - \text{O}}{\text{O}}$
Percent decrease	Percent decrease = $\frac{\text{Original value} - \text{New value}}{\text{Original value}}$, $x = \frac{\text{O} - \text{N}}{\text{O}}$

Example: A product increased production from ***1500 last month*** to ***1650 this month***. Find the ***percent increase.***

- New value (N): 1650 This month.
- Original value (O): 1500 Last month.
- Percent increase: $x = \frac{\text{N}-\text{O}}{\text{O}} = \frac{1650-1500}{1500} = 0.1 = 10\%$ A 10% increase.

Example: A product was ***reduced*** from ***$33*** to ***$29.*** What percent ***reduction*** is this?

Percent decrease: $x = \frac{\text{O}-\text{N}}{\text{O}} = \frac{33-29}{33} \approx 0.12 = 12\%$ A 12 % decrease.

Topic D: Similar Triangles

Similar Triangles

Similar triangles: triangles that have the same shape and proportions, but may have different sizes.

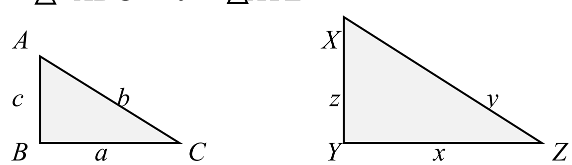
The symbol “△” is used for triangle; the symbol “<” is used for angle.

Sides and angles in a triangle △ :

- Sides are labeled with lower case letters.
- Angles (<) are labeled with uppercase letters.

Corresponding (matching) angles and corresponding sides of two similar triangles:

$\triangle ABC \sim \triangle XYZ$ “∼” means “is similar to”.



- The corresponding angles of two similar triangles are equal.

$< A = < X$ $< B = < Y$ $< C = < Z$

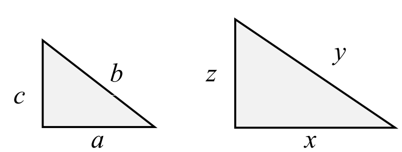
- The corresponding sides of two similar triangles are proportional in length.
  - Side ***a*** corresponds to side ***x***.
  - Side ***b*** corresponds to side ***y***.
  - Side ***c*** corresponds to side ***z***.

The formula for similar triangles:

$$\boxed{\frac{a}{x} = \frac{b}{y} = \frac{c}{z}}$$

This includes three proportions:

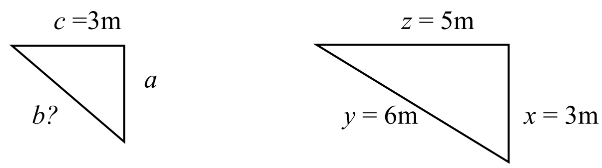
$\boxed{\frac{a}{x} = \frac{b}{y}}$ $\boxed{\frac{a}{x} = \frac{c}{z}}$ $\boxed{\frac{b}{y} = \frac{c}{z}}$



Solving Similar Triangles

Example: Find the value of the missing side in the following figures (the two triangles are similar).

1)



$\frac{b}{y} = \frac{c}{z}$ or $\frac{b}{6m} = \frac{3m}{5m}$

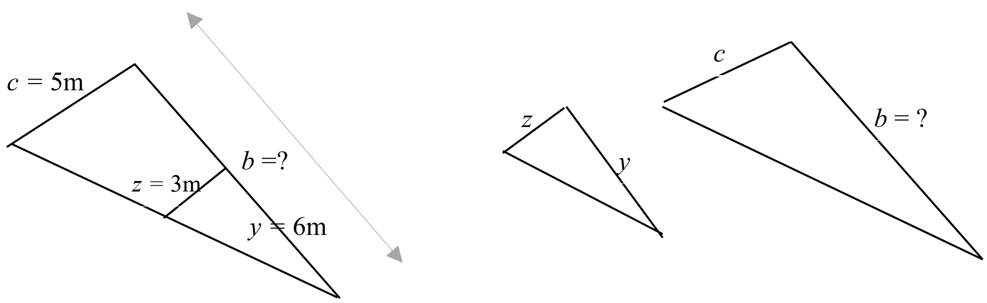
b and y are corresponding sides.

c and z are corresponding sides.

$b = \frac{(3\text{m})(6\text{m})}{5\text{m}} = \boxed{3.6\text{m}}$

Multiply both sides by 6m.

2)



$\frac{b}{y} = \frac{c}{z}$ or $\frac{b}{6\text{m}} = \frac{5\text{m}}{3\text{m}}$

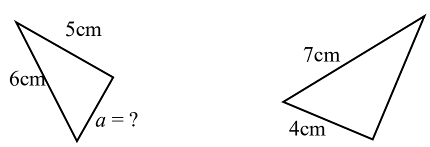
b and y are corresponding sides.

c and *z* are corresponding sides.

$b = \frac{(5\text{m})(6\text{m})}{3\text{m}} = \boxed{10\text{ m}}$

Multiply both sides by 6 m.

3)



$\frac{a}{4cm} = \frac{6cm}{7cm}$

a and 4cm are corresponding sides.

6cm and 7cm are corresponding sides.

$a = \frac{(4cm)(6cm)}{7cm} \approx \boxed{3.43\text{ cm}}$

Multiply both sides by 4cm.

Unit 9: Summary

Ratio, Proportion, and Percent

Ratio, rate and proportion:

	Representation		Example
Ratio	a to b or $a{:}b$ or $\frac{a}{b}$	with the same unit.	5 to 9 or 5:9 or $\frac{5\text{ m}}{9\text{ m}}$
Rate	a to b or $a{:}b$ or $\frac{a}{b}$	with different units.	3 to 7 or 3:7 or $\frac{3\text{ cm}}{7\text{ m}}$
Proportion	$\frac{a}{b}=\frac{c}{d}$	an equation with a ratio/rate on each side.	$\frac{x\text{ m}}{5\text{ km}}=\frac{3\text{m}}{8\text{ km}}$, $\frac{x\text{ m}}{7\text{ m}}=\frac{2\text{m}}{5\text{m}}$
Note: the units for both numerators must match and the units for both denominators must match.			

Unit rate: A rate in which the number in the **second term (denominator)** is 1.

Solving a proportion:

- Cross multiply: multiply along two diagonals. $\frac{a}{b}=\frac{c}{d}$
- Solve for the unknown.

Percent (%): one part per hundred, or per one hundred.

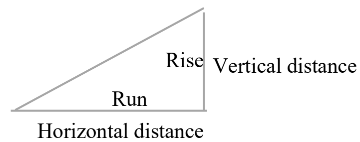
Converting between percent, decimal and fraction:

Conversion	Steps	Example
Percent → Decimal	Move the decimal point two places to the left, then remove %.	31% = 31.% = 0.31
Decimal → Percent	Move the decimal point two places to the right, then insert %.	0.317 = 0. 317 = 31.7 %
Percent → Fraction	Remove %, divide by 100, then simplify.	$15\% = \frac{15}{100} = \frac{3}{20}$
Fraction → Percent	Divide, move the decimal point two places to the right, then insert %.	$\frac{1}{4} = 1 \div 4 = 0.25 = 25\,\%$
Decimal → Fraction	Convert the decimal to a percent, then convert the percent to a fraction.	$0.35 = 35\% = \frac{35}{100} = \frac{7}{20}$

Grade and pitch

- Grade (or slope, pitch, incline etc.): the slope of a straight line is the rate of change in height over a distance. It is a measure of the "steepness" or incline" of a line.
- **The grade or slope formula:**

Formula
Grade or slope $= \dfrac{\text{vertical distance}}{\text{horizontal distance}} = \dfrac{\text{rise}}{\text{run}}$



Two methods to solve percent problems

- Percent proportion method
- Translation (translate the words into math symbols.)

Percent proportion method:

With the word "**is**"

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100} \quad \text{or} \quad \frac{\text{"is" number}}{\text{"of" numbere}} = \frac{\%}{100}$$

With the word "of"

Translation method (translate the words into mathematical symbols):

- What ⸻ x : the word "what" represents an unknown quantity x.
- Is ⸻ $=$: the word "is" represents an equal sign.
- of ⸻ $\times$: the word "of" represents a multiplication sign.
- % to decimal: always change the percent to a decimal.

Percent increase or decrease:

Application	Formula
Percent increase	Percent increase $= \dfrac{\text{New value} - \text{Original value}}{\text{Original value}}$, $x = \dfrac{N-O}{O}$
Percent decrease	Percent decrease $= \dfrac{\text{Original value} - \text{New value}}{\text{Original value}}$, $x = \dfrac{O-N}{O}$

The symbol "△" is used for triangle; the symbol " < " is used for angle.

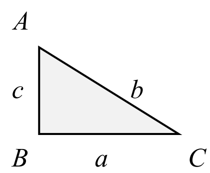
Similar ($\sim$) triangles: triangles that have the same shape and proportions, but may be of different sizes.

Sides and angles in a triangle:

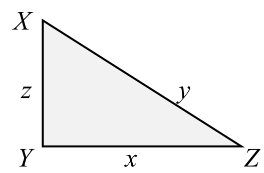
- Sides are labeled with lower case letters.
- Angles (<) are labeled with uppercase letters.

Corresponding angles and corresponding (matching) sides:

$\triangle ABC \sim \triangle XYZ$ $\sim$ means “is similar to”,



1st triangle



2nd triangle

- The corresponding angles of two similar triangles are equal.

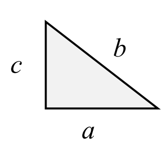
$< A = < X$ $< B = < Y$ $< C = < Z$

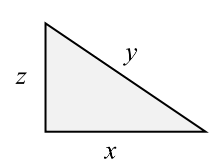
- The corresponding sides of two similar triangles are proportional in length.
  - Side ***a*** corresponds to side ***x***.
  - Side ***b*** corresponds to side ***y***.
  - Side ***c*** corresponds to side ***z***.

Solve similar triangles:

$\boxed{\frac{a}{x} = \frac{b}{y} = \frac{c}{z}}$ This includes three proportions:

$\boxed{\frac{a}{x} = \frac{b}{y}}$ $\boxed{\frac{a}{x} = \frac{c}{z}}$ $\boxed{\frac{b}{y} = \frac{c}{z}}$





Unit 9: Self-Test

Ratio, Proportion, and Percent

Topic A

1. Write the following as a ratio or rate in lowest terms.

a) 15 nickels to 45 nickels.

b) 24 kilometers to 88 kilometers.

c) 350 people for 1500 tickets.

d) 0.33 centimetres to 0.93 centimetres.

e) 160 kilometres per 740 minutes.

2. Determine the grade (%) of a road that has a length of 2,500 m and a vertical height of 3.5m.

3. What is the grade (%) of a river that drops 9 meters over a distance of 720 meters?

4. A train travelled 459 km in 6 hours. What is the unit rate?

5. A 4 L bottle of milk sells for $4.47. A 2 L bottle of the same milk sells for $3.43. What is the best buy?

6. An 8-pound bag of apples costs $7.49. A 6-pound bag of the same apples costs $5.99. What is the best buy?

Topic B

7. Write the following sentence as a proportion.

a) 5 teachers is to 110 students as 15 teachers is to 330 students.

b) 24 hours is to 1,940 kilometers as 12 hours is to 985 kilometers.

8. 4 liters of juice cost $7.38, how much do 2 liters cost?

9. Todd's height is 5.44 feet, and his shadow is 8.5 feet long. A building's shadow is 25 feet long at the same time. How high is the building?

10. Sarah earns $4,500 in 30 days. How much does she earn in 120 days?

Topic C

11. What is 45% of 260?

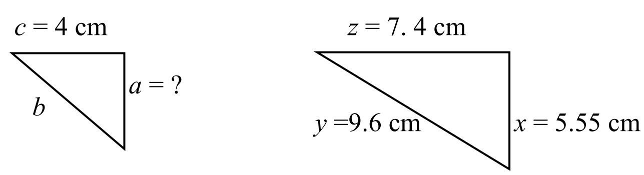
12. 36 is 12% of what number?

13. A product increased production from 2,800 last year to 3,920 this year. Find the percent increase.

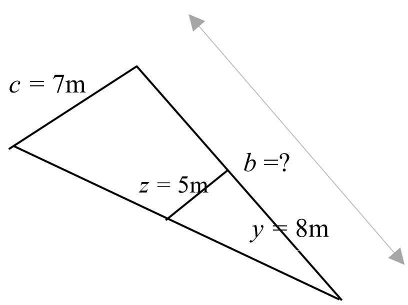
14. A product was reduced from \$199 to \$159. What percent reduction is this?

15. Find the value of the missing side in the following figures (the two triangles are similar).

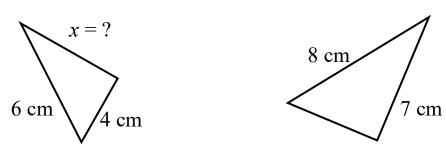
a)



b)



c)



Unit 10

Trigonometry

Topic A: Angles and triangles

- Angles
- Triangles
- Find the missing measurement

Topic B: Trigonometric functions

- Sides and angles
- Trigonometric functions
- Sine, cosine, and tangent

Topic C: Solving right triangles

- Trigonometry using a calculator
- Solving triangles
- Angles of depression and elevation
- Applications of trigonometry

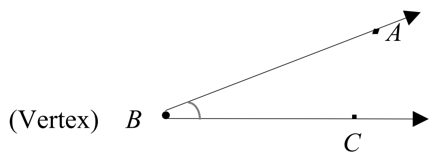
Unit 10 Summary

Unit 10 Self-test

Topic A: Angles and Triangles

Angles

Angle: two rays (sides) that have a common point (the vertex).

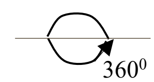


< ***B*** = < *A**B**C* = < *C**B**A*

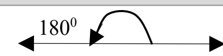
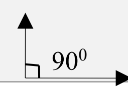
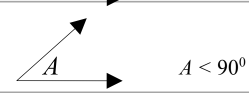
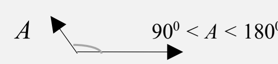
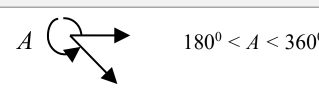
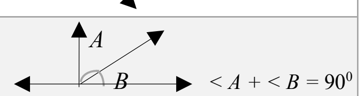
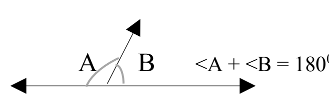
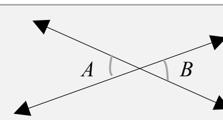
Vertex

The angle *B* in the figure above could be called < ***B*** or < ***ABC*** or < ***CBA***.

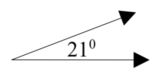
An angle can vary from 0 to 360 degrees (360^0).

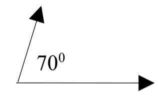


Classifying angles:

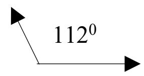
Angle	Definition	Figure
Straight angle	An angle of exactly 180 degrees.	 180^0
Right angle	An angle of exactly 90 degrees.	 90^0
Acute angle	An angle between 0 and 90 degrees. (Less than 90^0)	 A; $A < 90^0$
Obtuse angle	An angle between 90 and 180 degrees.	 A; $90^0 < A < 180^0$
Reflex angle	An angle between 180 and 360 degrees.	 A; $180^0 < A < 360^0$
Complementary angles	Two angles whose sum is exactly 90 degrees.	 A, B; $< A + < B = 90^0$
Supplementary angles	Two angles whose sum is exactly 180 degrees.	 A, B; $<A + <B = 180^0$
Vertical angles	Two angles formed by the intersection of two straight lines. $<A$ and $< B$ are vertical angels.	 A, B

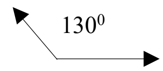
Example: Label each of the following angles.

1) 

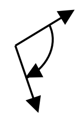
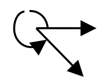


Acute angles.

2) 

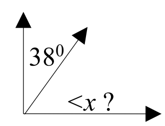


Obtuse angles.

3) 


Obtuse angles. Reflex angle.

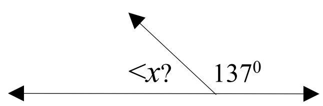
Example: What is the complementary angle to 38 degrees?



$< x + 38^0 = 90^0$

$< x = 90^0 - 38^0 = \boxed{52^0}$

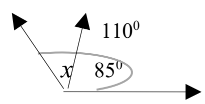
Example: What is the supplementary angle to 137^0?



$< x + 137^0 = 180^0$

$< x = 180^0 - 137^0 = \boxed{43^0}$

Example: What is the size of the angle x?

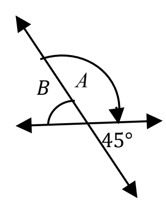


$< x = 110^0 - 85^0 = \boxed{25^0}$

Example **1)** Two angles A and 45^0 that add together to measure 180^0 are said to be____?

supplementary

2) What is the size of angle A and B?

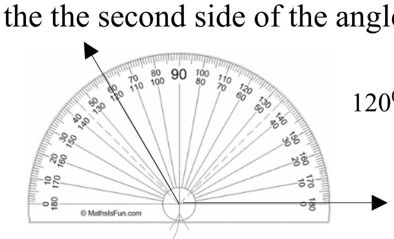


$< A + 45^0 = 180^0$, $< A = 180^0 - 45^0$, $\boxed{< A = 135^0}$

$< A + < B = 180^0$, $< B = 180^0 - <A$, $\boxed{< B = 45^0}$

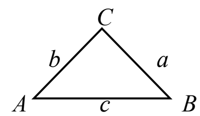
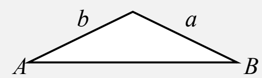
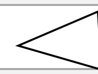
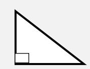
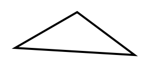
How to use a protractor:

- Place the protractor so that the center hole is over the angle's vertex.
- Line up the base line of the protractor with one of the sides of the angle.
- Read the angle over the the second side of the angle.

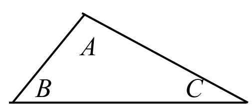


Triangles

Classify triangles:

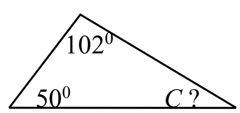
Name of triangle	Definition	Figure
Equilateral triangle	A triangle that has three equal sides and three equal angles. $a = b = c, \quad < A = < B = < C = 60^0$	
Isosceles triangle	A triangle that has two equal sides and two equal angles. $a = b, \quad < A = < B$	
Acute triangle	A triangle that has three acute angles ($< 90^0$).	
Right triangle	A triangle that has a right angle ($= 90^0$). (A right angle is usually marked on the figure as a small square.)	
Obtuse triangle	A triangle that has an obtuse angle ($> 90^0$).	
Scalene triangle	A triangle that has three unequal sides.	

Angles in a triangle: the sum of the three angles in a triangle is always 180^0.



$\boxed{< A + < B + < C = 180^0}$

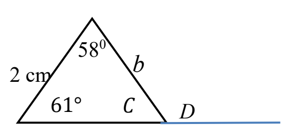
Example: What is the size of angle *C* in the following figure?



$102^0 + 50^0 + < C = 180^0$

$< C = 180^0 - (102^0 + 50^0) = \boxed{28^0}$

Example: What is the size of angle *C, D* and the side *b* in the following figure?



$61^0 + 58^0 + < C = 180^0$

$< C = 180^0 - (61^0 + 58^0) = \boxed{61^0}$

$< D = 180^0 - < C = 180^0 - 61^0 = \boxed{119^0}$

$b = 2$ cm (An isosceles triangle)

Example: Match the following triangles to the letter with the best definition.

______Scalene triangle — a. has three equal sides $\boxed{\text{c.}}$

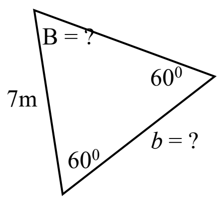
______Equilateral triangle — b. has two equal sides $\boxed{\text{a.}}$

______Isosceles triangle — c. has three unequal sides $\boxed{\text{b.}}$

Find the Missing Measurement

Example: Find the missing measurement and then name the kind of triangle.

1)



$< B = 180^0 - (60^0 + 60^0)$ — $< A + < B + < C = 180^0$

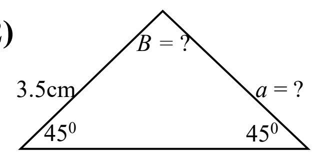
$= \boxed{60^0}$

It is an equilateral triangle. — It has three equal angles.

(An acute triangle: $60^0 < 90^0$.)

$b = \boxed{7 \text{ m}}$ — It is an equilateral triangle.

2)



$< B = 180^0 - (45^0 + 45^0)$ — $< A + < B + < C = 180^0$

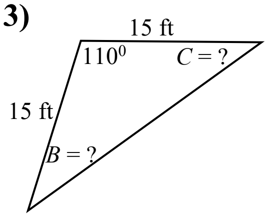
$= \boxed{90^0}$

It is an isosceles triangle. — It has two equal angles.

(An right triangle: it has a 90^0 angle.)

$a = \boxed{3.5 \text{ cm}}$ — It is an isosceles triangle.

3)



It is an isosceles triangle. — It has two equal sides.

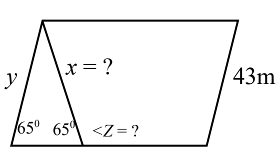
$< B + < C = 180^0 - 110^0$ — $< A + < B + < C = 180^0$

$\boxed{= 70^0}$

$< B = < C = 70^0 \div 2 = \boxed{35^0}$ — It is an isosceles triangle.

(An obtuse triangle: it has an angle $> 90^0$.)

4)



It is an isosceles triangle. — It has two equal angles.

$y = 43$ m — A parallelogram.

$x = y = \boxed{43 \text{ m}}$ — It is an isosceles triangle.

$< Z = 180^0 - 65^0 = \boxed{115^0}$ — Supplementary angles

(An acute triangle: $65^0 < 90^0$)

Topic B: Trigonometric Functions

Sides and Angles

Trigonometry: the study of the relationships between sides and angles of right triangles and trigonometric functions.

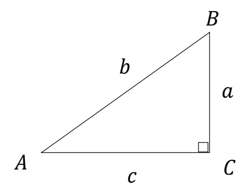
Right triangle review: a triangle that has a 90° angle (right-angled triangle).

Sides and angles:

- < *C* is a right angle (90^0).
- Sides are labeled with lower case letters (or two capital letters).

  Example: The side *a* or *BC,* the side *b* or *AB,* the side *c* or *AC*.
- Angles are labeled with uppercase letters.

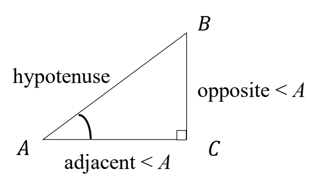
  Example: < *A*, < *B*, < *C*
- Side *a* will be the side opposite angle *A*; side *b* will be the side opposite angle *B*; and side *c* will be the side opposite angle *C*.

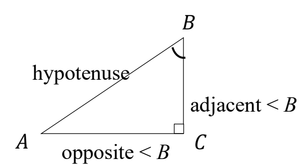


Hypotenuses, adjacent, and opposite:

- The longest side of the triangle is the hypotenuses (the side opposite the 90° angle).
- “Opposite” and “adjacent” refer to sides that are opposite or adjacent to the two acute angles (< *A* and < *B*) of the triangle.
- Adjacent side: the side next to the acute angle.
- Opposite side: the side opposite the acute angle.

  (An acute angle < 90^0.)





Example: Fill in the blanks in each of the following

1) Side *EG* (or *f*) is ________angle *F*. opposite

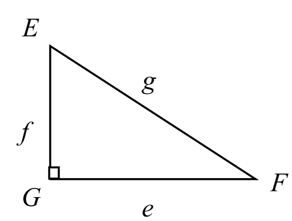
2) Side *FG* (or *e*) is ________angle *F*. adjacent

3) Side *EF* (or *g*) is the ___________ . hypotenuse

4) Side *EG* (or *f*) is ________angle *E*. adjacent

5) Side *FG* (or *e*) is ________angle *E*. opposite

6) Side *EG* is opposite to angle _____ . *F*



Trigonometric Functions

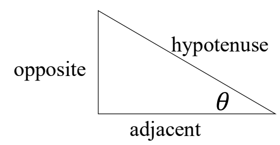
Trigonometric functions (of right triangles):

- There are six trigonometric functions (or ratios): sine (sin), cosine (cos), tangent (tan), secant (sec), cosecant (csc), and cotangent (cot).
- The lengths of the sides are used to define the trigonometric functions (or ratios).

Sine, cosine, and tangent (three main trigonometric functions):

- The sine of the angle $\theta = \dfrac{\text{the length of the opposite side}}{\text{the length of the hypotenuse}}$

  $\boxed{\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}}$



θ is a Greek letter that uses for an angle.

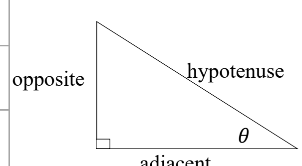
- The cosine of the angle $\theta = \dfrac{\text{the length of the adjacent side}}{\text{the length of the hypotenuse}}$
- $\boxed{\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}}$
- The tangent of the angle $\theta = \dfrac{\text{the length of the opposite side}}{\text{the length of the adjacent}}$

  $\boxed{\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent}}}$

Secant, cosecant, and cotangent: the inverse trigonometric functions.

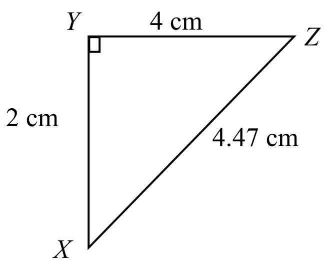
- Secant is the inverse of cosine: $\boxed{\sec\theta = \dfrac{1}{\cos\theta}}$
- Cosecant is the inverse of sine: $\boxed{\csc\theta = \dfrac{1}{\sin\theta}}$
- Cotangent is the inverse of tangent: $\boxed{\cot\theta = \dfrac{1}{\tan\theta}}$

Six trigonometric functions:

Trigonometric function	Read	Diagram	Memory aid
$\boldsymbol{\sin\theta = \dfrac{\textbf{opposite side}}{\textbf{hypotenuse}}}$	Sine of θ		Soh
$\boldsymbol{\cos\theta = \dfrac{\textbf{adjacent side}}{\textbf{hypotenuse}}}$	Cosine of θ		Cah
$\boldsymbol{\tan\theta = \dfrac{\textbf{opposite side}}{\textbf{adjacent side}}}$	Tangent of θ		Toa
$\boldsymbol{\sec\theta = \dfrac{1}{\cos\theta}}$	Secant of θ		Inverse of cosine
$\boldsymbol{\csc\theta = \dfrac{1}{\sin\theta}}$	Cosecant of θ		Inverse of sine
$\boldsymbol{\cot\theta = \dfrac{1}{\tan\theta}}$	Cotangent of θ		Inverse of tangent

Sine, Cosine, and Tangent

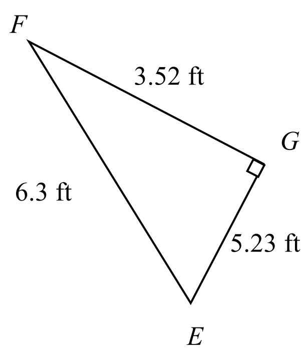
Example: Find the sine, cosine, and tangent for each of the following.



$\sin X = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{4\ \cancel{\text{cm}}}{4.47\ \cancel{\text{cm}}} \approx \boxed{0.8949}$	Soh
$\cos X = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{2\text{ cm}}{4.47\text{ cm}} \approx \boxed{0.4474}$	Cah
$\tan X = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{4\text{ cm}}{2\text{ cm}} = \boxed{2}$	Toa
$\sin Z = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{2\text{ cm}}{4.47\text{ cm}} \approx \boxed{0.4474}$	
$\cos Z = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{4\text{ cm}}{4.47\text{ cm}} \approx \boxed{0.8949}$	
$\tan Z = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{2\text{ cm}}{4\text{ cm}} = \dfrac{1}{2} = \boxed{0.5}$	

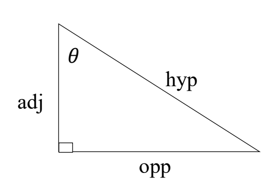
The sine of one angle in the right triangle is equal to the cosine of the other angle in that same right triangle.

Example: Find the sine, cosine, and tangent for each of the following.



$\sin F = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{5.23\ \cancel{\text{ft}}}{6.3\ \cancel{\text{ft}}} \approx \boxed{0.8302}$	Soh
$\cos F = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{3.52\text{ ft}}{6.3\text{ ft}} \approx \boxed{05587}$	Cah
$\tan F = \dfrac{\text{opp}}{\text{adj}} = \dfrac{5.23\text{ ft}}{3.52\text{ ft}} = \boxed{1.4858}$	Toa
$\sin E = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{3.52\text{ ft}}{6.3\text{ ft}} \approx \boxed{0.5587}$	
$\cos E = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{5.23\text{ ft}}{6.3\text{ ft}} \approx \boxed{0.8302}$	
$\tan E = \dfrac{\text{opp}}{\text{adj}} = \dfrac{3.52\text{ ft}}{5.23\text{ ft}} = \boxed{0.673}$	

Memory Aid:

Sine, cosine, and tangent	Trigonometric function	Memory aid	Diagram
Sine	$\sin \boldsymbol{\theta} = \dfrac{\textbf{opp}}{\textbf{hyp}}$	Soh	
Cosine	$\cos \boldsymbol{\theta} = \dfrac{\textbf{adj}}{\textbf{hyp}}$	Cah	
Tangent	$\tan \boldsymbol{\theta} = \dfrac{\textbf{opp}}{\textbf{adj}}$	Toa	

Topic C: Solving Right Triangles

Trigonometry Using a Calculator

Find the trigonometric functions of an angle:

Example: Find each of the following using a scientific calculator.

1) $\sin 132^0 = ?$

Type in: $\boxed{\sin}$ 132 Display: **0.7431**… $\sin 132^0 \approx 0.7431$

Or 132 $\boxed{\sin}$ with some calculators.

2) $\cos 25^0 = ?$

Type in: $\boxed{\cos}$ 25 $\boxed{=}$ Display: **0.9063 …** $\cos 25^0 \approx 0.9063$

Or 25 $\boxed{\cos}$ with some calculators.

3) $\tan 48^0 = ?$

Type in: $\boxed{\tan}$ 48 Display: **1.11061**… $\tan 48^0 \approx 1.1106$

Or 48 $\boxed{\tan}$ with some calculators.

Find an angle when given the trigonometric function (ratio):

Example: Find each of the following using a scientific calculator.

1) $\sin A = 0.5446$, $< A = ?$

Type in: $\boxed{2ndF}$ $\boxed{\sin^{-1}}$ 0.5446 $\boxed{=}$ Display: **32.997333**… $< A \approx 33^0$

Or $\boxed{INV}$ with some calculators.

2) $\tan B = 0.57736$, $< B = ?$

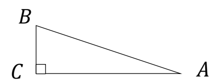
Type in: $\boxed{2ndF}$ $\boxed{\tan^{-1}}$ 0.57736 $\boxed{=}$ Display: **30.000418**… $< B \approx 30^0$

Or $\boxed{INV}$ with some calculators.

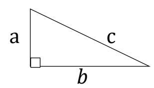
Solving Triangles

Angles in a triangle: the sum of the three internal angles in a triangle is always 180^0.

$\boxed{< A + < B + < C = 180^0}$



Pythagorean theorem review: a relationship between the three sides of a right triangle.



$\boxed{c = \sqrt{a^2 + b^2}}$

There are six elements (or parts) in a triangle, that is, three sides and three internal angles.

Solving a triangle: to solve a triangle means to know all three sides and all three angles.

Example: 1) Solve for the variable.

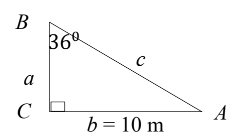
$\tan 32^0 = \frac{7}{x}$, $\quad x = ?$

$x \cdot \tan 32^0 = \frac{7}{\cancel{x}} \cdot \cancel{x}$ Multiply both sides by x.

$\frac{x \cdot \cancel{\tan 32^0}}{\cancel{\tan 32^0}} = \frac{7}{\tan 32^0}$ Divide both sides by $\tan 32^0$.

$\boldsymbol{x} = \frac{7}{\tan 32^0} \approx \boxed{11.2}$ Use a calculator.

2) Find side c if $b = 10\text{m}$ and $< B = 36^0$.



$\sin 36^0 = \frac{10\text{m}}{c}$ $\quad \sin = \frac{\text{opp}}{\text{hyp}}$

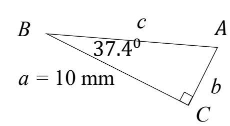
$\sin 36^0 \cdot c = \frac{10\text{m}}{\cancel{c}} \cdot \cancel{c}$ Multiply both sides by c.

$\frac{\cancel{\sin 36^0}}{\cancel{\sin 36^0}} \cdot c = \frac{10\text{m}}{\sin 36^0}$ Divide both sides by $\sin 36^0$.

$c = \frac{10\text{m}}{\sin 36^0} \approx \boxed{17.01\text{ m}}$

Example: Solve the triangle ($< A = ?\quad b = ?\quad c = ?$). Find all unknown sides and angles.

$< A + < B + < C = 180^0$



- $< \boldsymbol{A} = 180^0 - (< C + < B)$

  $= 180^0 - (90^0 + 37.4^0)$

  $= \boxed{52.6^0}$

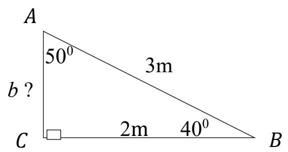
- $\tan B = \frac{b}{a}$ $\quad \tan = \frac{\text{opp}}{\text{adj}}$

  $b = a\,(\tan B) = 10\,(\tan 37.4) \approx \boxed{7.65\text{ mm}}$ Multiply both sides by a and reverse the sides.

- $\boldsymbol{c} = \sqrt{a^2 + b^2} = \sqrt{10^2 + 7.65^2} \approx \boxed{12.59\text{ mm}}$ Pythagorean theorem.

Example: Find the missing part of each triangle.

1)



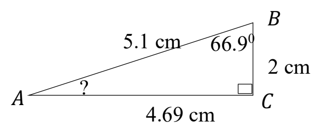
$\cos 50^0 = \frac{b}{3}$ $\qquad$ $\cos = \frac{\text{adj}}{\text{hyp}}$

$3 \cdot \cos 50^0 = \frac{b}{\cancel{3}} \cdot \cancel{3}$ $\qquad$ Multiply both sides by 3.

$3\,(\cos 50^0) = b$

$\boldsymbol{b} = 3\,(\cos 50^0) \approx \boxed{1.928\text{ m}}$ $\qquad$ Reverse the sides of the equation.

2)



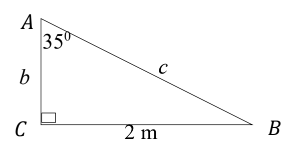
$\sin A = \frac{2}{5.1}$ $\qquad$ $\sin = \frac{\text{opp}}{\text{hyp}}$

$< \boldsymbol{A} = \sin^{-1}\left(\frac{2}{5.1}\right) \approx \boxed{23.1^0}$ $\qquad$ $\boxed{\text{2nd F}}$ $\boxed{\sin^{-1}}$

Example: Solve the right triangle. $\qquad$ Find all unknown sides and angles.

1) **< B:** $< \boldsymbol{B} = 180^0 - (< C + < A)$ $\qquad$ $(< B = ? \quad b = ? \quad c = ?)$

$= 180^0 - (90^0 + 35^0) = \boxed{55^0}$



b: $\tan 35^0 = \frac{2}{b}$ $\qquad$ $\tan = \frac{\text{opp}}{\text{adj}}$

$b \cdot \tan 35^0 = \frac{2}{\cancel{b}} \cdot \cancel{b}$ $\qquad$ Multiply both sides by *b*.

$\frac{\cancel{b \tan 35^0}}{\cancel{\tan 35^0}} = \frac{2}{\tan 35^0}$ $\qquad$ Divide both sides by $\tan 35^0$.

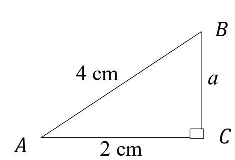
$\boldsymbol{b} = \frac{2}{\tan 35^0} \approx \boxed{2.856\text{ m}}$

c: $\boldsymbol{c} = \sqrt{a^2 + b^2} = \sqrt{2^2 + 2.856^2} \approx \boxed{3.487\text{ m}}$ $\qquad$ Pythagorean theorem.

2) ***a***: $\boldsymbol{a} = \sqrt{c^2 - b^2} = \sqrt{4^2 - 2^2} \approx \boxed{3.464\text{ m}}$ $\qquad$ $(a = ? \quad < B = ? \quad < A = ?)$

<A: $\cos A = \frac{2}{4} = 0.5$ $\qquad$ $\cos = \frac{\text{adj}}{\text{hyp}}$

$< \boldsymbol{A} = \cos^{-1} A = \cos^{-1} 0.5 = \boxed{60^0}$ $\qquad$ $\boxed{\text{2nd F}}$ $\boxed{\cos^{-1}}$



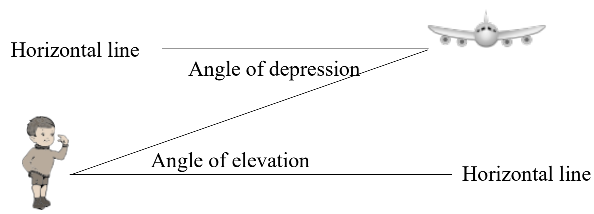
<B: $<\boldsymbol{B} = 180^0 - (90^0 + 60^0) = \boxed{30^0}$

Check: $< A + < B + < C \overset{?}{=} 180^0, \quad 60^0 + 30^0 + 90^0 \overset{\surd}{=} \boxed{180^0}$ $\qquad$ Correct!

Angles of Depression and Elevation

Angle of depression: the angle between a horizontal line and the line of sight for an object below the horizontal. The word "depression" means "fall" or "drop".

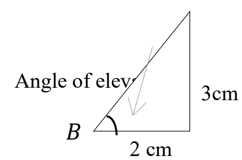
Angle of elevation: the angle between a horizontal line and the line of sight for an object above the horizontal. The word "elevation" means "rise" or "move up".



Example: 1) Find the angle of elevation.

$\tan B = \frac{3}{2} = 1.5$ $\qquad \tan = \frac{\text{opp}}{\text{adj}}$

$< \boldsymbol{B} = \tan^{-1} B = \tan^{-1} \frac{3}{2} \approx \boxed{56.3^0}$ $\qquad$ 2nd F $\quad \tan^{-1}$

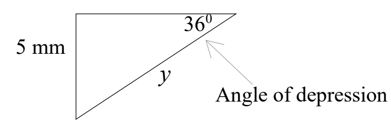


2) Find y if the angle of depression is 36^0.

$\sin 36^0 = \frac{5}{y}$ $\qquad \sin = \frac{\text{opp}}{\text{hyp}}$

$\boldsymbol{y} = \frac{5}{\sin 36^0} \approx \boxed{8.507 \text{ mm}}$

(Divide both sides by $\sin 36^0$ and multiply both sides by y.)



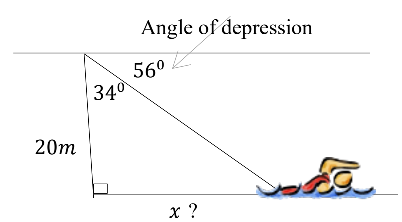
Example: From the top of a rock wall, the angle of depression to a swimmer is 56^0. If the wall is 20m high, how far from the base of the wall is the swimmer?

$90^0 - 56^0 = 34^0$

$\tan 34^0 = \frac{x}{20}$ $\qquad \tan = \frac{\text{opp}}{\text{adj}}$

$\boldsymbol{x} = 20 (\tan 34^0) \approx \boxed{13.49 \text{ m}}$

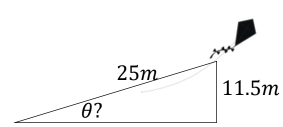
(Multiply both sides by 20 and reverse the sides of the equation.)



Example: Mike has let 25 m of string out on his kite. He is flying it 11.5 m above his eye level. Find the angle of elevation of the kite.

$\sin \theta = \frac{11.5}{25} \approx 0.46$ $\qquad \sin = \frac{\text{opp}}{\text{hyp}}$

$\theta = \sin^{-1} 0.46 \approx \boxed{27.4.^0}$



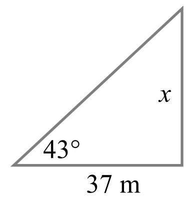
Applications of Trigonometry

Example: When Brandon stands 37 m from the base of a building and sights the top of the building, he is looking up at an angle of 43°. How high is the building?

$\tan 43^0 = \frac{x}{37}$ tan = $\frac{\text{opp}}{\text{adj}}$

$(37)\tan 43^0 = \frac{x}{\cancel{37}} \cdot \cancel{37}$ Multiply both sides by 37.

$x = (37)\tan 43^0 \approx \boxed{34.5\text{ m}}$



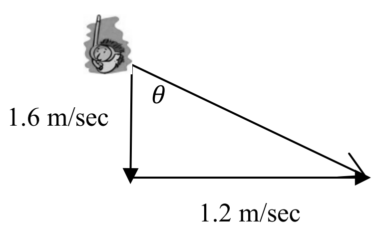
The building is approximately **34.5 m** high.

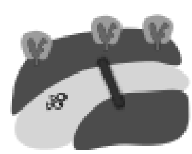
Example: Tom tries to swim straight across a river. He can swim at 1.6 m/sec, but the river is flowing at 1.2 m/sec. At what angle to his intended direction will Tom actually travel?

$\tan\theta = \frac{1.2}{1.6} = 0.75$ tan = $\frac{\text{opp}}{\text{adj}}$

$< \theta = \tan^{-1} 0.75 \approx \boxed{36.87^0}$ $\boxed{\text{2nd F}}$ $\boxed{\tan^{-1}}$

Tom will travel about **36.87°** off course.



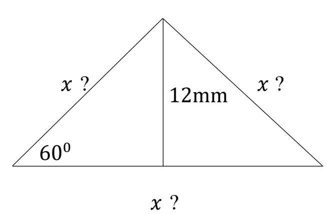


Example: An equilateral triangle has a height of 12 mm. Find the length of each side.

$\sin 60^0 = \frac{12}{x}$ Each angle = 60^0 (an equilateral triangle.)

$\boldsymbol{x} = \frac{12}{\sin 60^0} \approx \boxed{13.86\text{ mm}}$

(Multiply both sides by x and divide both sides by $\sin 60^0$.)



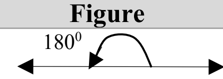
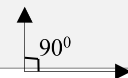
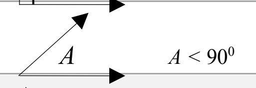
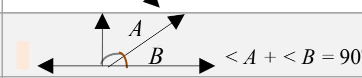
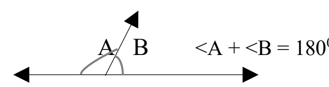
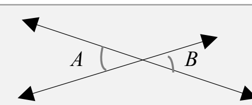
The length of each side is about **13.86 mm**.

Unit 10: Summary

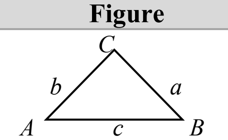
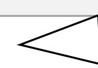
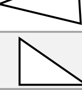
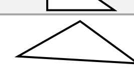
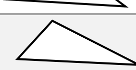
Trigonometry

An angle can vary from 0 to 360 degrees (360^0).

Classifying angles:

Angle	Definition	Figure
Straight angle	An angle of exactly 180^0.	
Right angle	An angle of exactly 90^0.	
Acute angle	An angle between 0 and 90^0.	
Obtuse angle	An angle between 90 and 180^0.	
Reflex angle	An angle between 180 and 360^0.	
Complementary angles	Two angles whose sum is exactly 90^0.	
Supplementary angles	Two angles whose sum is exactly 180^0.	
Vertical angles	Two angles formed by the intersection of two straight lines. $<A$ and $< B$ are vertical angels.	

Classify triangles:

Name of triangle	Definition	Figure
Equilateral triangle	A triangle that has three equal sides and three equal angles. $a = b = c, \quad < A = < B = < C = 60^0$	
Isosceles triangle	A triangle that has two equal sides and two equal angles. $a = b, \quad < A = < B$	
Acute triangle	A triangle that has three acute angles ($< 90^0$).	
Right triangle	A triangle that has a right angle ($= 90^0$).	
Obtuse triangle	A triangle that has an obtuse angle ($> 90^0$).	
Scalene triangle	A triangle that has three unequal sides.	

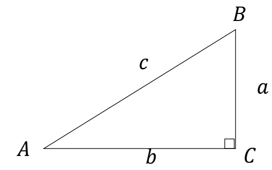
Angles in a triangle: the sum of the three angles in a triangle is always 180^0.

$$< X + < Y + < Z = 180^0$$

How to use a protractor:

- Place the protractor so that the center hole is over the angle's vertex.
- Line up the base line of the protractor with one of the sides of the angle.
- Read the angle over the second side of the angle.

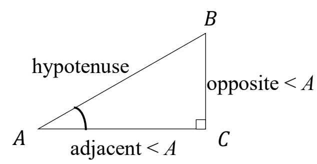
Sides and angles:

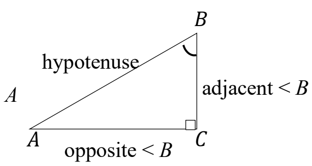


- Sides are labeled with lower case letters (or two capital letters).
- Angles are labeled with uppercase letters.
- Side a will be the side opposite angle A; side b will be the side opposite angle B; and side c will be the side opposite angle C.

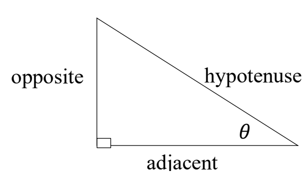
Hypotenuses, adjacent, and opposite:

- The longest side of the triangle is the hypotenuses (the side opposite the 90° angle).
- "Opposite" and "adjacent" refer to sides that are opposite or adjacent to the two acute angles (< A and < B) of the triangle.
- Adjacent side: the side next to the acute angle.
- Opposite side: the side opposite the acute angle.

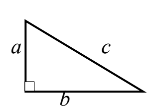




Six trigonometric functions:

Trigonometric function	Diagram	Memory aid
$\sin \boldsymbol{\theta} = \frac{\textbf{opposite}}{\textbf{hypotenuse}}$		Soh
$\cos \boldsymbol{\theta} = \frac{\textbf{adjacent}}{\textbf{hypotenuse}}$		Cah
$\tan \boldsymbol{\theta} = \frac{\textbf{opposite}}{\textbf{adjacent side}}$		Toa
$\csc \boldsymbol{\theta} = \frac{\mathbf{1}}{\mathbf{sin}\ \boldsymbol{A}}$		Inverse of sine
$\sec \boldsymbol{\theta} = \frac{\mathbf{1}}{\mathbf{cos\ A}}$		Inverse of cosine
$\cot \boldsymbol{\theta} = \frac{\mathbf{1}}{\mathbf{tan}\ \boldsymbol{A}}$		Inverse of tangent

Pythagorean theorem review: a relationship between the three sides of a right triangle.

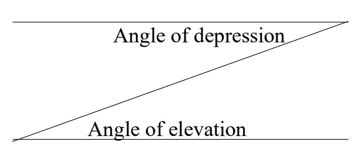


$$c = \sqrt{a^2 + b^2}$$

Solving a triangle: to solve a triangle means to know all three sides and all three angles.

Angle of depression: the angle between a horizontal line and the line of sight for an object below the horizontal.

Angle of elevation: the angle between a horizontal line and the line of sight for an object above the horizontal.



Unit 10: Self-Test

Trigonometry

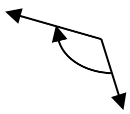
Topic A

1. Label each of the following angles.

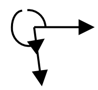
a)

b)

c)



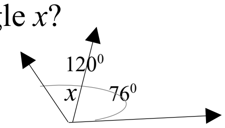
d)



2. What is the complementary angle to 42 degrees?

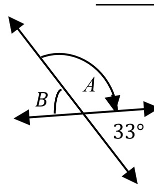
3. What is the supplementary angle to 146 degrees?

4. What is the size of the angle *x*?

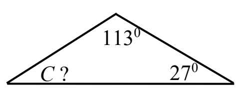


5. **a)** Two angles *A* and 33^0 that add together to measure 180^0 are said to be______?

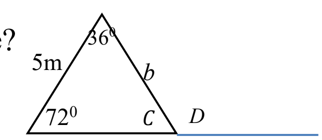
b) What is the size of angle *A* and *B*?



6. What is the size of angle *C* in the following figure?



7. What is the size of angle *C, D* and the side *b* in the following figure?

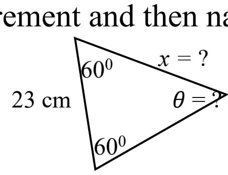


8. Match the following triangles to the letter with the best definition.

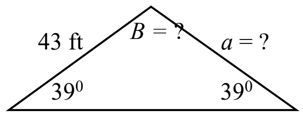
a) Equilateral triangle	i. has two equal sides
b) Isosceles triangle	ii. has three unequal sides
c) Supplementary angles	iii. Two angles whose sum is exactly 180^0.
d) Scalene triangle	iv. has three equal sides

9. Find the missing measurement and then name the kind of triangle.

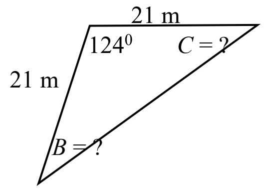
a)



b)



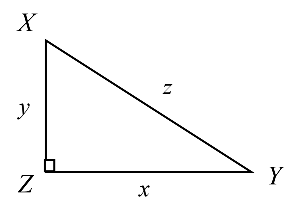
c)



d)

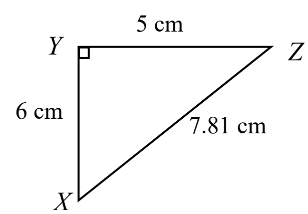
Topic B

10. Fill in the blanks in each of the following

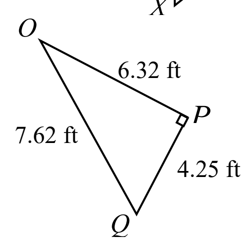


- **a)** Side ZY (or x) is ________angle X.
- **b)** Side XZ (or y) is ________angle X.
- **c)** Side XY (or z) is the ___________ .
- **d)** Side ZY (or x) is ________angle Y.
- **e)** Side XZ (or y) is ________angle Y.
- **f)** Side XZ (or y) is opposite to angle _____ .

11. Find the sine, cosine, and tangent of each acute angle.



12. Find the sine, cosine, and tangent of each acute angle.



Topic C

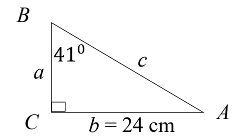
13. Use a calculator to find the trigonometric value of each angle.

- **a)** sin 57° = ?
- **b)** cos 36^0 = ?
- **c)** tan 87^0 = ?
- **d)** sin (   ) = 0.2165
- **e)** cos (   ) = 0.4567
- **f)** tan (   ) = 1.2356

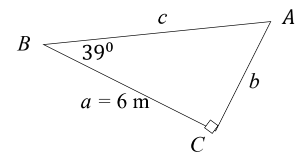
14. Solve for the variable.

$\tan 57^0 = \frac{12}{x}$, $\quad x = ?$

15. Find side c if $b = 24$ cm and $< B = 41^0$.

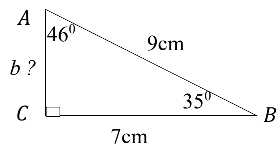


16. Solve the triangle ($< A = ?$ $\;b = ?$ $\;c = ?$).

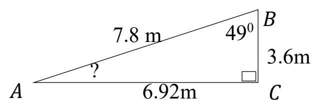


17. Find the missing part of each triangle.

a)

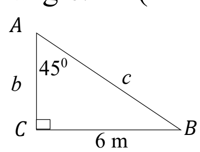


b)

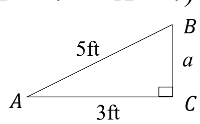


18. Solve the right triangle. ($< B = ?$ $\;b = ?$ $\;c = ?$)

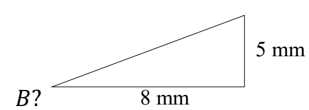
a)



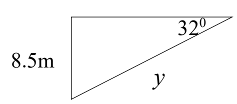
b) ($a = ?$ $\;< B = ?$ $\;< A = ?$)



19. **a)** Find the angle of elevation.



b) Find y if the angle of depression is 32^0.



20. From the top of a wall, the angle of depression to a boy is 43^0. If the wall is 24 m high, how far from the base of the wall is the boy?

21. Todd has let 34 m of string out on his kite. He is flying it 22.4 m above his eye level. Find the angle of elevation of the kite.

22. When Ann stands 28 m from the base of a building and sights the top of the building, she is looking up at an angle of 39°. How high is the building?

23. Damon tries to swim straight across a river. He can paddle at 1.3 m/sec, but the river is flowing at 1.5 m/sec. At what angle to his intended direction will Damon actually travel?

24. An equilateral triangle has a height of 41 cm. Find the length of each side.

Unit 11

Exponents, Roots and Scientific Notation

Topic A: Exponents

Basic Exponent Properties Review

Exponent review: a^n or Base$^{\text{Exponent}}$

Exponential notation	Example
Power **Exponent** $\boxed{a^n} = a \cdot a \cdot a \cdot a \ldots a$ Base Read "*a* to the *n*th" or "the *n*th power of *a*."	$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$ Read "2 to the 4th."

Exponents - basic properties:

Name	Property
Zero Exponent a^0	$a^0 = 1$ (0^0 is undefined)
One Exponent a^1	$a^1 = a$
	$1^n = 1$

Example: Write the following exponential expressions in expanded form.

	Exponential expressions	Expanded form	
1)	$\mathbf{4^3}$	$\boxed{4 \cdot 4 \cdot 4}$	$a^n = a \cdot a \cdot a \ldots$
2)	$(-\boldsymbol{u})^3$	$\boxed{(-u)\ (-u)\ (-u)}$	
3)	$-\boldsymbol{u}^3$	$\boxed{-u \cdot u \cdot u}$	
4)	$(2x^3y^0)^2$	$\boxed{(2x^3y^0)\ (2x^3y^0)}$	
5)	$\left(\frac{-5}{7}\boldsymbol{w}\right)^3$	$\boxed{\left(\frac{-5}{7}w\right)\left(\frac{-5}{7}w\right)\left(\frac{-5}{7}w\right)}$	

Example: Write each of the following in the exponential form.

	Expanded form	Exponential notation	
1)	**(0.3) (0.3) (0.3)**	$\boxed{(0.3)^3}$	
2)	$(4t)\ (4t)\ (4t)\ (4t)$	$\boxed{(4t)^4}$	
3)	$(3x)\ (2y)\ (x)\ (2y)$	$\boxed{12x^2\ y^2}$	$12(x \cdot x)\ (y \cdot y)$

Example: Evaluate.

1) $\mathbf{2x^3 + y}$, **for** $\boldsymbol{x = 2}$, $\boldsymbol{y = 3}$

$2x^3 + y = 2 \cdot 2^3 + 3$ — Substitute *x* for 2 and *y* for 3.

$= 2\ (8) + 3 = \boxed{19}$

2) $\mathbf{(2a)^4 - b}$, **for** $\boldsymbol{a = 1}$, $\boldsymbol{b = 4}$ — Substitute *a* for 1 and *b* for 4.

$(2a)^4 - b = (2 \cdot 1)^4 - 4$

$= 2^4 - 4 = \boxed{12}$

Degree of a Polynomial

The degree of a *term* with *one* variable: the exponent of its variable.

Example: $9x^3$ degree: $\boxed{3}$

$-7u^5$ degree: $\boxed{5}$

$2a$ degree: $\boxed{1}$ $2a = 2a^1$, $a^1 = a$

The degree of a *term* with *more* variables: the sum of the exponents of its variables.

Example: $-8x^2 y^4 z^3$ degree: $2 + 4 + 3 = \boxed{9}$

The degree of a *polynomial* with more variables: the highest degree of any individual term.

Example: $\underbrace{9t^2u}_{3} + \underbrace{4t^3u^2v^5}_{10} - \underbrace{6t}_{1} + 5$ degree: $\boxed{10}$ $3 + 2 + 5 = 10$, $a^1 = a$

Examples of degree of a polynomial:

Polynomial	$5x^3 - x^2 + 21$	$2x^2y - 5z + 7x^4y^2z$
Term	$5x^3$, $-x^2$, 21	$2x^2y$, $-5z$, $7x^4y^2z$
Degree of the term	3 , 2, 0	3, 1 , 7
Degree of the polynomial	3	7

Example: What is the degree of the following term / polynomial?

1) $3xy^3$ degree: $\boxed{4}$

2) $\underbrace{2bc^3d^5}_{9} + \underbrace{5e^2}_{2} - \underbrace{fg^2}_{3} + \underbrace{2e^0}_{0}$ degree: $\boxed{9}$

Descending order: the exponent of a variable ***decreases*** for each succeeding term.

Example: $9x^4 - 7x^3 + x^2 - x + 2$ $a^1 = a$

$21uv^3 - uv^2 + 4v - 67$ The descending order of exponent v.

Ascending order: the exponent of a variable ***increases*** for each succeeding term.

Example: $13 - 8a + 34a^2 - 12a^3$ $a^1 = a$

$-7 + \frac{3}{5}wz + 3.5w^2z^2 - 5z^3 + z^4$ The ascending order of power z.

Topic B: Properties of Exponents

Properties of Exponents

Properties of exponents:

Name	Rule	Example
Product rule	$a^m\, a^n = a^{m+n}$	$2^3\, 2^2 = 2^{3+2} = 2^5 = 32$
Quotient rule (the same base)	$\frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$	$\frac{y^4}{y^2} = y^{4-2} = y^2$
Power of a power	$(a^m)^n = a^{mn}$	$(x^3)^2 = x^{3\cdot 2} = x^6$
Power of a product (different bases)	$(a \cdot b)^n = a^n\, b^n$	$(2 \cdot 3)^2 = 2^2\, 3^2 = 4 \cdot 9 = 36$
	$(a^m \cdot b^n)^p = a^{mp}\, b^{np}$	$(t^3 \cdot s^4)^2 = t^{3\cdot 2}\, s^{4\cdot 2} = t^6\, s^8$
Power of a quotient (different bases)	$(\frac{a}{b})^n = \frac{a^n}{b^n}$ $(b \neq 0)$	$(\frac{2}{3})^2 = \frac{2^2}{3^2} = \frac{4}{9}$
	$(\frac{a^m}{b^n})^p = \frac{a^{mp}}{b^{np}}$ $(b \neq 0)$	$(\frac{q^2}{p^4})^3 = \frac{q^{2\cdot 3}}{p^{4\cdot 3}} = \frac{q^6}{p^{12}}$
Negative exponent a^{-n}	$a^{-n} = \frac{1}{a^n}$ $(a \neq 0)$	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
	$\frac{1}{a^{-n}} = a^n$ $(a \neq 0)$	$\frac{1}{4^{-2}} = 4^2 = 16$
Zero exponent a^0	$a^0 = 1$	$15^0 = 1$
One exponent a^1	$a^1 = a$ (But $1^n = 1$)	$7^1 = 7$, $1^{13} = 1$

Properties of exponents explained:

- **Product rule** (multiplying the same base): when multiplying two powers with the same base, keep the base and add the exponents. $\boxed{a^m\, a^n = a^{m+n}}$

  a^n or $\text{Base}^{\text{Exponent}}$

  Example: $\mathbf{2^3\, 2^2} = (2 \cdot 2 \cdot 2)(2 \cdot 2) = 2^5 = \boxed{32}$

  Or $\mathbf{2^3\, 2^2} = 2^{3+2} = 2^5 = \boxed{32}$

  A short cut, $a^m\, a^n = a^{m+n}$

- **Quotient rule** (dividing the same base): when dividing two powers with the same base, keep the base and subtract the exponents. $\boxed{\frac{a^m}{a^n} = a^{m-n}}$

  Example: $\frac{\mathbf{2^4}}{\mathbf{2^2}} = \frac{2 \cdot 2 \cdot \cancel{2} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{2}} = 2^2 = \boxed{4}$

  Or $\frac{\mathbf{2^4}}{\mathbf{2^2}} = 2^{4-2} = 2^2 = \boxed{4}$

  A short cut, $\frac{a^m}{a^n} = a^{m-n}$

  This law can also show that why $a^0 = 1$ (zero exponent a^0): $\frac{a^2}{a^2} = a^{2-2} = a^0 = 1$

- **Power rule:**
  - Power of a power: when raise a power to a power, just multiply the exponents.

    $\boxed{(a^m)^n = a^{mn}}$

    Example: $\mathbf{(4^3)^2} = (4^3)(4^3) = (4 \cdot 4 \cdot 4)(4 \cdot 4 \cdot 4) = 4^6 = \boxed{4096}$

    Or $\mathbf{(4^3)^2} = 4^{3 \cdot 2} = 4^6 = \boxed{4096}$ A short cut, $(a^m)^n = a^{mn}$

  - Power of a product: when raise a power to different bases, distribute the exponent to each base. $\boxed{(a \cdot b)^n = a^n b^n}$

    Example: $\mathbf{(2 \cdot 3)^2} = (2 \cdot 3)(2 \cdot 3) = 6 \cdot 6 = \boxed{36}$

    Or $\mathbf{(2 \cdot 3)^2} = 2^2 3^2 = 4 \cdot 9 = \boxed{36}$ A short cut, $(a \cdot b)^n = a^n b^n$

  - Power of a product (different bases): when raise a power to a power with different bases, multiply each exponent inside the parentheses by the power outside the parentheses. $\boxed{(a^m \cdot b^n)^p = a^{mp} b^{np}}$

    Example: $\mathbf{(2^2 \cdot 3^2)^2} = (2^2 \cdot 3^2)(2^2 \cdot 3^2) = (2^2 \cdot 2^2)(3^2 \cdot 3^2) = 16 \cdot 81 = \boxed{1296}$

    Or $\mathbf{(2^2 \cdot 3^2)^2} = 2^{2 \cdot 2} 3^{2 \cdot 2} = 2^4 3^4 = 16 \cdot 81 = \boxed{1296}$ A short cut, $(a \cdot b)^n = a^n b^n$

- **Power of a quotient** (different bases):
  - When raise a fraction to a power, distribute the exponent to the numerator and denominator of the fraction. $\boxed{(\frac{a}{b})^n = \frac{a^n}{b^n}}$

    Example: $\mathbf{(\frac{2}{3})^3} = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} = \frac{2^3}{3^3} = \boxed{\frac{8}{27}}$

    Or $\mathbf{(\frac{2}{3})^3} = \frac{2^3}{3^3} = \boxed{\frac{8}{27}}$ A short cut, $(\frac{a}{b})^n = \frac{a^n}{b^n}$

  - When raise a fraction with powers to a power, multiply each exponent in the numerator and denominator by the power outside the parentheses. $\boxed{(\frac{a^m}{b^n})^p = \frac{a^{mp}}{b^{np}}}$

    Example: $\mathbf{(\frac{2^2}{3^4})^3} = \left(\frac{2^2}{3^4}\right)\left(\frac{2^2}{3^4}\right)\left(\frac{2^2}{3^4}\right) = \frac{4 \cdot 4 \cdot 4}{81 \cdot 81 \cdot 81} = \boxed{\frac{64}{531441}}$

    Or $\mathbf{(\frac{2^2}{3^4})^3} = \frac{2^{2 \cdot 3}}{3^{4 \cdot 3}} = \frac{2^6}{3^{12}} = \boxed{\frac{64}{531441}}$ A short cut, $(\frac{a^m}{b^n})^p = \frac{a^{mp}}{b^{np}}$

- **Negative exponent:** a negative exponent is the reciprocal of the number with a positive exponent. $\boxed{a^{-n} = \frac{1}{a^n}}$, $\boxed{\frac{1}{a^{-n}} = a^n}$ a^{-n} is the reciprocal of a^n.

  Example: $\mathbf{3^{-4}} = \frac{1}{3^4} = \boxed{\frac{1}{81}}$ $a^{-n} = \frac{1}{a^n}$

  Example: $\mathbf{\frac{1}{3^{-4}}} = 3^4 = \boxed{81}$ $\frac{1}{a^{-n}} = a^n$

Properties of Exponents − Examples

Example: Simplify (do not leave negative exponents in the answer).

1)	$(-4)^1 = \boxed{-4}$	$a^1 = a$
2)	$(-2345)^0 = \boxed{1}$	$a^0 = 1$
3)	$(-0.3)^3 = \boxed{-0.027}$	$a^n = a \cdot a \cdot a \ldots$
4)	$-5^2 = -(5^2) = \boxed{-25}$	
5)	$x^2\, x^3 = x^{2+3} = \boxed{x^5}$	$a^m\, a^n = a^{m+n}$
6)	$\frac{y^6}{y^4} = y^{6-4} = \boxed{y^2}$	$\frac{a^m}{a^n} = a^{m-n}$
7)	$(x^4)^{-3} = x^{4\,(-3)} = x^{-12} = \boxed{\frac{1}{x^{12}}}$	$(a^n)^m = a^{n\,m}$, $\frac{1}{a^{-n}} = a^n$
8)	$7b^{-1} = 7 \cdot \frac{1}{b^1} = \boxed{\frac{7}{b}}$	$a^{-n} = \frac{1}{a^n}$, $a^1 = a$
9)	$[(-4) \cdot (0.7)]^2 = (-4)^2 \cdot 0.7^2 = (16)\,(0.49) = \boxed{7.84}$	$(a \cdot b)^n = a^n\, b^n$
10)	$(2t^3 \cdot w^2)^4 = 2^4\, t^{3\cdot 4} \cdot w^{2\cdot 4} = \boxed{16\, t^{12}\, w^8}$	$(a^m \cdot b^n)^p = a^{mp}\, b^{np}$
11)	$\frac{1}{3^{-2}} = 3^2 = \boxed{9}$	$\frac{1}{a^{-n}} = a^n$
12)	$(\frac{u}{z})^{-2} = \frac{u^{-2}}{z^{-2}} = \boxed{\frac{z^2}{u^2}}$	$(\frac{a}{b})^n = \frac{a^n}{b^n}$, $a^{-n} = \frac{1}{a^n}$, $\frac{1}{a^{-n}} = a^n$
13)	$(\frac{x^4}{y^{-3}})^2 = \frac{x^{4\cdot 2}}{y^{(-3)(2)}} = \frac{x^8}{y^{-6}} = \boxed{x^8 y^6}$	$(\frac{a^m}{b^n})^p = \frac{a^{mp}}{b^{np}}$, $\frac{1}{a^{-n}} = a^n$
14)	$(2^{-3})(2^3) = \frac{1}{2^3} \cdot 2^3 = \boxed{1}$	$\frac{1}{a^{-n}} = a^n$
15)	$\frac{7x^4 y^{-5}}{9^0 \cdot x^2 y^3} = \frac{7x^{4-2} y^{-5-3}}{1} = 7x^2 y^{-8} = \boxed{\frac{7x^2}{y^8}}$	$a^0 = 1$, $\frac{a^m}{a^n} = a^{m-n}$, $a^{-n} = \frac{1}{a^n}$
16)	$(\frac{e^{-3} f^2}{g^{-2}})^{-2} = \frac{e^{(-3)\,(-2)}\, f^{2(-2)}}{g^{(-2)(-2)}} = \frac{e^6\, f^{-4}}{g^4} = \boxed{\frac{e^6}{g^4\, f^4}}$	$(\frac{a^m}{b^n})^p = \frac{a^{mp}}{b^{np}}$, $\frac{1}{a^{-n}} = a^n$

Using a calculator: $4^2 = ?$ 4 $\boxed{x^2}$ $\boxed{=}$ (The display reads 16)

$3^4 = ?$ 3 $\boxed{x^y}$ 4 $\boxed{=}$ (The display reads 81)

(Or $\boxed{y^x}$ or $\boxed{\wedge}$ on some calculators.)

Simplifying Exponential Expressions

Steps for simplifying exponential expressions:

- Remove parentheses using "power rule" if necessary. $(a^m \cdot b^n)^p = a^{mp} b^{np}$
- Regroup coefficients and variables.
- Use "product rule" and "quotient rule". $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$
- Simplify.
- Use "negative exponent" rule to make all exponents positive if necessary.

Example: Simplify.

1) $(3x^3y^2)^2(2x^{-3}y^{-1})^3(-248z^{-19})^0$

$= 3^2x^{3\cdot2}y^{2\cdot2} \cdot 2^3x^{-3\cdot3} \cdot y^{-1\cdot3} \cdot 1$ — Remove brackets. $(a^m \cdot b^n)^p = a^{mp} b^{np}$, $a^0 = 1$

$= (3^2 \cdot 2^3)(x^6\, x^{-9})(y^4\, y^{-3})$ — Regroup coefficients and variables.

$= 72\, x^{-3}\, y^1$ — Simplify. $a^m a^n = a^{m+n}$

$= \boxed{\frac{72y}{x^3}}$ — Make exponent positive. $a^{-n} = \frac{1}{a^n}$, $a^1 = a$

2) $\left(\frac{(2x^4)(y^5)}{3x^3y^2}\right)^2 = \frac{(2x^4)^2(y^5)^2}{(3x^3y^2)^2}$ — $\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}$

$= \frac{2^2x^{4\cdot2}y^{5\cdot2}}{3^2x^{3\cdot2}y^{2\cdot2}}$ — Remove brackets. $(a \cdot b)^n = a^n b^n$

$= \frac{4}{9} \cdot \frac{x^8}{x^6} \cdot \frac{y^{10}}{y^4}$ — Regroup coefficients and variables.

$= \boxed{\frac{4}{9}x^2y^6}$ — Simplify. $\frac{a^m}{a^n} = a^{m-n}$

Example: Evaluate for $a = 2$, $b = 1$, $c = -1$.

1) $(-29a^{-5}b^4c^{-7})^0 = \boxed{1}$ — $a^0 = 1$

2) $\left(\frac{a}{b}\right)^{-4} = \left(\frac{2}{1}\right)^{-4}$ — Substitute 2 for *a* and 1 for *b*,

$= \frac{2^{-4}}{1^{-4}} = \frac{1^4}{2^4} = \boxed{\frac{1}{16}}$ — $\frac{a^m}{a^n} = a^{m-n}$, $a^{-n} = \frac{1}{a^n}$, $\frac{1}{a^{-n}} = a^n$

3) $(a + b - c)^a = [2 + 1 - (-1)]^2 = 4^2 = \boxed{16}$ — Substitute 2 for *a*, 1 for *b*, and -1 for *c*.

Topic C: Scientific Notation and Square Roots

Scientific Notation

Scientific notation is a special way of concisely expressing very *large* and *small* numbers.

Example: $300{,}000{,}000 = 3 \times 10^8$ m/sec — The speed of light.
$0.00000000000000000016 = 1.6 \times 10^{-19}$ C — An electron.

Scientific notation: a product of a number between 1 and 10 and power of 10.

$N \times 10^{\pm n}$

Scientific notation	Example
$N \times 10^{\pm n}$ — $1 \le N < 10$, n - integer	$67504.3 = 6.75043 \times 10^4$ (Standard form = Scientific notation)

Scientific vs. non-scientific notation:

Scientific notation	Not scientific notation		
7.6×10^3	76×10^2	$76 > 10$	76 is not between 1 and 10.
8.2×10^{13}	0.82×10^{14}	$0.82 < 1$	0.82 is not between 1 and 10.
5.37×10^7	53.7×10^6	$53 > 10$	53 is not between 1 and 10.

Writing a number in scientific notation:

Step	Example
▪ Move the decimal point ***after*** the ***first nonzero digit***.	0.0079 37213000.
▪ Determine n (the power of 10) by counting the number of places you moved the decimal.	$n = 3$ $n = 7$
▪ If the decimal point is moved to the ***right***: $\times 10^{-n}$	$0.0079 = \boxed{7.9 \times 10^{-3}}$ 3 places to the right
▪ If the decimal point is moved to the ***left***: $\times 10^{n}$	$37213000. = \boxed{3.7213 \times 10^7}$ 7 places to the left

Example: Write in scientific notation.

1) **2340000** = 2340000. = $\boxed{2.34 \times 10^6}$ — 6 places to the left, $\times 10^n$
2) **0.000000439** = $\boxed{4.39 \times 10^{-7}}$ — 7 places to the right, $\times 10^{-n}$

Example: Write in standard (or ordinary) form.

1) $\mathbf{6.4275 \times 10^4} = \boxed{64275}$ 2) $2.9 \times 10^{-3} = \boxed{0.0029}$

Example: Simplify and write in scientific notation.

1) $(\mathbf{4.9 \times 10^{-3}})(\mathbf{3.82 \times 10^{8}}) = (4.9 \times 3.82)(10^{-3+8})$ — Multiply coefficients of $10^{\pm n}$, $a^m a^n = a^{m+n}$

$= (18.718 \times 10^5)$ — $18 > 10$, this is not in scientific notation.

$\boxed{= (1.8718 \times 10^6)}$ — N = 1, this is in scientific notation.

2) $\dfrac{(\mathbf{5 \times 10^5})(\mathbf{2.3 \times 10^{-2}})}{\mathbf{4.5 \times 10^7}} = \dfrac{5 \times 2.3}{4.5} \times \dfrac{(10^5 \times 10^{-2})}{10^7}$ — Regroup coefficients of $10^{\pm n}$

$\approx \boxed{2.556 \times 10^{-4}}$ — $a^m a^n = a^{m+n}$, $\dfrac{a^m}{a^n} = a^{m-n}$

Square Roots

Square root ($\sqrt{\ }$): a number with the symbol $\sqrt{\ }$ that is the opposite of the square of a number, such as $\sqrt{9} = 3$ and $3^2 = 9$, respectively.

$$3 \xrightarrow{\text{Square } (3^2)} 9$$
$$3 \xleftarrow{\text{Square root } (\sqrt{9})} 9$$

Perfect square: a number that is the exact square of a whole number.

Perfect square	Square root
$1\times1 = 1^2 = \mathbf{1}$	$\sqrt{1} = 1$
$2\times2 = 2^2 = \mathbf{4}$	$\sqrt{4} = 2$
$3\times3 = 3^2 = \mathbf{9}$	$\sqrt{9} = 3$
$4\times4 = 4^2 = \mathbf{16}$	$\sqrt{16} = 4$
$5\times5 = 5^2 = \mathbf{25}$	$\sqrt{25} = 5$
$6\times6 = 6^2 = \mathbf{36}$	$\sqrt{36} = 6$
$7\times7 = 7^2 = \mathbf{49}$	$\sqrt{49} = 7$
$8\times8 = 8^2 = \mathbf{64}$	$\sqrt{64} = 8$
$9\times9 = 9^2 = \mathbf{81}$	$\sqrt{81} = 9$
…	…

Examples:

Square root	Square
$\sqrt{100} = 10$	$10^2 = 100$
$\sqrt{49} = 7$	$7^2 = 49$
$\sqrt{121} = 11$	$11^2 = 121$
$\sqrt{169} = 13$	$13^2 = 169$
$\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$	$4^2 = 16$ $5^2 = 25$

Using a calculator: $\sqrt{81} = ?$

$\boxed{\sqrt{\ }}$ 81 $\boxed{=}$ (The display reads 9) Or 81 $\sqrt{\ }$ $\boxed{=}$ for some calculators.

Example: Find the square roots.

1) $\sqrt{\mathbf{144}} = \sqrt{12^2} = \boxed{12}$ $\boxed{\sqrt{\ }}$ 144 $\boxed{=}$

2) $\frac{\sqrt{\mathbf{64}}}{\sqrt{\mathbf{225}}} = \frac{\sqrt{8^2}}{\sqrt{15^2}} = \boxed{\frac{8}{15}}$ $\boxed{\sqrt{\ }}$ 225 $\boxed{=}$

Simplifying Square Roots

Order of operations:

Order of operations	
1. the brackets or parentheses and absolute values (innermost first)	( ) , [ ] , { } , | |
2. exponent or square root (from left-to-right)	a^n , $\sqrt{\ }$
3. multiplication or division (from left-to-right)	× and ÷
4. addition or subtraction (from left-to-right)	+ and -

Memory aid - BEDMAS

B	E (R)	D M	A S
Brackets	**E**xponents or Square **R**oot	**D**ivide or **M**ultiply	**A**dd or **S**ubtract

Example: Calculate.

1) $\mathbf{6 - 2\sqrt{81}} = 6 - 2 \cdot 9$ $\qquad 81 = 9^2$

$= 6 - 18 = \boxed{-12}$

2) $\mathbf{3.2^2 - 3\sqrt{2 + 3^2}} = 10.24 - 3\sqrt{11}$

$\approx 10.24 - 3\ (3.32)$

$= 10.24 - 9.96 = \boxed{0.28}$

3) $\dfrac{\sqrt{64}}{\sqrt{250-249}} = \dfrac{8}{\sqrt{1}} = \boxed{8}$ $\qquad 64 = 8^2$, $\sqrt{1} = \sqrt{1^2} = 1$

Simplifying square roots:

- Factor the number inside the square root sign.
  (Find the perfect square(s) that will divide the number).
- Rewrite the square root as a multiplication problem.
- Reduce the perfect squares ("pulling out" the integer(s)).

Example

$\sqrt{\mathbf{75}}$

25 3

$\sqrt{75} = \sqrt{25 \cdot 3}$

$\sqrt{75} = \sqrt{5^2 \cdot 3} = \boxed{5\sqrt{3}}$

Example: Simplify.

1) $\sqrt{\mathbf{180}} = \sqrt{45 \cdot 4} = \sqrt{9 \cdot 5 \cdot 4} = \sqrt{3^2 \cdot 5 \cdot 2^2} = 3 \cdot \sqrt{5} \cdot 2 = \boxed{6\sqrt{5}}$

45 4

9 5

2) $\sqrt{\dfrac{\mathbf{92}}{\mathbf{64}}} = \dfrac{\sqrt{4 \times 23}}{\sqrt{8^2}} = \dfrac{2\sqrt{23}}{8} = \boxed{\dfrac{\sqrt{23}}{4}}$

Unit 11: Summary

Exponents, Roots & Scientific Notation

The degree of a term with one variable: the exponent of its variable.

The degree of a term with more variables: the sum of the exponents of its variables.

The degree of a polynomial with more variables: the highest degree of any individual term.

Descending order: the exponent of a variable decreases for each succeeding term.

Ascending order: the exponent of a variable increases for each succeeding term.

Properties of exponents:

Name	Rule	Example
Product rule	$a^m a^n = a^{m+n}$	$2^3\, 2^2 = 2^{3+2} = 2^5 = 32$
Quotient rule (the same base)	$\frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$	$\frac{y^4}{y^2} = y^{4-2} = y^2$
Power of a power	$(a^m)^n = a^{mn}$	$(x^3)^2 = x^{3 \cdot 2} = x^6$
Power of a product (different bases)	$(a \cdot b)^n = a^n b^n$	$(2 \cdot 3)^2 = 2^2\, 3^2 = 4 \cdot 9 = 36$
	$(a^m \cdot b^n)^p = a^{mp} b^{np}$	$(t^3 \cdot s^4)^2 = t^{3 \cdot 2} s^{4 \cdot 2} = t^6 s^8$
Power of a quotient (different bases)	$(\frac{a}{b})^n = \frac{a^n}{b^n}$ $(b \neq 0)$	$(\frac{2}{3})^2 = \frac{2^2}{3^2} = \frac{4}{9}$
	$(\frac{a^m}{b^n})^p = \frac{a^{mp}}{b^{np}}$ $(b \neq 0)$	$(\frac{q^2}{p^4})^3 = \frac{q^{2 \cdot 3}}{p^{4 \cdot 3}} = \frac{q^6}{p^{12}}$
Negative exponent a^{-n}	$a^{-n} = \frac{1}{a^n}$ $(a \neq 0)$	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
	$\frac{1}{a^{-n}} = a^n$ $(a \neq 0)$	$\frac{1}{4^{-2}} = 4^2 = 16$
Zero exponent a^0	$a^0 = 1$	$15^0 = 1$
One exponent a^1	$a^1 = a$ (But $1^n = 1$)	$7^1 = 7$, $1^{13} = 1$

Steps for simplifying exponential expressions:

- Remove parentheses using "power rule" if necessary. $(a^m \cdot b^n)^p = a^{mp} b^{np}$
- Regroup coefficients and variables.
- Use "product rule" and "quotient rule". $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$
- Simplify.
- Use "negative exponent" rule to make all exponents positive if necessary.

Scientific notation: a product of a number <u>between 1 and 10</u> and <u>power of 10</u>.

$N \times 10^{\pm n}$

Scientific notation	Example
$N \times 10^{\pm n}$ $1 \le N < 10$, n - integer	$67504.3 = 6.75043 \times 10^4$ Standard form Scientific notation

Writing a number in scientific notation:

Step	Example
▪ Move the decimal point ***after*** the ***first nonzero digit***.	0.0079 37213000.
▪ Determine n (the power of 10) by counting the number of places you moved the decimal.	$n = 3$ $n = 7$
▪ If the decimal point is moved to the ***right***: $\times 10^{-n}$	$0.0079 = \boxed{7.9 \times 10^{-3}}$ 3 places to the right
▪ If the decimal point is moved to the ***left***: $\times 10^{n}$	$37213000. = \boxed{3.7213 \times 10^{7}}$ 7 places to the left

Square root ($\sqrt{\ }$): a number with the symbol $\sqrt{\ }$ that is the opposite of the square of a number.

Perfect square: a number that is the exact square of a whole number.

Order of operations

Order of Operations	
1. the brackets or parentheses and absolute values (innermost first)	( ) , [ ] , { } , | |
2. exponent or square root (from left-to-right)	a^n , $\sqrt{\ }$
3. multiplication or division (from left-to-right)	× and ÷
4. addition or subtraction (from left-to-right)	+ and -

Memory aid - BEDMAS

B	E (R)	D M	A S
<u>B</u>rackets	<u>E</u>xponents or Square <u>R</u>oot	<u>D</u>ivide or <u>M</u>ultiply	<u>A</u>dd or <u>S</u>ubtract

Simplifying square roots	Example
▪ Factor the number inside the square root sign. (Find the perfect square(s) that will divide the number).	$\sqrt{75}$ 25 3
▪ Rewrite the square root as a multiplication problem.	$\sqrt{75} = \sqrt{25 \times 3}$
▪ Reduce the perfect squares ("pulling out" the integer(s)).	$\sqrt{75} = \sqrt{5^2 \times 3} = 5\sqrt{3}$

Unit 11: Self-Test

Exponents, Roots & Scientific Notation

Topic A

1. Write the following exponential expressions in expanded form.

a) 7^4

b) $(-t)^3$

c) $(5a^4b^0)^2$

d) $\left(\frac{-7}{11}x\right)^3$

2. Write each of the following in the exponential form.

a) (0.5) (0.5) (0.5) (0.5)

b) $(6w)\,(6w)\,(6w)$

c) $(7u)\,(3v)\,(u)\,(2v)$

3. Evaluate.

a) $4x^2 + 5y$, for $x = 1$, $y = 4$

b) $(2a)^3 - 3b$, for $a = 5$, $b = 6$

4. What is the degree of the following term / polynomial?

a) $15ab^4$

b) $6xy^2z^4 + 5y^6 - xz + 2z^0$

5. Arranging polynomials in descending order:

a) $x^2 + 2 - 7x^3 - x + 9x^4$

b) $4v - 67 + 21uv^3 - uv^2$

6. Arranging polynomials in ascending order.

a) $26x^2 - 17x^3 - 5x + 43$

b) $4.3t^2w^2 + \frac{4}{7}tw + w^4 - 8w^3 - 9$

Topic B

7. Simplify (do not leave negative exponents in the answer).

a) $(-92)^1$

b) $(-38076)^0$

c) $(-0.4)^3$

d) -8^2

e) $y^4\,y^3$

f) $\frac{x^9}{x^6}$

g) $(t^4)^{-5}$
h) $13a^{-1}$
i) $[(-4) \cdot (0.2)]^3$
j) $(3a^2 \cdot b^3)^4$
k) $\frac{1}{4^{-3}}$
l) $(\frac{w}{u})^{-3}$
m) $(\frac{a^3}{b^{-4}})^2$
n) $(3^{-4})(3^4)$
o) $\frac{5x^5y^{-6}}{11^0 \cdot x^3y^4}$
p) $(\frac{u^{-2}v^3}{w^{-4}})^{-3}$
q) $(2x^2y^3)^3(3x^{-1}y^{-2})^2(-2345w^{-34})^0$
r) $(\frac{(3x^3)(y^4)}{4x^2y^3})^3$

8. Evaluate for $x = 3$, $y = 2$, $z = -2$.

a) $(-145x^{-6}y^5z^{-8})^0$
b) $\left(\frac{x}{y}\right)^{-3}$
c) $(x - y + 2z)^y$

Topic C

9. Write in scientific notation.
a) 45,600,000
b) 0.00000523

10. Write in standard (or ordinary) form.
a) 3.578×10^3
b) 4.3×10^{-5}

11. Simplify and write in scientific notation.
a) $(5.42 \times 10^{-2})(4.38 \times 10^7)$
b) $\frac{(5\times10^5)(2.4\times10^{-3})}{3.2\times10^8}$

12. Simplify.
a) $\sqrt{196}$
b) $\frac{\sqrt{121}}{\sqrt{225}}$
c) $\sqrt{320}$
d) $\sqrt{\frac{117}{81}}$

Unit 12

Solving Word Problems

Topic A: Value Mixture Problems

Solving Value Mixture Problems

Steps for solving word problems:

Steps for Solving Word Problems
▪ Organize the ***facts*** given from the problem (make a table).
▪ Identify and label the unknown quantity (***let x = unknown***).
▪ Draw a ***diagram*** if it will make the problem clearer.
▪ Convert words into a mathematical ***equation***.
▪ ***Solve*** the equation and find the solution(s).
▪ ***Check*** and state the ***answer***.

Table for value mixture problems:

Item	Value of the item	Number of items	Total value
Item A	value of A	# of A	(value of A) × (# of A) = amount of A
Item B	value of B	# of B	(value of B) × (# of B) = amount of B
Item C	value of C	# of C	(value of C) × (# of C) = amount of C
…	…	…	…
Total or mixture			total value

Let x = unknown

Value of item A + Value of item B + Value of item C + … = Total value of the mixture

Example: Susan has ***$5.95*** in nickels, dimes and quarters. If she has two ***less*** than ***three times quarters of dimes***, and ***three more nickels than quarters***. How many of each coin does she have?

- Let x = number of quarters
- Organize the facts:

Coin	Value of the coin	Number of coins	Total value (in cents)	
Quarter	25 ₵	x	$25x$	(value of 25¢) × (# of 25¢)
Dime	10 ₵	$3x - 2$	$10(3x - 2)$	(value of 10¢) × (# of 10¢)
Nickel	5 ₵	$x + 3$	$5(x + 3)$	(value of 5 ¢) × (# of 5¢)
Total			$5.95 = 595 ¢	

Convert $ to ¢

- Equation: $\mathbf{25x + 10(3x - 2) + 5(x + 3) = 595}$ value of quarters + value of dimes + value of nickels = 595¢
- Solve for x: $25x + 30x - 20 + 5x + 15 = 595$ Remove parentheses.

  $60x - 5 = 595$ Combine like terms.

  $60x = 600$ Solve for x.

  $\boldsymbol{x} = \boxed{10}$

- Check:

Number of quarters	$\mathbf{x = 10}$
Number of dimes	$\mathbf{3x - 2} = 3(10) - 2 = \mathbf{28}$
Number of nickels	$\mathbf{x + 3} = 10 + 3 = \mathbf{13}$

$25x + 10(3x - 2) + 5(x + 3) = 595$ Equation

$25 \cdot 10 + 10(3 \cdot 10 - 2) + 5(10 + 3) \overset{?}{=} 595$ Substitute x for 10.

$250 + 280 + 65 \overset{?}{=} 595$ Check LS $\overset{?}{=}$ RS

$595 \overset{\surd}{=} 595$ Correct! LS = RS

- State the answer:

Number of quarters	$\boxed{10}$
Number of dimes	$\boxed{28}$
Number of nickels	$\boxed{13}$

Example: Damon purchased \$1.00, \$1.19, and \$1.20 Canadian stamps with a ***total value of \$23.72.*** If the number of \$1.19 stamps is ***7 more than the number of \$1.00*** stamps, and the number of ***\$1.20*** stamps is ***8 more than three times*** of ***\$1.00*** stamps. How many of each did Damon receive?

- Let x = number of \$1.00 stamps
- Organize the facts:

Stamps	Value of the stamps	Number of stamps	Total value	
\$1.00	\$1.00	x	$1.00\,x$	(value of \$1.00) × (# of \$1.00)
\$1.19	\$1.19	$7 + x$	$1.19\,(7 + x)$	(value of \$1.19) × (# of \$1.19)
\$1.20	\$1.20	$8 + 3x$	$1.20\,(8 + 3x)$	(value of \$1.20) × (# of \$1.20)
Total			\$23.72	

Value of \$1.00 + value of \$1.19 + value of \$1.20 = \$20.68

- Equation: $\mathbf{1.00x + 1.19\,(7 + x) + 1.20\,(8 + 3x) = 23.72}$
- Solve for x: $1x + 8.33 + 1.19x + 9.6 + 3.6x = 23.72$ Remove parentheses.

$5.79x + 17.93 = 23.72$ Combine like terms.

$579x + 1793 = 2372$ Remove decimals (× 100)

$579x = 579$ Divide both sides by 579.

$\mathbf{x} = \boxed{1}$

- State the answer:

Number of \$1.00	$\mathbf{x} = \boxed{1}$
Number of \$1.19	$\mathbf{7 + x} = 7 + 1 = \boxed{8}$
Number of \$1.20	$\mathbf{8 + 3x} = 8 + 3 \cdot 1 = \boxed{11}$

Topic B: Concentration Mixture Problems

Solving Mixture Problems

Table of concentration mixture:

Iterm	Concentration	Volume	Amount
Item A	concentration of A	volume of A	(concentration of A) × (volume of A) = amount of A
Item B	concentration of B	volume of B	(concentration of B) × (volume of B) = amount of B
…	…	…	…
Mixture	concentration of mixture	volume of mixture	(concentration of mixture) × (volume of mixture) = amount of mixture

Let x = unknown

Amount of item A + Amount of item B + … = Amount of the mixture

Example: A **shrimp meal** is **35%** protein and a **fish meal** is **25%** protein. Susan wants a **750 grams mixture** that is **30%** protein. How many grams of protein each meal should she have?

- \- Let x = the protein volume of the shrimp meal

  \- The protein volume of fish meal = $750 - x$

  The protein volume of ***mixture*** – The protein volume of ***shrimp*** meal = The protein volume of ***fish*** meal

  (If there is a total mixture protein volume of 750 g, then $750 - x$ must be the protein volume of fish meal.)

- Organize the facts:

Meal	Concentration	Protein volume	Amount	
Shrimp meal	35% = 0.35	x	$0.35\,x$	(concentration of shrimp meal) × (volume of shrimp meal)
Fish meal	25% = 0.25	$750 - x$	$0.25\,(750 - x)$	(concentration of fish meal) × (volume of fish meal)
Mixture	30% = 0.30	750	0.3 (750)	(concentration of mixture) × (volume of mixture)

- Equation: $\mathbf{0.35\,x + 0.25\,(750 - x) = (0.3)\,(750)}$ Remove parentheses.

  Amount of shrimp meal + Amount of fish meal = Amount of mixture

- Solve for x: $0.35\,x + 187.5 - 0.25x = 225$ Combine like terms.

  $0.1\,x = 37.5$ Divide both sides by 0.1.

- State the answer: - Shrimp meal: $\boldsymbol{x} = \boxed{375 \text{ g}}$

  \- Fish meal: $\mathbf{750 - \boldsymbol{x}} = 750 - 375$

  $= \boxed{375 \text{ g}}$

Example: How much ***8% sugar solution*** must be added to ***15 liters of 27% solution*** to make a ***20% solution***?

- - Let x = volume of 8% solution
  - Volume of 20% = $x + 15$

  Volume of 20% = Volume of 8% + Volume of 27%

  Mixture

- Organize the facts:

Solution	Concentration	Volume	Amount	
8%	0.08	x	$0.08\,x$	(concentration of 8%) × (volume of 8%)
27%	0.27	15	$(0.27)\,(15)$	(concentration of 27%) × (volume of 27%)
20%	0.2	$x + 15$	$0.2\,(x + 15)$	(concentration of 20%) × (volume of 20%)

- Equation: $\mathbf{0.08\,x + (0.27)\,(15) = 0.2\,(x + 15)}$ Amount of 8% + Amount of 27% = Amount of 20%
- Solve for x: $0.08\,x + 4.05 = 0.2\,x + 3$ Combine like terms.

  $-\,0.12\,x = -1.05$ Divide both sides by -0.12.

  $\boldsymbol{x} = \boxed{8.75}$

- State the answer: 8.75 liters of 8% sugar solution must be added to 15 liters of 27% solution.

Topic C: Motion and Business Problems

Distance, Speed and Time Problems

Formulas of motion:

- Distance = Speed · Time $\boxed{d = r\,t}$
- Speed = $\frac{\text{Distance}}{\text{Time}}$ $\boxed{r = \frac{d}{t}}$
- Time = $\frac{\text{Distance}}{\text{Speed}}$ $\boxed{t = \frac{d}{r}}$

Example: Adam walks for ***4.4 hours*** at a rate of ***2 km per hour***. ***How far*** does he walk?

Equation: $\boldsymbol{d = r\,t}$ $t = 4.4$ h, $r = 2$ km/h, $d = ?$

$= (2 \text{ km/h})\ (4.4 \text{ h}) = \boxed{8.8 \text{ km}}$ km/h: km per hour

Table of motions:

Condition	Speed (r)	Time (t)	Distance (d)
Condition A	r	t	$d = r\,t$
Condition B	r	t	$d = r\,t$
…	…	…	…
Total			

Example: Two cyclists are ***60 km apart*** and are travelling towards each other. Their ***speeds differ*** by ***1.5 km*** per hour. What is the ***speed*** of ***each*** cyclist if they meet after ***2 hours***?

Condition	Speed (r)	Time (t)	Distance ($d = r\,t$)
Bike A	r	2	$2\,r$
Bike B	$r - 1.5$	2	$2\,(r - 1.5)$
Total			60 km

- Equation: $\boldsymbol{2r + 2(r - 1.5) = 60}$ Distance of A + Distance of B = 60km

  $2r + 2r - 3 = 60$ Remove parentheses.

  $4r = 63$ Combine like terms.

- Bike A: $\boldsymbol{r =}$ $\boxed{15.75 \text{ km/h}}$ Divide both sides by 4.
- Bike B: $\boldsymbol{r - 1.5 =} 15.75 - 1.5 = \boxed{14.25 \text{ km/h}}$

Example: Mike ***boats*** at a ***speed*** of ***28 km*** per hour in still water. The **river** flows at a **speed** of ***5 km*** per hour. ***How long*** will it take Mike to boat ***3 km downstream***? ***3 km upstream*****?**

Condition	Speed (r)	Distance (d)	Time ($t = \frac{d}{r}$)
Downstream	$r = 28 + 5 = 33$ km/h	$d = 3$ km	$t = \frac{d}{r} = \frac{3 \text{ km}}{33 \text{ km/h}}$
Upstream	$r = 28 - 5 = 23$ km /h	$d = 3$ km	$t = \frac{d}{r} = \frac{3 \text{ km}}{23 \text{ km/h}}$

Downstream (fast): speed of boat + speed of river
Upstream (slower): speed of boat - speed of river

- Downstream: $\boldsymbol{t = \frac{d}{r}} = \frac{3 \text{ km}}{33 \text{ km/h}} \approx \boxed{0.091 \text{ h}}$
- Upstream: $\boldsymbol{t = \frac{d}{r}} = \frac{3 \text{ km}}{23 \text{ km/h}} \approx \boxed{0.13 \text{ h}}$

Business Problems

Business math formulas:

Business problems	Formulas
Percent increase	Percent increase = $\frac{\text{New value} - \text{Original value}}{\text{Original value}}$, $x = \frac{N-O}{O}$
Percent decrease	Percent decrease = $\frac{\text{Original value} - \text{New value}}{\text{Original value}}$, $x = \frac{O-N}{O}$
Sales tax	Sales tax = Sales × Tax rate
Commission	Commission = Sales × Commission rate
Discount	Discount = Original price × Discount rate Sale price = Original price – Discount
Markup	Markup = Selling price × Markup rate Original price = Selling price – Markup
Simple interest	Interest = Principle · Interest rate · Time , $I = P\,r\,t$ Balance = Principle + Interest
Compound interest	Balance = Principle (100% + Interest rate) ^{t} Balance = P (100% + r) ^{t}

Example: A product increased production from ***230 last month*** to ***250 this month***. Find the ***percent increase.***

- New value (N): 250 — This month.
- Original value (O): 230 — Last month.
- Percent increase: $x = \frac{N-O}{O} = \frac{250-230}{230} \approx 0.087 = \boxed{8.7\%}$ — About 8.7% increase.

Example: A product was ***reduced*** from ***$59*** to ***$39.*** What was the percent ***reduction***?

Percent decrease: $x = \frac{O-N}{O} = \frac{59-39}{59} \approx 0.339 = \boxed{33.9\%}$ — 33.9 % decrease.

Example: Find the ***sales tax*** for a ***$999*** laptop with a ***tax rate*** of ***7%***.

Sales tax = Seles × Tax rate

$= (\$999)\,(7\%) = (\$999)\,(0.07) = \boxed{\$69.93}$

Example: Find the ***commission*** for a ***$950,000*** house with a ***commission rate*** of ***5%***.

Commission = Sales × Commission rate

$= (\$950{,}000)\,(5\%) = (\$950{,}000)\,(0.05) = \boxed{\$47{,}500}$

Example: A men's coat was ***originally*** priced at ***$159,*** and is on sale at a ***25% discount***. Find the ***discount*** and ***sale price***.

- **Discount** = Original price × Discount rate
  = ($159) (25%)
  = ($159) (0.25)
  = $\boxed{\$39.75}$
- **Sale price** = Original price – Discount
  = $159 – $39.75
  = $\boxed{\$119.25}$

Example: A condo was sold at ***$399,000***, with a ***markup rate*** of ***8%***. What was the ***markup*** and ***original price***?

- **Markup** = Selling price × Markup rate
  = ($399,000) (8%)
  = ($399,000) (0.08)
  = $\boxed{\$31,920}$
- **Original price** = Selling price – Markup
  = $399,000 – $31,920
  = $\boxed{\$367,080}$

Example: Jo borrowed ***$150,000*** mortgage from a bank. Find the interest at ***3%*** per year for ***3.5 years***, and also find the ***total*** amount that Jo paid the bank.

- Interest = Principle · Interest rate · Time
  I = $P\ r\ t$ = ($150,000) (3%) (3.5)
  = ($150,000) (0.03) (3.5)
  = $\boxed{\$15,750}$
- **Balance** = Principle + Interest
  = $150,000 + $15,750
  = $\boxed{\$165,750}$

Example: David deposited ***$3,000*** in an account at ***4.5% interest compounded*** per year for ***5 years***. How much was in the account at the end of ***5 years***?

Balance = Principle (100% + Interest rate) ^{t} Compound interest

$$= P\,(100\% + r)^t$$
$$= \$3,000\,(100\% + 4.5\%)^5$$
$$= \$3,000\,(1+ 0.045)^5$$
$$\approx \boxed{\$\,3738.55}$$

Topic D: Mixed Problems

Solving Mixed Problems

Example: After a ***ten percent reduction***, a toy is on sale for ***twenty-nine dollars***. What was the ***original price***?

- Let x = original price
- Equation**:** $\boldsymbol{x - 10\%\, x = 29}$ Original price – Reduction = Sale price

  $1\,x - 0.1\,x = 29$ $x = 1 \cdot x$

  $0.9x = 29$
- Answer: $\boldsymbol{x} = \frac{29}{0.9} \approx \boxed{\$\,32.22}$ The original price was \$32.22.

Example: William receives a ***1.5% raises*** bring his salary ***to \$39,000***. What was his salary ***before*** the ***raise***?

- Let x = Tom's salary before the raise Raise = (1.5%)(Previous salary) = 1.5% x
- Equation: $\boldsymbol{x + 1.5\%\, x = 39{,}000}$ Previous salary + Raise = Current salary

  $1\,x + 0.015\,x = 39{,}000$

  $1.015\,x = 39{,}000$
- Answer: $\boldsymbol{x} = \frac{39000}{1.015} \approx \boxed{\$38423.65}$ Tom's salary before the raise was \$38423.65.

Example: Bob deposits a certain amount of money in a ***chequing account*** that earns ***2.5%*** in annual interest, and deposits ***\$2000 less than*** that in a ***saving account*** that pays ***1.5%*** in annual interest. If the total interest from ***both*** accounts at the end of the year is ***\$95***, how much is deposited in each account?

- Let x = money deposited in the saving account

Account	Deposit	Interest rate	Interest
Chequing account	x	2.5%	$0.025x$
Saving account	$x - 2000$	1.5%	$0.015\,(x - 2000)$

Total interest = \$95

- Equation: $\boldsymbol{0.025x + 0.015\,(x - 2000) = 95}$ 2.5% of saving + 1.5% of checking = \$95

  $0.025x + 0.015x - 30 = 95$ Combine like terms.

  $0.04x = 125$
- Answer: Chequing account: $\boldsymbol{x} = \frac{125}{0.04} = \boxed{\$3125}$ \$3125 in the chequing account.

  Saving account: $\boldsymbol{x - 2000} = 3125 - 2000 = \boxed{\$1125}$ \$1125 in the saving account.

Example: A string ***103 meters*** long is cut into ***four pieces***. The ***second*** is ***four times*** as long ***as*** the ***first***. The ***third*** piece is ***five*** meters ***longer*** than the ***first***. ***The fourth*** piece is ***twice*** as long ***as*** the ***third***. How long is each piece of string?

- Let x = the length of the first piece.

1st piece	x
2nd piece	$4x$
3rd piece	$x + 5$
4th piece	$2(x + 5)$

- Equation: $\mathbf{x + 4x + (x + 5) + 2(x + 5) = 103}$ — 1st + 2nd + 3rd + 4th = 103

  $x + 4x + x + 5 + 2x + 10 = 103$ — Combine like terms.

  $8x + 15 = 103$

  $8x = 88$

  $x = \boxed{11 \text{ m}}$

- Answer:

1st piece	$x = \boxed{11 \text{ m}}$
2nd piece	$4x = 4(11) = \boxed{44 \text{ m}}$
3rd piece	$x + 5 = 11 + 5 = \boxed{16\text{m}}$
4th piece	$2(x + 5) = 2(11 + 5) = \boxed{32 \text{ m}}$

Example: A fruit punch that contains ***25% fruit juice***. ***How much water*** would you have to add to ***1 liter*** of punch to get a new drink that contains ***10% fruit juice***?

- Let x = water to add to 1 L of punch to get a 10% fruit juice.

	Concentration	Volume	Amount
Fruit punch	25 %	1 (L)	0.25(1)
New drink	10 %	$x + 1$	$0.1(x + 1)$

- Equation: $\mathbf{0.25(1) = 0.1(x + 1)}$ — Amount of 25% = Amount of 10%

  $0.25 = 0.1x + 0.1$ — Multiply 100 for each term.

  $25 = 10x + 10$ — Combine like terms.

  $15 = 10x$ — Divide both sides by 10.

- Answer: $x = \boxed{1.5 \text{ L}}$

  It needs to add 1.5 L of water to get a new drink that contains 10% fruit juice.

Unit 12: Summary

Solving Word Problems

Steps for solving word problems:

Steps for solving word problems
▪ Organize the ***facts*** given from the problem (make a table).
▪ Identify and label the unknown quantity (***let x = unknown***).
▪ Draw a ***diagram*** if it will make the problem clearer.
▪ Convert words into a mathematical ***equation***.
▪ ***Solve*** the equation and find the solution(s).
▪ ***Check*** and state the ***answer***.

Table for value mixture problems:

Let x = unknown

Item	Value of the item	Number of items	Total value
Item A	value of A	# of A	(value of A) × (# of A) = amount of A
Item B	value of B	# of B	(value of B) × (# of B) = amount of B
Item C	value of C	# of C	(value of C) × (# of C) = amount of C
…	…	…	…
Total or mixture			total value

Value of item A + Value of item B + Value of item C + … = Total value of the mixture

Formulas of motion:

Distance = Speed · Time $\quad d = r\,t \quad t = \frac{d}{r} \quad r = \frac{d}{t}$

Table of motions:

Condition	Speed (r)	Time (t)	Distance (d)
Condition A	r	t	$d = r\,t$
Condition B	r	t	$d = r\,t$
…	…	…	…
Total			

- Downstream (fast): speed of boat + speed of river
- Upstream (slower): speed of boat - speed of river

Table of concentration mixture:

Let x = unknown

Iterm	Concentration	Volume	Amount
Item A	concentration of A	volume of A	(concentration of A) × (volume of A) = amount of A
Item B	concentration of B	volume of B	(concentration of B) × (volume of B) = amount of B
…	…	…	…
Mixture	concentration of mixture	volume of mixture	(concentration of mixture) × (volume of mixture) = amount of mixture

Amount of item A + Amount of item B + … = Amount of the mixture

Business math formulas:

Business problems	Formulas
Percent increase	Percent increase = $\frac{\text{New value} - \text{Original value}}{\text{Original value}}$, $x = \frac{N - O}{O}$
Percent decrease	Percent decrease = $\frac{\text{Original value} - \text{New value}}{\text{Original value}}$, $x = \frac{O - N}{O}$
Sales tax	Sales tax = Sales × Tax rate
Commission	Commission = Sales × Commission rate
Discount	Discount = Original price × Discount rate Sale price = Original price − Discount
Markup	Markup = Selling price × Markup rate Original price = Selling price − Markup
Simple interest	Interest = Principle · Interest rate · Time , $I = P\,r\,t$ Balance = Principle + Interest
Compound interest	Balance = Principle (100% + Interest rate) ^{t} Balance = $P\,(100\% + r)^{t}$

Unit 12: Self-Test

Solving Word Problems

Topic A

1. Robert has $2.50 in nickels, dimes and quarters. If he has two more than five times quarters of dimes, and two less nickels than quarters. How many of each coin does he have?

2. William purchased $1.00, $1.19, and $1.20 Canadian stamps with a total value of $27.13. If the number of $1.19 stamps is 5 more than the number of $1.00 stamps, and the number of $1.20 stamps is 6 more than four times of $1.00 stamps. How many of each did Damon receive?

Topic B

3. A lamb meal is 36% protein and a pork meal is 25% protein. Peter wants an 860 grams mixture that is 28% protein. How many grams of protein each meal should he have?

4. How much 5% salt solution must be added to 18 liters of 32% solution to make a 25% solution?

Topic C

5. Two cyclists are 72 km apart and are travelling towards each other. Their speeds differ by 2 km per hour. What is the speed of each cyclist if they meet after 3 hours?

6. Linda boats at a speed of 17 km per hour in still water. The river flows at a speed of 3 km per hour. How long will it take Linda to boat 4 km downstream? 4 km upstream?

7. A product increased production from 400 last month to 420 this month. Find the percent increase.

8. A product was reduced from $80 to $62. What was the percent reduction?

9. Find the sales tax for a $ 679 laptop with a tax rate of 9%.

10. Find the commission for a $699,000 townhouse with a commission rate of 4%.

11. A women's dress was originally priced at $199, and is on sale at a 15% discount. Find the discount and sale price.

12. A condo was sold at $469,000, with a markup rate of 5%. What was the markup and original price?

13. Smith borrowed $100,000 mortgage from a bank. Find the interest at 4% per year for 5 years, and also find the total amount that Smith paid the bank.

14. Susan deposited $2,500 in an account at 3.2% interest compounded per year for 2 years. How much was in the account at the end of 2 years?

Topic D

15. After a five percent reduction, a toy is on sale for thirty-nine dollars. What was the original price?

16. Ruth receives a 2.5% raises bring her salary to $34,000. What was her salary before the raise?

17. Amy deposits a certain amount of money in a chequing account that earns 1.5% in annual interest, and deposits $1500 less than that in a saving account that pays 1.2% in annual interest. If the total interest from both accounts at the end of the year is $76.50, how much is deposited in each account?

18. A string that is 52 meters long is cut into four pieces. The second is three times as long as the first. The third piece is seven meters longer than the first. The fourth piece is three times as long as the third. How long is each piece of string?

19. A fruit punch is 45% fruit juice. How much water would you have to add to 1.5 liter of punch to get a new drink that is 25% fruit juice?

Unit 13

More about Polynomials

Topic A: Adding and Subtracting Polynomials

Polynomials Review

Review of basic algebraic terms:

Algebraic term	Definition	Example
Algebraic expression	A mathematical phrase that contains numbers, variables, and arithmetic operations (+, – , ×, ÷, etc.).	$5x + 2$ $3a - (4b + 6)$ $\frac{2x}{3} + 4y - z^2 + 11$
Constant	A number.	$x + \mathbf{2}$ constant: 2
Variable	A letter that can be assigned different values.	$3 - \boldsymbol{x}$ variable: x
Coefficient	The number that is in front of a variable.	**-6** x coefficient: -6 xz^3 coefficient: 1
Term	A term can be a constant, a variable, or the product of a number and variable(s). (Terms are separated by a plus or minus sign.)	$3x - \frac{2}{5} + 13y^2 + 73xy$ Terms: $3x$, $-\frac{2}{5}$, $13y^2$, $73xy$
Like terms	The terms that have the same variables and exponents.	$2x - y^2 - \frac{2}{5} + 5x - 7 + 13y^2$ Like terms: $2x$ and $5x$ $-y^2$ and $13y^2$, $-\frac{2}{5}$ and -7
Factor	A number or variable that multiplies with another. A number or expression can have many factors.	$24 = 2 \cdot 3 \cdot 4$ factors: 2, 3, 4 $5xy = 5 \cdot x \cdot y$ factors: 5, x, y

Polynomial	Example	Coefficient
Monomial (one term)	$7a$	7
Binomial (two terms)	$3x - 5$	3
Trinomial (three terms)	$-4x^2 + xy + 7$	-4, 1
Polynomial (one or more terms)	$2pq + 4p^3 + p + 11$	2, 4, 1

The degree of a term with more variables: the sum of the exponents of its variables.

Example: $-3x^3 y^5 z^2$ degree: $3 + 5 + 2 = \boxed{10}$

The degree of a polynomial with more variables: the highest degree of any individual term.

Example: $\underbrace{4ab^3}_{4} + \underbrace{3a^2b^2c^3}_{7} - \underbrace{5a}_{1} + 1$ degree: $\boxed{7}$ $2 + 2 + 3 = 7$

Additive (or negative) inverse or opposite: the opposite of a term (two terms whose sum is 0).

Example: **1)** The additive inverse of **5** is $\boxed{-5}$ $5 + (-5) = 0$

2) The additive inverse of $-\frac{3}{4}y$ is $\boxed{\frac{3}{4}y}$ $-\frac{3}{4}y + \frac{3}{4}y = 0$

3) The additive inverse of $4ab^3 - 3a^2 + b^3$ is $\boxed{-4ab^3 + 3a^2 - b^3}$

Adding and Subtracting Polynomials

Add or subtract polynomials:

Example: Add $4x^3 - 5x^2 - x + 3$ and $3x^3 + 3x^2 - 5x + 2$.

Steps	Solution
	$\mathbf{(4x^3 - 5x^2 - x + 3) + (3x^3 + 3x^2 - 5x + 2)}$
▪ Regroup like terms:	$= (4x^3 + 3x^3) + (-5x^2 + 3x^2) + (-x - 5x) + (3 + 2)$
▪ Combine like terms:	$= \boxed{7x^3 - 2x^2 - 6x + 5}$

Example: Subtract $6x^2 + 7x - 5$ and $3x^2 - 4x + 16$.

Steps	Solution
	$\mathbf{(6x^2 + 7x - 5) - (3x^2 - 4x + 16)}$
▪ Remove parenthesis:	$= 6x^2 + 7x - 5 \mathbf{-} 3x^2 \mathbf{+} 4x \mathbf{-} 16$ (Reverse each sign in second parenthesis.)
▪ Regroup like terms:	$= (6x^2 - 3x^2) + (7x + 4x) + (-5 - 16)$
▪ Combine like terms:	$= \boxed{3x^2 + 11x - 21}$

Add or subtract polynomials using the column method:

Example: Add $4x^3 - 3x^2 + 6x - 5$ and $3x^3 + 2x + 3$.

Steps	Solution
▪ Line up like terms in columns:	$\mathbf{4x^3 - 3x^2 + 6x - 5}$
▪ Add:	+) $\mathbf{3x^3 \qquad + 2x + 3}$
Leave spaces for missing terms.	$\boxed{7x^3 - 3x^2 + 8x - 2}$

Example: Subtract $(7x^2 - 3x + 4)$ and $(3x^2 - 1)$.

Steps	Solution	
▪ Line up like terms in columns:	$\mathbf{7x^2 - 3x + 4}$	← Subtrahend
▪ Change signs in the minuend	+) $\mathbf{-3x^2 \qquad + 1}$	← Minuend
and **add**:	$\boxed{4x^2 - 3x + 5}$	← Difference

Or $(7x^2 - 3x + 4) - (3x^2 - 1) = 7x^2 - 3x + 4 - 3x^2 + 1$

$= 4x^2 - 3x + 5$

Topic B: Multiplication of Polynomials

Multiplying Polynomials

Multiplying monomials

Example: $(-4a^2 b^3)(5a^3 b^5) = (-4 \cdot 5)(a^2 \cdot a^3)(b^3 \cdot b^5)$ — Multiply the coefficients and add the exponents.

$= \boxed{-20\, a^5 b^8}$ — $a^m a^n = a^{m+n}$

Multiplying monomial and polynomial

Example: $5x^2(4x^3 - 3x) = (5x^2)(4x^3) - (5x^2)(3x)$ — Distributive property: $a(b + c) = ab + ac$

$= (5 \cdot 4)(x^{2+3}) - (5 \cdot 3)(x^{2+1})$ — Multiply the coefficients and add the exponents.

$= \boxed{20x^5 - 15x^3}$ — $a^m a^n = a^{m+n}$

Example: $3xy^3(4xy^2 + x^3y - y)$ — Distribute

$= (3xy^3)(4xy^2) + (3xy^3)(x^3y) + (3xy^3)(-y)$ — Multiply the coefficients and add the exponents.

$= \boxed{12x^2y^5 + 3x^4y^4 - 3xy^4}$ — $a^m a^n = a^{m+n}$

Multiplying binomials (2 terms × 2 terms)

Example: Find the following product.

$$(4a - 5)(2a - 3) = \overset{F}{4a \cdot 2a} + \overset{O}{4a(-3)} - \overset{I}{5 \cdot 2a} - \overset{L}{5(-3)}$$ FOIL

$= 8a^2 - 12a - 10a + 15$ — $a^m a^n = a^{m+n}$

$= \boxed{8a^2 - 22a + 15}$ — Combine like terms.

Multiplying binomial and polynomial

Example: Multiply: $2x - 3x^2$ and $x^2 + x - 4$

Steps	Solution
	$(2x - 3x^2)(x^2 + x - 4)$
▪ Use the distributive property:	$= 2x \cdot x^2 + 2x \cdot x + 2x(-4) - 3x^2 \cdot x^2 - 3x^2 \cdot x - 3x^2(-4)$
▪ Multiply coefficients and add exponents:	$= 2x^3 + 2x^2 - 8x - 3x^4 - 3x^3 + 12x^2$
▪ Combine like terms and write in descending order:	$= \boxed{-3x^4 - x^3 + 14x^2 - 8x}$

Multiplying polynomials mentally (no need to write out each step).

Example: Multiply.

a) $2x^3(3x^2 - 2) = \boxed{6x^5 - 4x^3}$ — $a(b + c) = ab + ac$, $a^n a^m = a^{n+m}$

b) $(a - 3)(2a - 1) = \boxed{2a^2 - 7a + 3}$ — FOIL

Special Binomial Products

Special binomial products – squaring binominals

Special products	Formula	Initial expansion	Example
Difference of squares	$(a + b)(a - b) = a^2 - b^2$ It does not matter if $(a - b)$ comes first	$(a + b)(a - b) = a^2 - ab + ba - b^2$ $= a^2 - b^2$	$(x + 3)(x - 3) = x^2 - 3^2 = x^2 - 9$ $(a = x,\ b = 3)$ or $(x - 3)(x + 3) = x^2 - 3^2 = x^2 - 9$
Square of sum	$(a + b)^2 = a^2 + 2ab + b^2$ A perfect square trinomial	$(a + b)^2 = (a + b)(a + b)$ $= a^2 + ab + ba + b^2$ $= a^2 + 2ab + b^2$	$(y + 2)^2 = y^2 + 2 \cdot y \cdot 2 + 2^2$ $= y^2 + 4y + 4$
Square of difference	$(a - b)^2 = a^2 - 2ab + b^2$ A perfect square trinomial	$(a - b)^2 = (a - b)(a - b)$ $= a^2 - ab - ba + b^2$ $= a^2 - 2ab + b^2$	$(z - 5)^2 = z^2 - 2 \cdot z \cdot 5 + 5^2$ $= z^2 - 10z + 25$

Special binomial products: special forms of binomial products that are worth memorizing.

Memory aid: $(a \pm b)^2 = (a^2 \pm 2ab + b^2)$ Notice the reversed plus or minus sign in the second term.

Example: Find the following products.

1) $\mathbf{(5x + 3)(5x - 3)} = \overset{a}{(5x)}^2 - \overset{b}{3}^2$ $\qquad (a + b)(a - b) = a^2 - b^2$

$= \boxed{25x^2 - 9}$ $\qquad a = 5x \ , \quad b = 3$

2) $\mathbf{(2t - 1)^2} = (2t)^2 - 2(2t) + 1^2$ $\qquad (a - b)^2 = a^2 - 2ab + b^2$

$= \boxed{4t^2 - 4t + 1}$ $\qquad a = 2t \ , \quad b = 1$

3) $\mathbf{(3w + \frac{1}{3})^2} = (3w)^2 + 2(3w)(\frac{1}{3}) + (\frac{1}{3})^2$ $\qquad (a + b)^2 = a^2 + 2ab + b^2$

$= \boxed{9w^2 + 2w + \frac{1}{9}}$ $\qquad a = 3w \ , \quad b = \frac{1}{3}$

4) $\mathbf{(5u - \frac{1}{2}v)^2} = (5u)^2 - 2(5u)(\frac{1}{2}v) + (\frac{1}{2}v)^2$ $\qquad (a - b)^2 = a^2 - 2ab + b^2$

$= \boxed{25u^2 - 5uv + \frac{1}{4}v^2}$ $\qquad a = 5u \ , \quad b = \frac{1}{2}v$

5) $\mathbf{(\frac{1}{3}t - \frac{1}{2})(\frac{1}{3}t + \frac{1}{2})} = (\frac{1}{3}t)^2 - (\frac{1}{2})^2$ $\qquad (a + b)(a - b) = a^2 - b^2$

$= \boxed{\frac{1}{9}t^2 - \frac{1}{4}}$ $\qquad a = \frac{1}{3}t \ , \quad b = \frac{1}{2}$

Topic C: Polynomial Division

Dividing Polynomials

Dividing a monomial by a monomial

Example: $\frac{\mathbf{14a^5}}{\mathbf{a^2}} = 14\,a^{5-2}$ Apply $\frac{a^m}{a^n} = a^{m-n}$

$= \boxed{14\,a^3}$

Example: $\frac{-28u^6v^2}{7u^4v^5}$

Steps	Solution
▪ Divide the coefficients:	$\frac{\mathbf{-28u^6v^2}}{\mathbf{7u^4v^5}} = (\frac{-28}{7})\,(\frac{u^6v^2}{u^4v^5})$
▪ Divide like variables (apply $\frac{a^m}{a^n} = a^{m-n}$):	$= -4\,(\frac{u^6}{u^4})(\frac{v^2}{v^5})$ $\frac{v^2}{v^5} = v^{2-5} = v^{-3}$
	$= \boxed{-4\left(\frac{u^2}{v^3}\right)}$ $a^{-m} = \frac{1}{a^m}$

Dividing a polynomial by a monomial

Example: $\frac{15a^2 + 5a - 4}{5a}$

Steps	Solution
▪ Split the polynomial into three parts:	$\frac{\mathbf{15a^2 + 5a - 4}}{\mathbf{5a}} = \frac{15a^2}{5a} + \frac{5a}{5a} - \frac{4}{5a}$
▪ Divide a monomial by a monomial:	$= \boxed{3a + 1 - \frac{4}{5a}}$

Example: $\frac{4x^2+8x+2x+4}{x+2}$

Steps	Solution
▪ Group:	$\frac{\mathbf{4x^2+8x+2x+4}}{\mathbf{x+2}} = \frac{(4x^2+8x)+(2x+4)}{x+2}$
▪ Factor out the greatest common factor (GCF):	$= \frac{4x(x+2)+2(x+2)}{x+2}$
▪ Split the polynomial into two parts:	$= \frac{4x\cancel{(x+2)}}{\cancel{x+2}} + \frac{2\cancel{(x+2)}}{\cancel{x+2}}$
▪ Divide a monomial by a monomial:	$= \boxed{4x + 2 = 2\,(2x + 1)}$

Long Division of Polynomials

Long division for numbers:

Example:

$$\begin{array}{r} \underline{\text{Quotient}} \\ \text{Divisor})\ \text{Dividend} \\ -\underline{\qquad\qquad} \\ \text{Remainder} \end{array} \qquad\qquad \begin{array}{r} \underline{7} \\ 3\ )22 \\ -\ \underline{21} \\ 1 \end{array}$$

Polynomial long division: a method used for dividing a polynomial by another polynomial of the same or lower degree (it is very similar to long division for numbers).

Example: $\frac{4x^2+8x+1}{2x}$

Steps	Solution	Long division for numbers
▪ Write in ***divisor) Dividend*** form:	$\mathbf{2x\ )\ \overline{4x^2+8x+1}}$	$2)\ \overline{481}$
▪ Divide the first term:	$\begin{array}{r} 2x \qquad\qquad \\ 2x\ )\ \overline{4x^2+8x+1} \\ -\ \underline{4x^2} \quad (2x)(2x)=4x^2 \end{array}$	$\begin{array}{r} 2 \\ 2)\ \overline{481} \\ -\ \underline{4} \end{array}$ $2 \cdot 2 = 4$
▪ Divide the second term:	$\begin{array}{r} 2x\ +\ 4 \qquad \\ 2x\ )\ \overline{4x^2+8x+1} \\ \underline{4x^2} \qquad\qquad \\ 8x \qquad \\ -\ \underline{8x} \qquad \\ 1 \end{array}$ Bring $8x$ down; $(2x)(4) = 8x$	$\begin{array}{r} 240 \\ 2)\overline{481} \\ \underline{4} \quad \\ 8 \quad \\ -\ \underline{8} \quad \\ 1 \end{array}$ Bring 8 down; $2 \cdot 4 = 8$

Remainder

- Quotient = $\boxed{2x + 4}$, remainder = $\boxed{1}$
- Tip: continue until the degree of the remainder is less than the degree of the divisor.
  (i.e. $1 = 1 \cdot x^{0}$ and $2x = 2x^{1}$, $0 < 1$)

$$\begin{array}{r} \underline{\text{Quotient}} \\ \text{Divisor})\ \text{Dividend} \\ -\underline{\qquad\qquad} \\ \text{Remainder} \end{array}$$

- Check: $\boxed{\text{Dividend} = \text{Quotient} \cdot \text{Divisor} + \text{Remainder}}$

$$4x^2 + 8x + 1 \overset{?}{=} (2x + 4)\ (2x) + 1 \qquad \text{Distribute}$$

$$4x^2 + 8x + 1 \overset{\surd}{=} 4x^2 + 8x + 1 \qquad \text{Correct!}$$

Missing Terms in Long Division

If there is a missing consecutive power term in a polynomial (i.e. if there are x^3 and x but not x^2), add in the missing term with a coefficient of 0.

Example: $\frac{7-4x^2+x^3}{1+x}$

Steps	Solution
▪ Rewrite both polynomials in descending order: Descending order: $Ax^3 + Bx^2 + Cx + D$, $Ax + B$	$\frac{x^3-4x^2+7}{x+1}$
▪ Write in **divisor) $\overline{\textbf{Dividend}}$** form and insert a 0 coefficient for the missing power term.	$x + 1\,\overline{)\,x^3 - 4x^2 + \mathbf{0x} + 7}$ Missing power
▪ Divide as usual:	see below

$$\begin{array}{rl}
 & x^2 - 5x + 5 \\
\boldsymbol{x+1}\ ) & \boldsymbol{x^3 - 4x^2 + 0x + 7} \\
-) & x^3 + x^2 \\
\hline
 & -5x^2 + 0x \\
-) & -5x^2 - 5x \\
\hline
 & 5x + 7 \\
-) & 5x + 5 \\
\hline
 & 2
\end{array}$$

$(x^2)(x) = x^3$
$(x^2)(1) = x^2$
$(-5x)(x) = -5x^2$
$(-5x)(1) = -5x$
$(5)(x) = 5x$
$(5)(1) = 5$

▪ Quotient = $\boxed{x^2 - 5x + 5}$, remainder = $\boxed{2}$

▪ Check: Dividend = Quotient · Divisor + Remainder

$$\begin{array}{r}
\text{Quotient} \\
\text{Divisor) Dividend} \\
- \qquad\qquad \\
\hline
\text{Remainder}
\end{array}$$

$$7 - 4x^2 + x^3 = (x^2 - 5x + 5)(x + 1) + 2$$ Distribute

$$7 - 4x^2 + x^3 \stackrel{?}{=} (x^3 + x^2 - 5x^2 - 5x + 5x + 5) + 2$$ Combine like terms.

$$7 - 4x^2 + x^3 \stackrel{\surd}{=} x^3 - 4x^2 + 7$$ Correct!

Unit 13: Summary

More about Polynomials

Basic algebraic terms:

Algebraic term	Definition
Algebraic expression	A mathematical phrase that contains numbers, variables, and arithmetic operations.
Constant	A number.
Variable	A letter that can be assigned different values.
Coefficient	The number that is in front of a variable.
Term	A term can be a constant, a variable, or the product of a number and variable(s). (Terms are separated by a plus or minus sign.)
Like terms	The terms that have the same variables and exponents.
Factor	A number or variable that multiplies with another.

Polynomial	Description
Monomial	One term.
Binomial	Two terms.
Trinomial	Three terms.
Polynomial	One or more terms.

The degree of a term with more variables: the sum of the exponents of its variables.

The degree of a polynomial with more variables: the highest degree of any individual term.

Additive (or negative) inverse or opposite: the opposite of a term.

Add or subtract polynomials:

- Regroup like terms.
- Combine like terms.

Add polynomials using the column method:

- Line up like terms in columns.
- Add.

Subtract polynomials using the column method:

- Line up like terms in columns.
- Change signs in minuend and add.

Multiplying binomial and polynomial:

- Use the distributive property. $a\,(b + c) = ab + ac$
- Multiply coefficients and add exponents. Apply $\frac{a^m}{a^n} = a^{m-n}$
- Combine like terms and write in descending order.

Special binomial products – squaring binominals

Special products	Formula
Difference of squares	$(a + b)(a - b) = a^2 - b^2$
Square of sum	$(a + b)^2 = a^2 + 2ab + b^2$
Square of difference	$(a - b)^2 = a^2 - 2ab + b^2$

Memory aid: $(a \pm b)^2 = (a^2 \pm 2ab + b^2)$

Dividing a monomial by a monomial

- Divide coefficients.
- Divide like variables (apply $\frac{a^m}{a^n} = a^{m-n}$).

Dividing a polynomial by a monomial

- Split the polynomial into parts.
- Divide a monomial by a monomial.

Polynomial long division: a method used for dividing a polynomial by another polynomial of the same or lower degree (it is very similar to long division for numbers).

Example: $\frac{8-3x+x^3}{2+x}$

Steps	Solution
Rewrite both polynomials in descending order: Descending order: $Ax^3 + Bx^2 + Cx + D$, $Ax + B$	$\frac{x^3-3x+8}{x+2}$
Write in **divisor) $\overline{\textbf{Dividend}}$** form and insert a 0 coefficient for the missing power term.	$x + 2\ \overline{)\ x^3 + \mathbf{0}x^2 - 3\boldsymbol{x} + 8}$ Missing power
Divide as usual:	see below

$$
\begin{array}{rll}
 & x^2 - 2x + 1 & \\
x+2\ ) & \overline{x^3 + \mathbf{0}x^2 - 3\boldsymbol{x} + 8} & (x^2)(x) = x^3 \\
-) & x^3 + 2x^2 & (x^2)(2) = 2x^2 \\
 & \overline{-2x^2 - 3x} & (-2x)(x) = -2x^2 \\
-) & -2x^2 - 4x & (-2x)(2) = -4x \\
 & \overline{x + 8} & (1)(x) = x \\
-) & x + 2 & (1)(2) = 2 \\
 & \overline{6} &
\end{array}
$$

- Quotient = $x^2 - 2x + 1$, remainder = 6
- Tip: continue until the degree of the remainder is less than the degree of the divisor.
- Check: $\boxed{\text{Dividend} = \text{Quotient} \cdot \text{Divisor} + \text{Remainder}}$

$$
\begin{array}{r}
\underline{\text{Quotient}} \\
\text{Divisor) Dividend} \\
-\underline{\qquad\qquad} \\
\text{Remainder}
\end{array}
$$

Unit 13: Self-Test

More about Polynomials

Topic A

1. Determine the degree of the following.

a) $-8x^4 y^3 z^5$

b) $21x^5y + 32x^2y^3z + 6x^3y^4z^2$

c) $3.5a^4b + 6.1a^4b^3c - 7.3a + 5.4$

2. Determine the additive inverse.

a) 8y

b) $-\frac{5}{8}x$

c) $9xy^2 - 4x^2 + y^3$

3. Add $5x^4 - 3x^3 - x + 7$ and $4x^4 + 2x^3 - 7x + 3$.

4. Subtract $8x^2 + 5x - 4$ and $4x^2 - 2x + 14$.

5. Add or subtract polynomials using the column method:

a) Add $7a^3 - 4a^2 + 3a - 6$ and $4a^3 + 6a + 8$.

b) Subtract $(9x^2 - 4x + 8)$ and $(4x^2 - 3)$.

Topic B

6. Multiply.

a) $(-6x^3 y^2)(4x^4 y^3)$

b) $4a^2(3a^4 - 6a)$

c) $7xy^2(2xy^4 + x^3y - 3y)$

d) $(3x - 4)(4x - 5)$

e) $(3a - 2a^2)(a^2 + a - 5)$

7. Find the following product.

a) $4t^4(2t^3 - 5)$

b) $(x - 5)(3x - 2)$

c) $(6a + 5)(6a - 5)$

d) $(3w - 1)^2$

e) $(5u + \frac{1}{2})^2$

f) $(6x - \frac{1}{3}y)^2$

g) $(\frac{1}{5}z - \frac{1}{4})(\frac{1}{5}z + \frac{1}{4})$

Topic C

8. Divide the following.

a) $\frac{56x^6}{x^3}$

b) $\frac{-81a^5b^3}{9a^3b^6}$

c) $\frac{28y^2+7y-3}{7y}$

d) $\frac{6a^2+18a+3a+9}{a+3}$

9. Use long division to divide the following.

a) $\frac{9x^2+6x+2}{3x}$

b) $\frac{30-3x^2+2x^3}{2+x}$

Unit 14

Factoring Polynomials

Topic A: Factoring

- Highest / greatest common factor
- Factoring polynomials by grouping
- Factoring difference of squares

Topic B: Factoring trinomials

- Factoring $x^2 + b\,x + c$
- Factoring $ax^2 + b\,x + c$
- More on factoring $ax^2 + b\,x + c$
- Factoring trinomials: AC method
- Factoring special products

Topic C: Application of factoring

- Quadratic equations
- Solving quadratic equations
- Application of quadratic equations

Unit 14 Summary

Unit 14 Self-test

Topic A: Factoring

Highest / Greatest Common Factor

Factoring whole numbers: write the number as a product (multiply) of its prime factors.

Prime factor: it is a prime number that has only two factors, 1 and itself.

Example: Factor 42.

$\mathbf{42} = \boxed{2 \cdot 3 \cdot 7}$ 2, 3 and 7 are prime factors.

Common factor: a number or an expression that is a factor of each term of a group of terms.

Greatest / highest common factor (GCF or HCF): the product of the common factors.

Examples:

Expression	Factors	Common factor	GCF or GCF
30 42	$\mathbf{2} \cdot \mathbf{3} \cdot 5$ $\mathbf{2} \cdot \mathbf{3} \cdot 7$	2, 3	6
$2xy^3$ $6xy^2$	$\mathbf{2} \cdot \boldsymbol{x} \cdot y \cdot \boldsymbol{y^2}$ $\mathbf{2} \cdot 3 \cdot \boldsymbol{x} \cdot \boldsymbol{y^2}$	2, x, y^2	$2xy^2$

$2 \cdot 3 = 6$

$2 \cdot x \cdot y^2 = 2xy^2$

Factoring a polynomial: express a polynomial as a product of other polynomials. (Factoring is the reverse of multiplication.)

$$(a + b)\, c \xrightarrow{\text{Multiplying (or expanding)}} ac + bc$$

$$(a + b)\, c \xleftarrow{\text{Factoring}} ac + bc$$

Distributive property.

The common factor is c.

Example:

Multiplying	Factoring	GCF or HCF
$3xy\,(2x - 4xy + 3)$ $= 6x^2y - 12x^2y^2 + 9xy$	$6x^2y - 12x^2y^2 + 9xy$ $= \mathbf{3xy}\,(2x) - \mathbf{3xy}\,(4xy) + \mathbf{3xy} \cdot 3$ $= \mathbf{3xy}\,(2x - 4xy + 3)$	$3xy$

Examples

Expression	Factoring	GCF or HCF
$6a^2 - 9a$	$\mathbf{3a} \cdot 2a - \mathbf{3a} \cdot 3 = \mathbf{3a}\,(2a - 3)$	$3a$
$4x^4y + 12x^3y - 16xy$	$\mathbf{4xy} \cdot x^3 + \mathbf{4xy} \cdot 3x^2 - \mathbf{4xy} \cdot 4 = \mathbf{4xy}\,(x^3 + 3x^2 - 4)$	$4xy$
$13z^2\,(z + 2) - (3z + 6)$	$13z^2\,\mathbf{(z + 2)} - 3\,\mathbf{(z + 2)} = \mathbf{(z + 2)}\,(13z^2 - 3)$	$z + 2$
$\frac{2}{3}w^2 - \frac{4}{3}wz^2 + \frac{1}{3}w$	$\frac{1}{3}\mathbf{w} \cdot 2w - \frac{1}{3}\mathbf{w} \cdot 4z^2 + \frac{1}{3}\mathbf{w} \cdot 1 = \frac{1}{3}\mathbf{w}\,(2w - 4z^2 + 1)$	$\frac{1}{3}w$
$-5x^4 - 10x^2 + 15x$	$\mathbf{-5x} \cdot x^3 \mathbf{- 5x} \cdot 2x + (\mathbf{-5x}) \cdot (-3) = \mathbf{-5x}\,(x^3 + 2x - 3)$	$-5x$

Tips: - Factor each term and pull out the GCF.
- If the first term is negative, factor out a negative GCF to make the first term positive.

Factoring Polynomials by Grouping

Steps for factoring polynomials by grouping:

Steps	Example
	Factor $\mathbf{16x^2 - 4x + 28x - 7}$.
▪ Regroup terms with the GCF.	$16x^2 - 4x + 28x - 7 = (16x^2 - 4x) + (28x - 7)$
▪ Factor out the GCF from each group.	$= 4x\,\mathbf{(4x - 1)} + 7\mathbf{(4x - 1)}$
▪ Factor out the GCF again from last step.	$= \boxed{(4x - 1)(4x + 7)}$

Factoring completely: continue factoring until no further factors can be found.

Example: Factor the following completely.

1) $\mathbf{35xy^2 - 7x^2y + 5y - x} = (35xy^2 - 7x^2y) + (5y - x)$ — Regroup terms with the GCF.

$= \mathbf{7xy}(5y - x) + (5y - x) \cdot 1$ — Factor out $7xy$.

$= \boxed{(5y - x)(7xy + 1)}$ — Factor out $(5y - x)$.

2) $\mathbf{3xy + yz - 5yz + 6xy} = (3\mathbf{xy} + 6\mathbf{xy}) + (\mathbf{yz} - 5\mathbf{yz})$ — Regroup.

$= \mathbf{3xy}\,(1 + 2) + \mathbf{yz}\,(1 - 5)$ — Factor out the GCF.

$= 3xy\,(3) + yz\,(-4)$ — Simplify.

$= \boxed{9xy - 4yz}$

3) $\mathbf{t^3 - t^2w - tw^2 + w^3} = (\mathbf{t}^3 - \mathbf{t}^2w) - (t\mathbf{w}^2 - \mathbf{w}^3)$ — Regroup.

$= \mathbf{t^2}\,(t - w) - \mathbf{w^2}\,(t - w)$ — Factor out $(t - w)$.

$= \mathbf{(t - w)}\,(t^2 - w^2)$ — Apply $a^2 - b^2 = (a + b)(a - b)$

$= \mathbf{(t - w)}\,(t + w)\,\mathbf{(t - w)}$

$= \boxed{(t - w)^2\,(t + w)}$

Tip: Identify patterns of common factors such as $5y - x$, $t - w$ …

Factoring Difference of Squares

Factoring difference of squares:

Formula	Example
$\boldsymbol{a^2 - b^2 = (a + b)\,(a - b)}$ or $\boldsymbol{a^2 - b^2 = (a - b)\,(a + b)}$	$x^2 - 49 = x^2 - 7^2 = (x + 7)\,(x - 7)$ $y^2 - 81 = y^2 - 9^2 = (y - 9)\,(y + 9)$

Note:
- $a^2 + b^2$ cannot be factored.
- Always factor out the greatest common factor (GCF) first.
- Determine the perfect square or the square root of each term.

Recall that **factoring is the reverse of multiplication**.

$$a^2 - b^2 \xrightarrow{\text{Factoring}} (a + b)\,(a - b)$$
$$a^2 - b^2 \xleftarrow[\text{Multiplying}]{} (a + b)\,(a - b)$$

Example: Factor the following completely.

1) $\mathbf{2x^2 - 18} = 2\,(x^2 - 9)$ — Factor out 2.

$= 2\,(x^2 - 3^2)$ — $9 = 3^2$ or $\sqrt{9} = 3$

$= \boxed{2\,(x + 3)\,(x - 3)}$ — $a^2 - b^2 = (a + b)\,(a - b)$: $a = x$, $b = 3$

2) $\mathbf{1 - 64u^2} = 1^2 - 8^2 u^2$ — $1 = 1^2$, $64 = 8^2$ or $\sqrt{64} = 8$

$= 1^2 - (8\,u)^2$ — $a^n\,b^n = (a\,b)^n$

$= \boxed{(1 + 8u)\,(1 - 8u)}$ — $a^2 - b^2 = (a + b)\,(a - b)$: $a = 1$, $b = 8u$

3) $\mathbf{100t^2 - 256} = 10^2 t^2 - 16^2$ — $256 = 16^2$ or $\sqrt{256} = 16$

$= (10t)^2 - 16^2$ — $a^n\,b^n = (a\,b)^n$

$= \boxed{(10t + 16)\,(10t - 16)}$ — $a^2 - b^2 = (a + b)\,(a - b)$: $a = 10t$, $b = 16$

4) $\mathbf{9x^2 - 16y^2} = 3^2x^2 - 4^2y^2 = (\overset{a}{\mathbf{3x}})^2 - (\overset{b}{\mathbf{4y}})^2$ — $a^n\,b^n = (a\,b)^n$

$= \boxed{(3x + 4y)\,(3x - 4y)}$ — $a^2 - b^2 = (a + b)\,(a - b)$: $a = 3x$, $b = 4y$

5) $\mathbf{36x^8 - 0.04} = 6^2\,(x^4)^2 - 0.2^2$ — $0.04 = 0.2^2$ or $\sqrt{0.04} = 0.2$, $x^8 = (x^4)^2$

$= (6x^4)^2 - 0.2^2$ — $a^n\,b^n = (a\,b)^n$

$= \boxed{(6x^4 + 0.2)\,(6x^4 - 0.2)}$ — $a^2 - b^2 = (a + b)\,(a - b)$: $a = 6x^4$, $b = 0.2$

Topic B: Factoring Trinomials

Factoring $x^2 + b\,x + c$

Factoring $x^2 + b\,x + c$: cross-multiplication method

Steps	Standard form	Example
	$x^2 + b\,x + c$	$x^2 + 7x + 12$
▪ Setting up two sets of parenthesis.	$= (\quad)(\quad)$	$= (\quad)(\quad)$
	$x^2 + b\,x + c$	$x^2 + 7x + 12$
▪ Factor the first term x^2: $x^2 = x \cdot x$	$x \quad c_1$	$x \quad 3$
▪ Factor the last term c (by trial and error): $c = c_1 \cdot c_2$	$x \quad c_2$	$x \quad 4$
	$x \cdot x = x^2 \quad c_1 \cdot c_2 = c$	$x \cdot x = x^2 \quad 3 \cdot 4 = 12$
▪ Cross multiply and then add up to the middle term.	$(c_1)(x) + (c_2)(x) = b\,x$	$3 \cdot x + 4 \cdot x = 7x$
▪ Complete the parenthesis with $x + c_1$ and $x + c_2$.	$x^2 + b\,x + c$	$x^2 + 3x + 2$
	$= \mathbf{(x + c_1)\,(x + c_2)}$	$= \boxed{(x + 3)\,(x + 4)}$

▪ Check using FOIL.

F O I L

$(x + 3)\,(x + 4) = x^2 + 4x + 3x + 12$

$(x + 3)\,(x + 4) = x^2 + 7x + 12 \quad \surd$

Factoring $x^2 + b\,x + c$ using the cross-multiplication method	
In general	**Example**
$x^2 + b\,x + c = (\quad)(\quad)$	$x^2 - 8\,x + 15 = (\quad)(\quad)$
$x \quad c_1$	$x \quad -5$
$x \quad c_2$	$x \quad -3$
$x \cdot x = x^2 \quad c_1 \cdot c_2 = c$	$x \cdot x = x^2 \quad (-3)(-5) = 15$
$(c_1)(x) + (c_2)(x) \stackrel{?}{=} b\,x$	$(-5)\,x + (-3)\,x \stackrel{?}{=} -8x$ yes!
$x^2 + b\,x + c = \mathbf{(x + c_1)(x + c_2)}$	$x^2 - 8x + 15 = \boxed{(x - 5)\,(x - 3)}$

Check: $-5 + (-3) = -8 \quad \surd$

Tips: - Cross multiply and then add up to the middle term.
- Write the factors with their appropriate signs (+ or –) to get the right middle term.

Summary: Factoring $x^2 + b\,x + c$	**Example: $x^2 - 8x + 15$**
$x^2 + (c_1 + c_1)\,x + c_1c_2 = (x + c_1)\,(x + c_2)$	$x^2 + [-5 + (-3)]\,x + 15 = \boxed{(x - 5)\,(x - 3)}$
$x \quad c_1$	$x \quad -5$
$x \quad c_2$	$x \quad -3$
Check: $c_1x + c_2\,x \stackrel{?}{=} b\,x$	Check: $-5x + (-3x) \stackrel{?}{=} -8x$ yes!

Example: Factor the following:

Trial and error process

1) $\mathbf{a^2 - 11a + 30} = (\quad)(\quad)$

$a \quad -5$

$a \quad -6$

$a \cdot a = a^2 \quad (-5)(-6) = 30$

$(-5)a + (-6)\,a \stackrel{?}{=} -11a$ yes! Check: $-5 + (-6) = -11 \quad \surd$

Answer: $a^2 - 11a + 30 = \boxed{(a - 5)\,(a - 6)}$

Trial and error:

$a^2 - 11a + 30$	$a^2 - 11a + 30$
$a \quad 3$	$a \quad 6$
$a \quad 10$	$a \quad 5$
$3a + 10a \stackrel{?}{=} -11a$ no	$6a + 5a \stackrel{?}{=} -11a$ no

2) $3x^2 + 24x - 27 = 3(x^2 + 8x - 9)$

$x \quad -1$

$x \quad 9$

$x \cdot x = x^2 \quad (-1)(9) = -9$

$(-1)\,x + 9x \stackrel{?}{=} 8x$ yes! Check: $-1 + 9 = 8 \quad \surd$

Answer: $3(x^2 + 8x - 9) = \boxed{3(x - 1)\,(x + 9)}$

Trial and error:

$x^2 + 8x - 9$	$x^2 + 8x - 9$
$x \quad 3$	$x \quad -3$
$x \quad -3$	$x \quad 3$
$3x + (-3)\,x \stackrel{?}{=} 8x$ no	$(-3)x + 3x \stackrel{?}{=} 8x$ no

Note: Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form ($ax^2 + b\,x + c$) first.

Factoring $ax^2 + bx + c$

Procedure for factoring $ax^2 + bx + c$ using the cross-multiplication method:

Steps	In general	Example
	$ax^2 + bx + c$	$3x^2 - 2x - 8$
▪ Setting up two sets of parenthesis.	$= (\quad)(\quad)$	$= (\quad)(\quad)$
	$ax^2 + bx + c$	$3x^2 - 2x - 8$
▪ Factor the first term ax^2: $ax^2 = a_1x \cdot a_2x$	$a_1x \quad c_1$	$x \quad -2$
▪ Factor the last term c (by trial and error): $c = c_1 \cdot c_2$	$a_2x \quad c_2$	$3x \quad 4$
	$a_1x \cdot a_2x = ax^2 \quad c_1 \cdot c_2 = c$	$3x^2 = x \cdot 3x \quad -8 = -2 \cdot 4$
▪ Cross multiply and then add up to the middle term.	$c_1(a_2x) + c_2(a_1x) = bx$	$(-2)(3x) + 4(x) = -2x$
▪ Complete the parenthesis with $(a_1x + c_1)$ and $(a_2x + c_2)$.	$ax^2 + bx + c$ $= \mathbf{(a_1x + c_1)(a_2x + c_2)}$	$3x^2 - 2x - 8$ $= \boxed{(x-2)(3x+4)}$

▪ Check using FOIL.

$$(x-2)(3x+4) = \overset{F}{3x^2} + \overset{O}{4x} - \overset{I}{6x} - \overset{L}{8}$$

$$(x-2)(3x+4) \overset{\surd}{=} 3x^2 - 2x - 8$$

(Original expression)

Tip: Write the factors with their appropriate signs (+ or –) to get the right middle term.

Factoring $ax^2 + bx + c$ using the cross-multiplication method	
In general	**Example**
$\mathbf{ax^2 + bx + c = (\quad)(\quad)}$	$\mathbf{4x^2 + 7x + 3 = (\quad)(\quad)}$
$a_1x \quad c_1$	$4x \quad 3$
$a_2x \quad c_2$	$x \quad 1$
$a_1x \cdot a_2x = ax^2 \quad c = c_1 \cdot c_2$	$4x \cdot x = 4x^2 \quad 3 \cdot 1 = 3$
$(c_1)(a_2x) + (c_2)(a_1x) \overset{?}{=} \mathbf{bx}$	$3 \cdot x + 4x \cdot 1 \overset{?}{=} \mathbf{7x}$ yes!
$ax^2 + bx + c = \mathbf{(a_1x + c_1)(a_2x + c_2)}$	$4x^2 + 7x + 3 = \boxed{(4x+3)(x+1)}$

Tip: Cross multiply and then add up to the middle term.

Summary: Factoring $ax^2 + bx + c$
$a_1a_2x^2 + (c_1a_2 + c_2a_1)x + c_1c_2 = (a_1x + c_1)(a_2x + c_2)$
$a_1x \quad c_1$
$a_2x \quad c_2$

Note: Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form ($ax^2 + bx + c$) first.

More on Factoring $ax^2 + bx + c$

Example: Factor $\mathbf{6y^2 - 17y - 14}$.

$\mathbf{6y^2 - 17y - 14} = (\quad)(\quad)$

$$\begin{array}{ccc} 3y & & 2 \\ 2y & \times & -7 \end{array}$$

$3y \cdot 2y = 6y^2 \qquad 2(-7) = -14$

$(2)(2y) + (-7)(3y) \stackrel{?}{=} \mathbf{-17y}$ yes!

$6y^2 - 17y - 14 = \boxed{(3y + 2)(2y - 7)}$

Trial and error process

1) $6y^2 - 17y - 14$

$$\begin{array}{ccc} y & & -7 \\ 6y & \times & 2 \end{array}$$

$(-7)(6y) + 2y \stackrel{?}{=} -17y$ no

2) $6y^2 - 17y - 14$

$$\begin{array}{ccc} 3y & & 7 \\ 2y & \times & -2 \end{array}$$

$7(2y) + (-2)(3y) \stackrel{?}{=} -17y$ no

3) $6y^2 - 17y - 14$

$$\begin{array}{ccc} 6y & & 2 \\ y & \times & -7 \end{array}$$

$2y + (-7)(6y) \stackrel{?}{=} -17y$ no

Check: $(3y + 2)(2y - 7) = \underset{F}{6y^2} \underset{O}{- 21y} + \underset{I}{4y} \underset{L}{- 14}$

$(2y + 2)(2y - 7) \stackrel{\surd}{=} \boxed{6y^2 - 17y - 14}$ Correct!

Example: Factor the following completely.

1) $\mathbf{28x - 24 + 20x^2} = 20x^2 + 28x - 24$ Rewrite in descending order or standard form ($ax^2 + bx + c$).

$= 4(5x^2 + 7x - 6) = 4(\quad)(\quad)$ Factor out 4.

$$\begin{array}{ccc} x & & 2 \\ 5x & \times & -3 \end{array}$$

$10x + (-3x) = 7x$ √

$4(5x^2 + 7x - 6) = \boxed{4(x + 2)(5x - 3)}$

Note: Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form ($ax^2 + bx + c$) first.

2) $\mathbf{8a^2 - 6ab - 5b^2} = (\quad)(\quad)$

$$\begin{array}{ccc} 2a & & b \\ 4a & \times & -5b \end{array}$$

$-10ab + 4ab = -6ab$ √

$8a^2 - 6ab - 5b^2 = \boxed{(2a + b)(4a - 5b)}$

3) $\mathbf{2t^4 + 14t^2 + 20} = 2(t^4 + 7t^2 + 10) = 2(\quad)(\quad)$ Factor out 2.

$$\begin{array}{ccc} t^2 & & 2 \\ t^2 & \times & 5 \end{array}$$

$2t^4 + 14t^2 + 20 = \boxed{2(t^2 + 2)(t^2 + 5)}$

$5t^2 + 2t^2 = 7t^2$ √

Factoring Trinomials: AC Method

AC method for factoring trinomials: $ax^2 + bx + c$

Factoring $ax^2 + bx + c = 0$ by Grouping	Example
Steps	**Solve** $14x + 6 = -8x^2$
▪ Convert to standard form (descending order) if necessary.	$8x^2 + 14x + 6 = 0$
▪ Factor out the greatest common factor (GCF).	$2(4x^2 + 7x + 3) = 0$
▪ Multiply ***a*** and ***c*** in $ax^2 + bx + c$.	$ac = 4 \cdot 3 = 12$
▪ Factor the product ***ac*** that sum to the middle coefficient ***b***.	$4 \cdot 3 = 12, \quad 4 + 3 = 7$
▪ Rewrite the middle term as the sum using the factors found in last step.	$2(4x^2 + 7x + 3) = 0$ $2(4x^2 + 4x + 3x + 3) = 0$
▪ Factor by grouping.	$2[4x(x + 1) + 3(x + 1)] = 0$ $\boxed{2(x + 1)(4x + 3) = 0}$ Factor out $(x + 1)$.

Example: Factor $6x^2 - 16 = 4x$ using *ac* method.

Steps	Solution
	$6x^2 - 16 = 4x$
▪ Write in standard form:	$6x^2 - 4x - 16 = 0$
▪ Factor out the greatest common factor:	$2(3x^2 - 2x - 8) = 0$
▪ Multiply ***a*** and ***c*** in $ax^2 + bx + c$:	$ac = 3 \cdot (-8) = -24$
▪ Factor the product ***ac*** that sum to the middle coefficient ***b***.	

(There are different pairs to get the product of ***ac*** of **-24**. Try to find two numbers that multiply to ***ac*** and add to obtain ***b* = -2**.)

Some factors of *ac* (-24)	Sum of factors (*b* = -2)
-3 & 8	-3 + 8 = 5
-4 & 6	-4 + 6 = 2
8 & -3	8 + (-3) = 5
4 & -6	4 + (- 6) = **-2** Correct!

The right choices are 4 an -6, since they both add up to *b* = -2. 4 (-6) = -24, 4 + (-6) = -2

- Rewrite the middle term as $4x - 6x$.

$$2(3x^2 - 2x - 8) = 0$$

$$2(3x^2 + 4x - 6x - 8) = 0$$

- Factor by grouping.

$$2[x(3x + 4) - 2(3x + 4) = 0$$ Factor out $(3x + 4)$

$$\boxed{2(3x + 4)(x - 2) = 0}$$

Factoring Special Products

Recall that **factoring is the reverse of multiplication**.

$$a^2 + 2ab + b^2 \xrightarrow{\text{Factoring}} (a+b)^2 \qquad (a+b)^2 \xrightarrow{\text{Multiplying}} a^2 + 2ab + b^2$$

Recognize some polynomials as special products can factor more quickly.

Special products:

Name	Formula	Example
Square of sum (perfect square trinomial)	$\mathbf{a^2 + 2ab + b^2 = (a + b)^2}$	$x^2 + 10x + 25 = (x + 5)^2$ $a = x,\ \ b = 5$ $x \quad 5$ $x \quad 5$ Check: $(x + 5)^2 = x^2 + 2 \cdot x \cdot 5 + 5^2 = x^2 + 10x + 25$ √
Square of difference (perfect square trinomial)	$\mathbf{a^2 - 2ab + b^2 = (a - b)^2}$	$9y^2 - 24y + 16 = (3y - 4)^2$ $a = 3y,\ \ b = 4$ $3y \quad -4$ $3y \quad -4$ Check: $(3y - 4)^2 = (3y)^2 - 2(3y)(4) + 4^2 = 9y^2 - 24y + 16$ √

Note: The quickest way to factor an expression is to recognize it as a special product.

Memory aid: $(a^2 \pm 2ab + b^2) = (a \pm b)^2$ ν Notice the reversed plus or minus sign in the second term.

To use perfect square trinomial formulas: use cross-multiplication method to factor a perfect square. Then use the square formula to check.

Example: Factor the following completely.

1) $\mathbf{28z + 49 + 4z^2} = 4z^2 + 28z + 49$ Rewrite in standard form: $ax^2 + bx + c$

$2z \quad 7$
$2z \quad 7$

$= (2z + 7)(2z + 7)$ $7(2z) + 7(2z) = 28x$

$= \boxed{(2z + 7)^2}$

Check: $(2z + 7)^2 = (2z)^2 + 2 \cdot 2z \cdot 7 + 7^2 = 4z^2 + 28z + 49$ √ $a^2 + 2ab + b^2 = (a + b)^2$: $a = 2z$, $b = 7$

2) $\mathbf{50p^2 - 40p + 8} = 2(25p^2 - 20p + 4)$ Factor out 2.

$2(25p^2 - 20p + 4) = \boxed{2(5p - 2)^2}$

$5p \quad -2$
$5p \quad -2$ $-2(5p) + -2(5p) = -20p$

Check: $2(5p - 2)^2 = 2[(5p)^2 - 2(5p)(2) + (2)^2] = 2(25p^2 - 20p + 4)$ √ $a^2 - 2ab + b^2 = (a - b)^2$: $a = 5p$, $b = 2$

3) $\mathbf{16n^{10} - 48n^5 + 36} = 4(4n^{10} - 12n^5 + 9)$ Factor out 4.

$2n^5 \quad -3$
$2n^5 \quad -3$ $(2n^5)(-3) + (2n^5)(-3) = -12n^5$

$= \boxed{4(2n^5 - 3)^2}$ $a^m a^n = a^{m+n}$

Check: $(2n^5 - 3)^2 = (2n^5)^2 - 2(2n^5)(3) + (3)^2 = 4n^{10} - 12n^5 + 9$ √ $a^2 - 2ab + b^2 = (a - b)^2$: $a = 2n^5$, $b = 3$

Topic C: Application of Factoring

Quadratic Equations

Quadratic equation: an equation that has a squared term, such as $7x^2 + 3x - 5 = 0$.

Quadratic equations in standard form	
$ax^2 + bx + c = 0$	$a \neq 0$

Incomplete quadratic equation

Incomplete quadratic equation		Example	a	b	c
$ax^2 + bx = 0$	$(c = 0)$	$4x^2 - 3x = 0$	4	-3	0
$ax^2 + c = 0$	$(b = 0)$	$8x^2 + 5 = 0$	8	0	5

Zero-product property:

Zero-product property
If $A \cdot B = 0$, then either $A = 0$ or $B = 0$ (or both) (A and B are algebraic expressions.)

Note: "or" means possibility of both.

Solving incomplete quadratic equations

Incomplete quadratic equation	Steps	Example
Use the zero-product property to solve $ax^2 + bx = 0$	- Express in $ax^2 + \boldsymbol{b}x = 0$ - Factor: $x(ax + b) = 0$ - Apply the zero-product property: $x = 0$ ¦ $ax + b = 0$ - Solve for x: $x = 0$ ¦ $x = -\frac{b}{a}$	**Solve $11x^2 = -6x$** $\mathbf{11}x^2 + \mathbf{6}x = 0$ Add $6x$. $x(11x + 6) = 0$ $x = 0$ ¦ $11x + 6 = 0$ $x = 0$ ¦ $x = -\frac{6}{11}$
Use the square root method to solve $ax^2 - c = 0$ (or $ax^2 = c$)	- Express in $\boldsymbol{a}x^2 = \boldsymbol{c}$ - Divide both sides by a: $x^2 = \frac{c}{a}$ - Take the square root of both sides: $x = \pm\sqrt{\frac{c}{a}}$	**Solve $64x^2 - 9 = 0$** $\mathbf{64}x^2 = \mathbf{9}$ $x^2 = \frac{9}{64}$ $x = \pm\sqrt{\frac{9}{64}} = \pm\frac{3}{8}$

Solving Quadratic Equations

Solve a quadratic equation: a quadratic equation $ax^2 + bx + c = 0$ can be written as:

	$(x + a)(x + b) = 0$	Factor.
Set each term equal to zero:	$x + a = 0 \quad \vert \quad x + b = 0$	Zero-product property.
Solutions:	$x = -a \quad \vert \quad x = -b$	Solve for x.

Example: Solve for x. $\quad \boldsymbol{(x + 6)(x - 11) = 0}$

$x + 6 = 0 \quad \vert \quad x - 11 = 0$ — Zero-product property.

$\boldsymbol{x} = \boxed{-6} \quad \vert \quad \boldsymbol{x} = \boxed{11}$ — Solve for x.

Example: Solve the quadratic equation $\boldsymbol{x^2 - x - 20 = 0}$.

1) $\boldsymbol{x^2 - x - 20 = 0}$

$$\begin{array}{cc} x & 4 \\ x & -5 \end{array}$$

Factor.

$4x + (-5)x = -x$

$(x + 4)(x - 5) = 0$

$x + 4 = 0 \quad \vert \quad x - 5 = 0$ — Zero-product property.

$\boldsymbol{x} = \boxed{-4} \quad \vert \quad \boldsymbol{x} = \boxed{5}$

2) $\boldsymbol{6x^2 - 13x = 15}$ — Rewrite in standard form: $ax^2 + bx + c = 0$

$6x^2 - 13x - 15 = 0$ — Set the equation equal to 0.

$$\begin{array}{cc} 6x & 5 \\ x & -3 \end{array}$$

Factor.

$5x + (-3)(6x) = -13x$

$(6x + 5)(x - 3) = 0$

$6x + 5 = 0 \quad \vert \quad x - 3 = 0$ — Zero-product property.

$\boldsymbol{x} = \boxed{-\frac{5}{6}} \quad \vert \quad \boldsymbol{x} = \boxed{3}$

3) $\boldsymbol{x^2 - \frac{2}{9} = \frac{1}{3}x}$ — Rewrite in standard form: $ax^2 + bx + c = 0$

$x^2 - \frac{1}{3}x - \frac{2}{9} = 0$ — Set the equation equal to 0.

$$\begin{array}{cc} x & \frac{1}{3} \\ x & -\frac{2}{3} \end{array}$$

Factor.

$\frac{1}{3}\left(-\frac{2}{3}\right) = -\frac{2}{9}$, $\quad \frac{1}{3}x + \left(-\frac{2}{3}x\right) = -\frac{1}{3}x$

$(x + \frac{1}{3})(x - \frac{2}{3}) = 0$

$x + \frac{1}{3} = 0 \quad \vert \quad y - \frac{2}{3} = 0$ — Zero-product property.

$\boldsymbol{x} = \boxed{-\frac{1}{3}} \quad \vert \quad \boldsymbol{x} = \boxed{\frac{2}{3}}$

Application of Quadratic Equations

Review number problems - examples

English phrase	Algebraic expression/equation
6 more than the difference of the square of a number and 11 is 32.	$(x^2 - 11) + 6 = 32$
The quotient of 5 and the product of 9 and a number is 7 less than the number.	$\frac{5}{9x} = x - 7$
The product of 9 and the square of a number decreased by 13 is 21.	$9x^2 - 13 = 21$
15 more than the quotient of $4x$ by 7 is 5 times the square of a number.	$15 + \frac{4x}{7} = 5x^2$

Let x = a number; y = a number

Review consecutive integers

English phrase	Algebraic expression	Example
Three consecutive **odd** integers	x, $x + 2$, $x + 4$	If $x = \mathbf{1}$, $x + 2 = \mathbf{3}$, $x + 4 = \mathbf{5}$
Three consecutive **even** integers	x, $x + 2$, $x + 4$	If $x = \mathbf{2}$, $x + 2 = \mathbf{4}$, $x + 4 = \mathbf{6}$
The product of two consecutive odd integers is 35.	$x(x + 2) = 35$	
Three consecutive even integers whose sum is 12.	$x + (x + 2) + (x + 4) = 12$	

Examples:

1) The product of a number and 4 more than the square of the number is 21. Find the number(s).

- Let x = the number
- Equation $\mathbf{x^2 + 4x = 21}$
- Solve for x: $x^2 + 4x - 21 = 0$ Rewrite in standard form.

$x \quad 7$

$x \quad -3$

Factor.
$7x + (-3)x = 4x$

$(x + 7)(x - 3) = 0$

$x + 7 = 0 \qquad x - 3 = 0$ Zero-product property.

$\mathbf{x} = \boxed{-7} \qquad \mathbf{x} = \boxed{3}$

2) The product of two consecutive even integers is 48. Find the integers.

- Let x = the first even integer
- Equation $\mathbf{x(x + 2) = 48}$ The 2nd integer is $x + 2$.
- Solve for x: $x^2 + 2x - 48 = 0$ Rewrite in standard form.

$x \quad -6$

$x \quad 8$

Factor.
$-6x + 8x = 2x$

$(x - 6)(x + 8) = 0$

$x - 6 = 0 \qquad x + 8 = 0$ Zero-product property.

$\mathbf{x} = \boxed{6} \qquad \mathbf{x} = \boxed{-8}$

If $x = 6$, $x + 2 = 8$ If $x = -8$, $x + 2 = -6$

Dimension (length and width) problems:

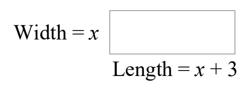
Example: Robert is going to replace the old carpet in his bedroom, which is a rectangle and has a ***length 3 meters greater than its width***. If the ***area*** of his bedroom is ***54*** square meters (***m²***), what will be the dimensions of the carpet?

Steps	Solution

- Organize the facts.

Facts	Area $A = 54\text{m}^2$ Length = Width + 3m
Unknowns	Width = x, Length = x + 3m

- Draw a diagram.



- Equation: **$x\,(x + 3) = 54$** — Area: $A = w\,l$
- Solve the equation. $x^2 + 3x = 54$ — Distribute.
  - Standard form: $x^2 + 3x - 54 = 0$ — $ax^2 + bx + c = 0$
  - Factor:
    x 9
    x -6 — $9x + (-6)\,x = 3x$
    $(x + 9)\,(x - 6) = 0$
  - Zero-product property: $x + 9 = 0$ | $x - 6 = 0$
  - Solutions: $x = -9$ | $x = 6$

Since the width of a rectangle cannot be negative, eliminate $x = -9$.

- Answer (the size of the carpet):
  Width = $\boldsymbol{x}$ = $\boxed{6\text{ m}}$
  Length = $\boldsymbol{x + 3}$ = 6 + 3 = $\boxed{9\text{ m}}$

Triangle problem:

Example: A triangle is 1 meter wider than it is tall. The area is 36 m². Find the base and the height.

- Organize the facts.

Facts	Area $A = 36\text{m}^2$ Base = Height + 1m
Unknowns	Height = x, Base = x + 1m

- Equation. $\frac{1}{2}(x + 1)\,x = 36$ — Area: $A = \frac{1}{2}bh$
- Solve the equation. $x^2 + x = 72$ — Multiply both sides by 2.
  - Standard form: $x^2 + x - 72 = 0$ — $ax^2 + bx + c = 0$
  - Factor:
    x 9
    x -8 — $9x + (-8)\,x = x$
    $(x + 9)\,(x - 8) = 0$
  - Zero-product property: $x + 9 = 0$ | $x - 8 = 0$
  - Solutions: $x = -9$ | $x = 8$
- Answer: **Height** = $\boldsymbol{x}$ = $\boxed{8\text{ m}}$
  (Since the height of a triangle cannot be negative, eliminate $x = -9$.)
  Base = $\boldsymbol{x + 1}$ = 8 + 1 = $\boxed{9\text{m}}$

Unit 14: Summary

Factoring Polynomials

Factoring whole numbers: write the number as a product of its prime factors.

Common factor: a number or an expression that is a factor of each term of a group of terms.

Greatest / highest common factor (GCF or HCF): the product of the common factors.

Factoring a polynomial: express a polynomial as a product of other polynomials. It is the reverse of multiplication.

Steps for factoring polynomials by grouping:

- Regroup terms with the GCF.
- Factor out the GCF from each group.
- Factor out the GCF again from last step.

Special products:

Name	Formula
Difference of squares	$a^2 - b^2 = (a + b)(a - b)$
Square of sum	$a^2 + 2ab + b^2 = (a + b)^2$
Square of difference	$a^2 - 2ab + b^2 = (a - b)^2$

Memory aid: $(a^2 \pm ab + b^2) = (a \pm b)^2$

Cross-multiplication method:

Factoring $x^2 + bx + c$ using the cross-multiplication method	
In general	**Example**
$x^2 + bx + c = (\quad)(\quad)$	$x^2 - 8x + 15 = (\quad)(\quad)$
$x \quad c_1$	$x \quad -5$
$x \quad c_2$	$x \quad -3$
$x \cdot x = x^2 \qquad c_1 \cdot c_2 = c$	$x \cdot x = x^2 \qquad (-3)(-5) = 15$
$(c_1)(x) + (c_2)(x) \overset{?}{=} bx$	$(-5)x + (-3)x \overset{?}{=} -8x$ yes!
$x^2 + bx + c = (x + c_1)(x + c_2)$	$x^2 - 8x + 15 = \boxed{(x - 5)(x - 3)}$

Tips:
- Cross multiply and then add up to the middle term.
- Write the factors with their appropriate signs (+ or –) to get the right middle term.

Factoring $ax^2 + bx + c$ using the cross-multiplication method	
In general	**Example**
$ax^2 + bx + c = (\quad)(\quad)$	$4x^2 + 7x + 3 = (\quad)(\quad)$
$a_1x \quad c_1$	$4x \quad 3$
$a_2x \quad c_2$	$x \quad 1$
$a_1x \cdot a_2x = ax^2 \qquad c = c_1 \cdot c_2$	$4x \cdot x = 4x^2 \qquad 3 \cdot 1 = 3$
$(c_1)(a_2x) + (c_2)(a_1x) \overset{?}{=} bx$	$3 \cdot x + 4x \cdot 1 \overset{?}{=} 7x$ yes!
$ax^2 + bx + c = (a_1x + c_1)(a_2x + c_2)$	$4x^2 + 7x + 3 = \boxed{(4x + 3)(x + 1)}$

Tips:
- Cross multiply and then add up to the middle term.
- Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form ($ax^2 + bx + c$) first.

Factoring polynomials:

Polynomial	Method
Two terms (binomial)	- If it is a perfect square: $a^2 - b^2 = (a+b)(a-b)$ - If not, use the distributive property: $ac + bc = c\,(a+b)$
Three terms (trinomial)	$ax^2 + b\,x + c$: use the cross-multiplication or AC methods.
Four terms	Factor by grouping

Quadratic equation: an equation that has a squared term.

Quadratic equations in standard form	
$ax^2 + b\,x + c = 0$	$a \neq 0$

Incomplete quadratic equation

Incomplete quadratic equation	
$ax^2 + b\,x = 0$	$(c = 0)$
$ax^2 + c = 0$	$(b = 0)$

Zero-product property:

Zero-product property
If $A \cdot B = 0$, then either $A = 0$ or $B = 0$ (or both) (A and B are algebraic expressions.)

Solving incomplete quadratic equations

Incomplete quadratic equation	Steps
Use the zero-product property to solve $ax^2 + bx = 0$	- Express in $ax^2 + \boldsymbol{b}\,x = 0$ - Factor: $x\,(ax + b) = 0$ - Apply the zero-product property: $x = 0$; $ax + b = 0$ - Solve for x: $x = 0$; $x = -\frac{b}{a}$
Use the square root method to solve $ax^2 - c = 0$	- Express in $\boldsymbol{a}x^2 = \boldsymbol{c}$ - Divide both sides by a: $x^2 = \frac{c}{a}$ - Take the square root of both sides: $x = \pm\sqrt{\frac{c}{a}}$

Unit 14: Self-Test

Factoring Polynomials

Topic A

1. Factor 60.

2. Find the greatest common factor (GCF) for the following.

a) $5x^2 - 20x$

b) $3a^3b + 15a^4b - 21ab$

c) $17y^2 (y + 4) - (2y + 8)$

d) $\frac{1}{4}x^3 - \frac{3}{4}xy^2 + \frac{5}{4}x$

e) $-4y^3 - 8y^2 + 20y$

3. Factor the following completely.

a) $25x^2 - 5x + 20x - 4$

b) $48ab^2 - 8a^2b + 6b - a$

c) $4uv + vw - 7vw + 21uv$

d) $x^3 - x^2y - xy^2 + y^3$

e) $5y^2 - 20$

f) $1 - 49w^2$

g) $81u^2 - 121$

h) $25a^2 - 36b^2$

i) $4y^6 - 0.09$

Topic B

4. Factor the following:

a) $x^2 + 9x + 20$

b) $x^2 - 10x + 24$

c) $x^2 - 3x - 18$

d) $2x^2 + 10x - 28$

e) $4x^2 - 7x - 15$

f) $5y^2 + 9y - 18$

g) $24ab^2 - 4a^2b + 6b - a$

h) $6uv + vs - 7vs + 11uv$

5. Factor the following using the *ac* method.

a) $6x^2 - 60 = 9x$

b) $6x^2 + 4x - 16$

6. Factor the following completely.

(Use the cross-multiplication method to factor a perfect square. Then use the square formula to check.)

a) $9x^2 + 30x + 25$

b) $27 + 12y^2 - 36y$

c) $18t^8 - 24t^4 + 8$

Topic C

7. Solve for x.

a) $23x^2 = -7x$

b) $81x^2 - 49 = 0$

c) $(x + 9)(x - 17) = 0$

8. Solve the following quadratic equations.

a) $x^2 - x - 42 = 0$

b) $7x^2 - 31x = 20$

c) $x^2 + \frac{3}{16} = x$

9. The product of a number and 5 more than the square of the number is 36. Find the number(s).

10. The product of two consecutive even integers is 24. Find the integers.

11. Lisa is going to replace old carpet in her living room, which is a rectangle and has a length 2 meters greater than its width. If the area of her living room is 63 square meters (m^2), what will be the dimensions of the carpet?

12. A triangle is 2 meters wider than it is tall. The area is $24m^2$. Find the base and the height.

Unit 15

Graphing Linear Equations

Topic A: Cartesian graphing

- The Coordinate plane
- Graphing linear equations

Topic B: The slope of a straight line

- Slope
- Vertical and horizontal lines

Topic C: Graphing a linear equation

- Slope-intercept equation of a line
- Graphing using the slope and the y-intercept
- Graphing linear equations
  - Intercept method

Topic D: Writing equations of lines

- Finding an equation of a line

Unit 15 Summary

Unit 15 Self-test

Topic A: Cartesian Graphing

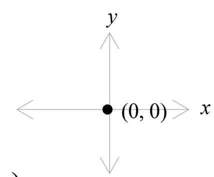
The Coordinate Plane

The coordinate plane (or Cartesian / rectangular coordinate system): a powerful tool to mark a point and solution of linear equations on a graph.

- Coordinate axes:

  x axis - the horizontal line.

  y axis - the vertical line.



- The origin: the intersection of the *x* and *y* axes (both lines are 0 at the origin).

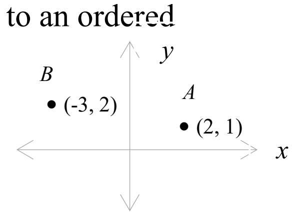
Ordered pair (*x*, *y*): a pair of numbers (each point on the plane corresponds to an ordered pair).

(x , y)

1st-coordinator (abscissa) 2nd-coordinator (ordinate)

Example: Point *A*: (2, 1)

Point *B*: (-3, 2)



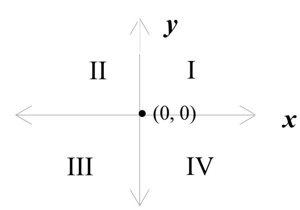
Example: (coke, \$0.90) , (juice, \$1.25)

Coordinate: the numbers in an ordered pair (the *x*-distance and the *y*-distance from a given origin).

Example: the coordinate of the point *A* is (2, 1) and the point *B* is (-3, 2).

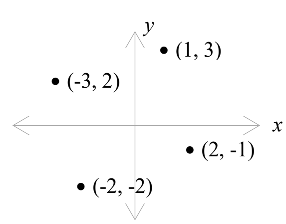
Four Quadrants:

Quadrant	(*x*, *y*)	Example
The 1st quadrant I	(+*x*, +*y*)	(+2, +3)
The 2nd quadrant II	(-*x*, +*y*)	(-2, +3)
The 3rd quadrant III	(-*x*, -*y*)	(-2, -3)
The 4th quadrant IV	(+*x*, -*y*)	(+2, -3)



Example: Plot the points and name the quadrants.

(1, 3) (-3, 2) (-2, -2) (2, -1)



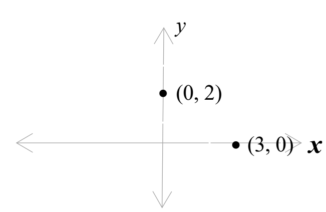
(1, 3): $\boxed{\text{I}}$, **(-3, 2):** $\boxed{\text{II}}$, **(-2, -2):** $\boxed{\text{III}}$, **(2, -1):** $\boxed{\text{IV}}$

***x* - intercept (*x*, 0):** the point at which the graph crosses the *x* - axis.

Example: $(x, y) = \boxed{(3, 0)}$

***y* - intercept (0, *y*):** the point at which the graph crosses the *y* - axis.

Example: $(x, y) = \boxed{(0, 2)}$



Points are on the axes.

Graphing Linear Equations

A linear (first-degree) equation: an equation whose graph is a straight line.

A linear equation in two variables: a linear equation that contains two variables, such as $2x + y = 3$.

The standard form of linear equation in two variables: $Ax + By = C$

Standard Form	Example
$Ax + By = C$	$5x - 7y = 4$

Solutions of equations: solutions for a linear equation in two variables are an ordered pair. They are the particular values of the variables in the equation that makes the equation true.

Example: Find the ordered pair solution of the given equation.

$\mathbf{2x - 3y = 7}$, **when $x = 2$.** — Replace x with 2.

$2(2) - 3y = 7$ $\quad$ $4 - 3y = 7$ — Subtract 4 from both sides.

$-3y = 3$ $\quad$ $y = -1$ — Divide -3 both sides.

Check: $2 \cdot 2 - 3(-1) \stackrel{?}{=} 7$, $7 = 7$. The ordered pair solution is $\boxed{(2, -1)}$. (x, y)

The graph of an equation is the diagram obtained by plotting the set of points where the equation is true (or satisfies the equation).

Procedure to graph a linear equation

Steps

- Choose two values of x, calculate the corresponding y, and make a table.
- Plot these two points on the coordinate plane.
- Connect the points with a straight line.
  (Any two points determine a straight line.)
- Check with the third point.

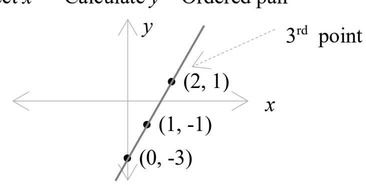
Is third point (2, 1) on the line? Yes. Correct!

Example: Graph $\mathbf{2x - y = 3}$

Isolate y.

x	$y = 2x - 3$	(x, y)	
0	$2 \cdot 0 - 3 = -3$	$(0, -3)$	y-intercept
1	$2 \cdot 1 - 3 = -1$	$(1, -1)$	

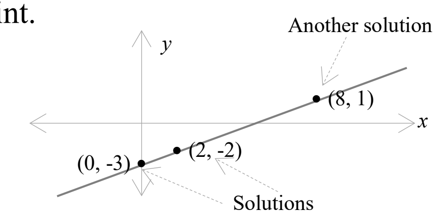
Select x — Calculate y — Ordered pair



x	$y = 2x - 3$	(x, y)
2	$2 \cdot 2 - 3 = 1$	$(2, 1)$

Example: Graph $y = \frac{1}{2}x - 3$ and determine another point.

x	y	(x, y)
0	- 3	(0, -3)
2	- 2	(2, -2)



Topic B: The Slope of a Straight Line

Slope

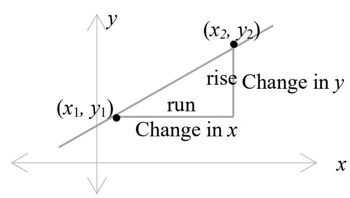
Recall: the graph of a linear equation is a straight line.

Slope (m) (grade or pitch): the slope of a straight line is the rate of change. It is a measure of the "steepness" or "incline" of the line and indicates whether the line rises or falls.

A line with a positive slope rises from left to right and a line with a negative slope falls.

The slope formula:

The slope formula	
$\text{Slope} = \dfrac{\text{the change in } y}{\text{the change in } x} = \dfrac{\text{rise}}{\text{run}}$	The slope of the straight line that passes through two points (x_1, y_1) and (x_2, y_2): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ or $m = \dfrac{y_1 - y_2}{x_1 - x_2}$ $\quad x_1 \neq x_2$



Example: Determine the slope containing points (3, -2) and (4, 1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-2)}{4-3} = \frac{3}{1} = \boxed{3}$$

$(x_1, y_1) = (3, -2)$, $(x_2, y_2) = (4, 1)$

or $$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-2-1}{3-4} = \frac{-3}{-1} = \boxed{3}$$

Example: Determine the slope of **$6x - y - 5 = 0$.**

x	$y = 6x - 5$	(x, y)
0	$6 \cdot 0 - 5 = -5$	$(x_1, y_1) = (0, -5)$
1	$6 \cdot 1 - 5 = 1$	$(x_2, y_2) = (1, 1)$

Choose Calculate

Solve for y from $6x - y - 5 = 0$

$6x - 5 = y$ (add y both sides.)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-5)}{1-0} = \frac{6}{1} = \boxed{6} \quad \text{or} \quad m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-5-1}{0-1} = \frac{-6}{-1} = \boxed{6}$$

Other points on the line will obtain the same slope m.

x	$y = 6x - 5$	(x, y)
2	7	(2, 7)
-1	-11	(-1, -11)

Choose Calculate

$$m = \frac{-11-7}{-1-2} = \frac{-18}{-3} = \boxed{6}$$

$(x_1, y_1) = (2, 7)$, $(x_2, y_2) = (-1, 11)$

Vertical and Horizontal Lines

Horizontal line: a line that is parallel to the x-axis.

- It has a zero slope ($m = 0$).
- With a y-intercept $y = b$ or $(0, b)$. Where b is any constant.

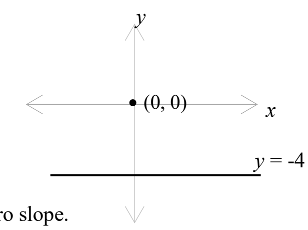
Example: $y = -4$

x	y	(x, y)
1	-4	(1, -4)
2	-4	(2, -4)

Choose Given

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(-4)}{2-1} = \frac{0}{1} = \boxed{0}$$

The horizontal line $y = -4$ has a zero slope.

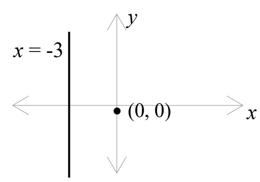


Vertical line: a line that is parallel to the y-axis.

- It has an infinite slope ($m = \infty$).
- With a x-intercept $x = a$ or $(a, 0)$.

Example: $x = -3$

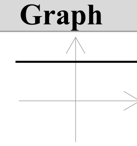
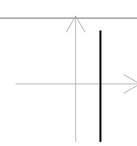
x	y	(x, y)
-3	3	(-3, 3)
-3	-1	(-3, -1)



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{-3-(-3)} = \frac{-4}{0} = \infty$$

The vertical line $x = -3$ has an infinite slope.

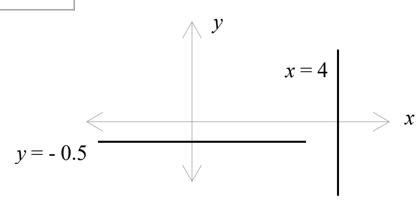
Summary of horizontal and vertical lines:

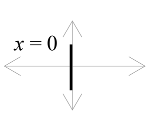
Line	Equation	Slope (m)	Example	Graph
Horizontal line	$y = b$	$m = 0$	$y = 2$	
Vertical line	$x = a$	$m = \infty$	$x = 1$	

Example: **1)** Graph $y = -0.5$

2) Graph $x = 4$

3) Graph $x = 0$



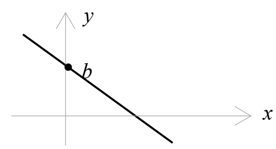


Topic C: Graphing a Linear Equation

Slope-Intercept Equation of a Line

Slope - intercept form of a linear equation

Slope - intercept equation of a line	
$y = \boldsymbol{m}x + \boldsymbol{b}$	m: the slope of the line b: y-intercept



Recall: *y* - intercept: the point at which the line crosses the y axis. $b = (0, y)$

Example: Identify the slope and y-intercept of the following equations.

1) $y = \mathbf{-3}\boldsymbol{x} - \mathbf{5}$

$y = \boldsymbol{m}x + \boldsymbol{b}$

$y = -3x - 5$

The slope: $\boldsymbol{m} = \boxed{-3}$

y-intercept: $\boldsymbol{b} = \boxed{-5}$ or $(0, -5)$

2) $\mathbf{3}\boldsymbol{y} - \mathbf{2}\boldsymbol{x} = \mathbf{1}$

$3y = 2x + 1$ Add $2x$ on both sides.

$y = \frac{2}{3}x + \frac{1}{3}$ Divide both sides by 3.

The slope: $\boldsymbol{m} = \boxed{\frac{2}{3}}$ $y = \boldsymbol{m}x + \boldsymbol{b}$

y-intercept: $\boldsymbol{b} = \boxed{\frac{1}{3}}$ or $(0, \frac{1}{3})$

3) $\boldsymbol{4x + \frac{1}{3}y = 5}$

$4x \cdot 3 + \frac{1}{3}y \cdot 3 = 5 \cdot 3$ Multiply 3 by each term.

$12x + y = 15$ Subtract $12x$ from both sides.

$y = -12x + 15$ $y = \boldsymbol{m}x + \boldsymbol{b}$

The slope: $\boldsymbol{m} = \boxed{-12}$

y-intercept: $\boldsymbol{b} = \boxed{15}$ or $(0, 15)$

Graphing Using the Slope and the y - Intercept

Slope-intercept equation: $y = mx + b$

$$\begin{cases} m = \text{slope} \\ b = y - \text{intercept} \end{cases}$$

The slope and a point can determine a straight line.

Example: Graph the equation using the slope and the y-intercept. $\boldsymbol{y = \frac{-2}{3}x + 5}$

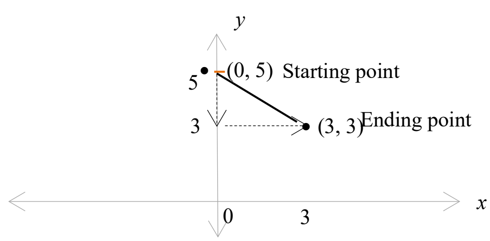
- Plot the y-intercept (0, 5).
- Determine the rise and run: $m = \frac{-2}{3}$

The change in y: the rise (move 2 units down, $\because$ y is negative).

The change in x: the run (move 3 units to the right, $\because$ x is positive).

- Plot another point by moving 2 units down and 3 units to the right (3, 3).
- Connect the two points with a straight line.

Starting point: y-intercept



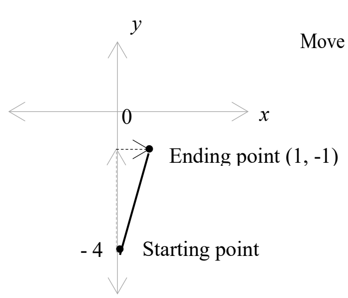
Example: Graph the equation using the slope and the y-intercept. $-9x + 12 = -3\boldsymbol{y}$

- Convert to the slope - intercept form. $3y = 9x - 12$ Divide each term by (-1).

$y = 3x - 4$ Divide both sides by 3.

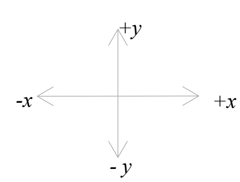
- y-intercept: (0, -4) $y = mx + b$
- Slope: $m = 3 = \frac{3}{1}$

Move 3 units up and 1 unit to the right (both x & y are positive).



Tip: $m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$

$$\begin{cases} +y: \text{ move up} \\ -y: \text{ move down} \\ +x: \text{ move to the right} \\ -x: \text{ move to the left} \end{cases}$$



Graphing Linear Equations
- Intercept Method

Recall: The x intercept is the point at which the line crosses the x axis. $(a, 0)$

The y intercept is the point at which the line crosses the y axis. $(0, b)$

Find the intercepts:

Example: Determine the intercepts of the line $5x - y = 6$.

- The x-intercept: let $y = 0$, and solve for x. $5x - 0 = 6$, $5x = 6$

  $x = \frac{6}{5} = 1.2$ Divide both sides by 5.

  $\boxed{(1.2,\ 0)}$

- The y-intercept: let $x = 0$, and solve for y. $5 \cdot 0 - y = 6$, $-y = 6$

  $y = -6$ Divide both sides by -1.

  $\boxed{(0, -6)}$

Procedure to graph a linear equation using the intercept method

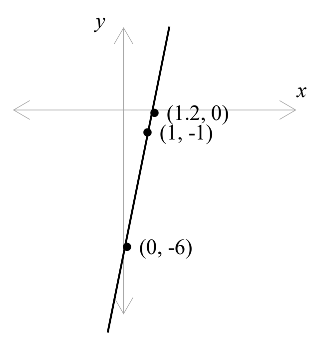
Steps

- Choose $x = 0$ and calculate the corresponding y.
- Choose $y = 0$ and calculate the corresponding x.
- Plot these two points on the coordinate plane.
- Connect the points with a straight line.
- Check with the third point.

  Is third point (1, -1) on the line? Yes. Correct!

Example: $5x - y = 6$

x	$y = 5x - 6$	(x, y)	Intercept
0	-6	(0, -6)	y-intercept
1.2	0	(1.2, 0)	x-intercept



x	$y = 5x - 6$	(x, y)
1	-1	(1, -1)

$(5 \cdot 1 - y = 6,\quad -y = 6 - 5,\quad y = -1)$

Topic D: Writing Equations of Lines

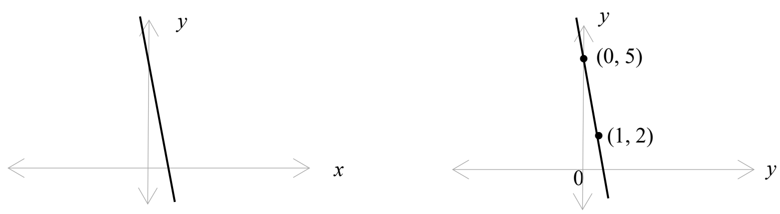
Finding an Equation of a Line

Equation of a straight line:

Straight-line equation	Equation	Example	
Point-slope form	$y - y_1 = m\,(x - x_1)$	$y - 3 = -4\,(x + 2)$	$m = -4 \quad y_1 = 3,\ x_1 = -2$
Slope-intercept form	$y = mx + b$	$y = 3x - \frac{4}{5}$	$m = 3\ ,\ \ b = -\frac{4}{5}$

Finding an equation of a line from the graph:

Example: Write the slope intercept equation of the given line. $y = mx + b$



- Choose two points on the given line, such as (0, 5) and (1, 2).
- The slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{1-0} = \frac{-3}{1} = -3$ $\quad (x_1, y_1) = (0, 5),\ (x_2, y_2) = (1, 2)$
- y-intercept: $b = 5$ The line crosses the y-axis at (0, 5).
- Equation of the line: $\boxed{y = -3x + 5}$ $\quad y = mx + b$: $m = -3, \quad b = 5$

Finding an equation of a line when the slope and a point are given:

Example: Write an equation for a line passing the point (5, 3) with slope $m = -4$.

- Start with: $y = mx + b$ Replace (x, y) by (5, 3) & m by -4.
- Solve for b: $3 = -4 \cdot 5 + b$ Add 20 on both sides.
- y-intercept: $b = 23$
- Equation of the line: $\boxed{y = -4x + 23}$ $\quad y = mx + b$: $m = -4, \quad b = 23$

Finding an equation of a line when two points are given:

Example: Write an equation for a line that passes through the points (2, 1) and (3, -5).

- The slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-1}{3-2} = \frac{-6}{1} = -6$ $\quad (x_1, y_1) = (2, 1),\ (x_2, y_2) = (3, -5)$.

  Substitute values into point-slope equation: $y - y_1 = m\,(x - x_1)$
- Point-slope equation: $y - 1 = -6\,(x - 2)$ Replace (x_1, y_1) with (2, 1) & m with – 6.
- Slope-intercept form: $y - 1 = -6x + 12$ Remove parentheses.

  $\boxed{y = -6x + 13}$ Add 1 on both sides, $y = mx + b$.

Unit 15: Summary

Graphing Linear Equations

The coordinate plane: a powerful tool to mark a point and solution of linear equation on a graph.

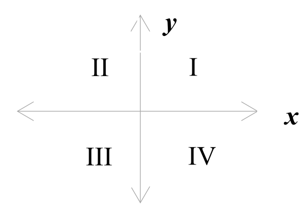
- Coordinate axes: x axis and y axis.
- The origin: the intersection of the x and y axes (both lines are 0 at the origin).

Ordered pair (x, y): a pair of numbers (each point on the plane corresponds to an ordered pair).

Coordinate: the numbers in an ordered pair (the x-distance and the y-distance from a given origin).

Four quadrants:

Quadrant	(x, y)
The 1st quadrant I	($+x$, $+y$)
The 2nd quadrant II	($-x$, $+y$)
The 3rd quadrant III	($-x$, $-y$)
The 4th quadrant IV	($+x$, $-y$)



x – intercept (x, 0): the point at which the graph crosses the x - axis.

y – intercept (0, y): the point at which the graph crosses the y - axis.

A linear (first-degree) equation: an equation whose graph is a straight line.

A linear equation in two variables: a linear equation that contains two variables, such as $5x + 2y = 7$.

The standard form of linear equation in two variables: $Ax + By = C$

Standard Form	Example
$Ax + By = C$	$4x - 9y = 11$

Solutions of equations: solutions for a linear equation in two variables are an ordered pair. They are the particular values of the variables in the equation that makes the equation true.

Procedure to graph a linear equation:

- Choose two values of x, calculate the corresponding y, and make a table.
- Plot these two points on the coordinate plane.
- Connect the points with a straight line.
- Check with the third point – is third point on the line?

Slope (m) (grade or pitch): the slope of a straight line is the rate of change. It is a measure of the "steepness" or "incline" of the line and indicates whether the line rises or falls.

- A line with a positive slope rises from left to right and a line with a negative slope falls.
- **The slope formula:**

The slope formula	
$\text{Slope} = \dfrac{\text{the change in } y}{\text{the change in } x} = \dfrac{\text{rise}}{\text{run}}$	The slope of the straight line that passes through two points (x_1, y_1) and (x_2, y_2): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ or $m = \dfrac{y_1 - y_2}{x_1 - x_2}$ $x_1 \neq x_2$

Horizontal and vertical lines:

Line	Equation	Slope (m)
Horizontal line	$y = b$	$m = 0$
Vertical line	$x = a$	$m = \infty$

The slope and a point can determine a straight line.

Procedure to graph a linear equation using the intercept method:

- Choose $x = 0$ and calculate the corresponding y.
- Choose $y = 0$ and calculate the corresponding x.
- Plot these two points on the coordinate plane.
- Connect the points with a straight line.
- Check with the third point - is third point on the line?

Equation of a straight line:

Straight-line equation	Equation
Point-slope form	$y - y_1 = m(x - x_1)$
Slope-intercept form	$y = mx + b$

$\begin{cases} \text{m = slope} \\ \text{b = y − intercept} \end{cases}$

Finding an equation of a line from the graph:

- Choose two points on the given line.
- Calculate the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Determine the y-intercept on the line: b or $(0, y)$ The line crosses the y-axis.
- Equation of the line: $y = m\,x + b$ Replace m and b with values.

Finding an equation of a line when the slope and a point are given:

- Start with: $y = mx + b$ Replace (x, y) & m with given values.
- Solve for b.
- Equation of the line: $y = mx + b$ Replace m and b with values.

Finding an equation of a line when two points are given:

- Calculate the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Point-slope equation: $y - y_1 = m(x - x_1)$ Replace (x_1, y_1) & m with values.
- Slope-intercept equation: $y = mx + b$ Solve for y.

Unit 15: Self-Test

Graphing Linear Equations

Topic A

1. Plot the points and name the quadrants.

(2, -1) (-4, 3) (-1, -3) (3, 2)

2. Graph the following.

a) $y = 3x$

b) $7x - y = 3$

c) $x + 3y = 6$

3. Graph $y = \frac{1}{3}x - 4$ and determine another point.

4. Find the ordered pair solution of the given equation.

a) $3x - 5y = 11$, when $x = 2$.

b) $x - 0.6y = -3$, when $x = -6$.

c) $\frac{3}{4}x - 4y = 5$, when $x = -4$.

Topic B

5. Determine the slope containing points (4, -1) and (3, 5).

6. Determine the slope of $8x - y - 3 = 0$.

7. Graph the following.

a) $y = -0.9$

b) $x = 3$

c) $y = 0$

Topic C

8. Identify the slope and y-intercept of the following equations.

a) $y = -7x - 11$

b) $5y - 3x = 2$

c) $7x + \frac{1}{5}y = 2$

9. Graph the equation using the slope and the y-intercept.

a) $\boldsymbol{y = \frac{-3}{4}x + 5}$

b) $-6x + 9 = -3y$

10. Determine the intercepts of the line $3x - y = 9$.

11. Graph the equation using the intercept method.

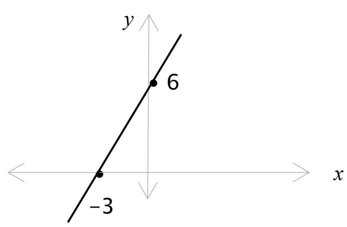
a) $4x - y = 8$

b) $y = \frac{-1}{2}x + 3$

Topic D

12. Write the slope intercept equation of the given line.

a)



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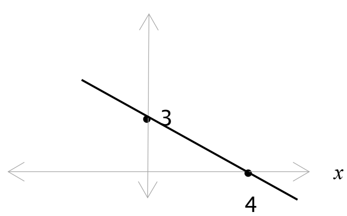
b)



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13. Write the equation for the following lines:

a) The line with a slope of -4 passing the point (-2, 5).

b) The line with a slope of $\frac{3}{5}$ passing the point (5, -7).

14. Write an equation of the line that passes through each pair of points.

a) (3, 2) and (4, -7).

b) (-3, 0) and (0, 6).

c) (0, 5) and (5, 3).

Answers for Self -Tests

Unit R

1. $2^2 \times 3^2$
2. a) Ten million, twenty-four thousand, five hundred twenty-six
   b) Forty-seven and two hundred sixty-eight thousandths
3. a) 6.439
   b) 8.025
   c) 2.7
4. a) $\frac{30}{7}$
   b) $1\frac{4}{5}$
5. $\frac{1}{4}$
6. 400
7. a) 0.45
   b) 43.6%
   c) $\frac{1}{4}$
   d) 20%
   e) $\frac{2}{5}$
   f) $\frac{3}{10}$
8. a) 192
   b) 105
9. a) $\frac{5}{6}$
   b) $\frac{1}{2}$
   c) $1\frac{5}{8}$
   d) $\frac{2}{3}$
   e) $6\frac{5}{7}$
   f) $2\frac{1}{12}$
   g) $1\frac{1}{4}$
   h) 2
   i) 12

j) $\frac{1}{6}$

k) $\frac{2}{3}$

Unit 1

1. 5

2. 9

3. a) 7

b) No mode.

4. a) 7

b) 8

5. a) 3

b) 1

c) 4

6. Let your instructor check your line graph.

7. Let your instructor check your circle graph.

8. a) 551

b) 311.64

c) 2839

d) $7\frac{31}{42}$

e) 248

9. a) 6,000,000

b) 570

c) 8,600

d) 48,000

10. a) 80,800

b) 9,600

c) 3,000,000

d) 20

Unit 2

1. a) Constant: -3 Coefficient: 2 Variable: x

   b) Constant: 13 Coefficient: -4 & $\frac{5}{7}$ Variable: t

2. a) $5x$, 3 , $-y$

   b) $2r$, $16r^2$, $-\frac{3}{14}r$, 1

3. a) $-\frac{5}{9}x$ and $5x$, $2y^2$ and $13y^2$, 7 and -1

   b) $0.6t$ and $-7t$, $9uv$ and $1.67uv$

4. a) 76
   b) 58
5. a) $10y$
   b) $\frac{t}{6}$
   c) $15 - (x + \frac{3}{7}) = 6$
   d) $6x - 7 = 15$
6. a) $\$375 + y$
   b) $175 - y$
   c) $45 - w$
   d) $\frac{x}{4}$, $\frac{x}{48}$
7. a) x
   b) 4
8. a) $9 \cdot 9 \cdot 9$
   b) $(-y)\,(-y)\,(-y)\,(-y)$
   c) $(0.5a^3b)\,(0.5a^3b)$
   d) $\frac{2}{7}x$
9. a) $(0.06)^4$
   b) $(12y)^3$
   c) $(\frac{-2}{9}x)^2$
10. 1440
11. a) y^8
    b) 5^3
12. a) 8
    b) 9
13. a) 133
    b) 63
    c) 8

Unit 3

1. 21 cm
2. 14.1 cm
3. a) 5.6 in
   b) 11 ft
   c) 35.2 cm
   d) $\frac{18}{19}$ yd
4. 7.85 in

5. a) 33 cm
   b) 26.85 cm
   c) 17.85 in
   d) 22.6 yd
6. 17.8 in
7. 18 m
8. a. 50 m
   b. $750
9. 36 m
10. a) $17.25 cm^2$
    b) $16.57 in^2$
    c) $23.85 m^2$
11. $47.47 m^2$
12. $281.2 m^2$
13. a) $50.65 cm^3$
    b) $45.144 mm^3$
    c) $3591.1 cm^3$
    d) $89.8 cm^3$
    e) $217.68 cm^3$
14. $14815.8 m^3$
15. $301.6 m^3$
16. No
17. $7263.4 cm^3$
18. $98.8 cm^2$
19. $32.74 in^2$
20. $LA \approx 93.12 yd^2$, $SA \approx 135.59 yd^2$
21. $LA \approx 73.39 cm^2$, $SA \approx 105.56 cm^2$
22. $10.18 m^2$
23. $1.72 m^2$
24. $273.3 m^2$

Unit 4

1. a) 0.439 m
   b) 223.6 g
   c) 0.0000483 kL
   d) 25 hg
2. a) 7.23 kg
   b) 520 mm

c) 0.34 L

d) 52000 cL

3. a) 4000 mm

b) 63006 g

c) 5290 mL

d) 28.87 km

4. a) 0.74 m^2

b) 90,000 m^2

c) 5,000,000 cm^3

d) 0.567 cm^3

5. a) 4

b) 38 g

c) 5000 cm^3

d) 2.7 cL

e) 76

f) 18,000 cm^3

g) 257 L

h) 0.039375 kL

6. a) 108 in

b) 94 pt

c) 7040 yd

d) 4.638 lb

7. a) 2.438 m

b) 7.6 kg

c) 93 tsp

d) 9 mi

e) 724.2 km

Unit 5

1. $\frac{1}{3}$, 7.3 (Answers may vary.)

2. a) 8

b) -3, 0, 8

c) -3, 0, 8, 4.7 , $\frac{3}{5}$, $2.\overline{56}$

d) 5.4259…, π, $\sqrt{5}$

3. a) Identity property of addition

b) Commutative property of addition

c) Associative property of addition

d) Inverse property of addition

e) Distributive property

f) Associative property of multiplication

g) Commutative property of multiplication

h) Inverse property of multiplication

i) Distributive property

j) Multiplicative property of zero

k) Commutative property of addition

l) Associative property of multiplication

4. a) $(12 + 88) + 45 = 145$

b) $(9 \cdot 8)\ 1000 = 72{,}000$

c) $(3 + 2997) + 56 = 3056$

5. a) $4y^2 + 1.2y$

b) $10 - 15y^2$

c) $\frac{2}{9} - \frac{1}{6}x$

6. a) $6 < 8$

b) $0 > -6$

c) $-4 < -2$

d) $-\frac{3}{7} < \frac{1}{7}$

e) $-0.6 > -0.8$

f) $1\frac{1}{2} > \frac{3}{8}$

7. a) $-17 < -9 < -4 < 0 < 8 < 23$

b) $-8 < -3.24 < 0.05 < \frac{2}{5} < \frac{3}{5}$

c) $-\frac{1}{3} < -\frac{1}{7} < \frac{2}{5} < 1\frac{3}{4}$

8. a) 67

b) 21

c) 0.45

d) -49

e) $\frac{1}{8}$

9. a) 116

b) 25

10. a) 37

b) -15

c) $-2\frac{3}{5}$

d) 5
e) -13
f) 7.5
g) -13
h) 2
i) $-\frac{5}{7}$
j) $1\frac{17}{28}$
k) 5
l) -0.6
m) 27
n) -8
0) 0
p) Undefined

11. a) 45
b) $-\frac{5}{8}$
c) 1

12. a) 4
b) 40
c) -41
d) Undefined

Unit 6

1. a) $5x^3$, $-8x^2$, $2x$
b) $-\frac{2}{3}y^4$, $9a^2$, a, -1

2. a) 2, -7, 9 Degree: 5
b) -8, $-\frac{2}{3}$, 11, 4, -23 Degree: 6

3. a) Binomial
b) Monomial
c) Trinomial

4. a) $15x^3 - 23x^2 + 8x + 3$
b) $\frac{2}{3}y^4 - 3y^3 - 45y^2 + 4y$

5. a) $-x + 19y$
b) $6a^2 - 31b$
c) $4uv^2 + 10u^2v$
d) $23t - 9r$
e) $9m^2 + 64n$

6. a) $10a^2 + 13$
   b) $-19x + 39y$
   c) $7z^2 - 16z + 31$
   d) $-20y^2 + 47y - 33$
   e) $17ab - 28xy$
7. a) a^9
   b) $\frac{1}{x^{11}}$
   c) $\frac{1}{t^6}$
   d) $-42a^7 b^{11}$
   e) $\frac{1}{4}x^4y^7z^9$
   f) $\frac{1}{6}y^5$
   g) $\frac{-9}{m}$
8. a) $-12x^7 + 28x^4$
   b) $27a^4b^3 + 18a^5b^3 - 9a^4b$
   c) $7a + 1 - \frac{4}{5a}$
   d) $40y^2 - 11y - 63$
   e) $21r^2 + 28rt^2 - 6rt - 8t^3$
   f) $10a^3 b^3 + 21a^2b^2 + 9ab$
   g) $x^2 - x + \frac{2}{9}$

Unit 7

1. a) Yes
   b) No
   c) Yes
2. a) $x = 19$
   b) $y = \frac{1}{4}$
   c) $m = 23$
   d) $t = 8$
   e) $x = \frac{3}{8}$
   f) $y = -52$
   g) $x = 28$
   h) $y = -\frac{7}{9}$
   i) $x = 7$
   j) $t = -2$

k) $y = -0.8$

l) $y = 7\frac{8}{9}$

3. a) $t = \frac{3}{14}$

b) $m = 9$

c) $x = 2$

d) $y = \frac{1}{2}$

e) $x \approx 0.069$

f) $t = -0.05$

g) $x = -\frac{4}{5}$

4. a) Contradiction equation

b) Identity equation

c) Conditional equation

d) Contradiction equation

e) Conditional equation

f) Identity equation

5. a) $(x - 7) + 9$

b) $\frac{7}{9x}$

c) $11x - 8$

6. a) $4xy - 13 = x + y + 6$

b) $x^2 + y^2 = xy - 26$

c) $5 + \frac{5x}{23} = 11x$

d) $(x + 1) - x = 1$

e) $x + (x + 2) + (x + 4) = 15$

f) $x(x + 2) = 48$

g) $x + (x + 2) + (x + 4) = 21$

7. a) $7x = 42$, $x = 6$

b) $4x - 3 = \frac{x}{4} - 9$, $x = -1.6$

c) $(5x - 3) + x + (4 + 5x - 3) = 20$, 2, 7, 11

d) $x + (x + 2) + (x + 4) = 27$, 7, 9, 11

e) $x + 7x + (30 + 7x) = 180^0$, 10^0, 70^0, 100^0

f) $128 = 2(l - 8) + 2l$, 36m, 28m

g) $x = 199.99 + 20\% x$, $x = \$249.99$

h) $x = 379.99 - (10\%)(379.99)$, $x = \$341.99$

Unit 8

1. 121.43

2. 195 km
3. 2 h
4. 186.13
5. $A = 385\ \text{cm}^2$, $P = 92$ cm
6. $696\ \text{ft}^2$
7. $C = 15.08$ ft , $A = 18.1\ \text{ft}^2$
8. \$337.50
9. 18¢
10. a) $r = \frac{d}{t}$

    b) $t = \frac{I}{P\,r}$

    c) $l = \frac{p-2w}{2}$

    d) $F = \frac{9}{5}C + 32$, 75.2

    e) $m = \frac{p-C}{C}$

    f) $z = \frac{x-35y^2}{y}$

    g) $b = \frac{2A}{h^2}$

    h) $z = y - x\,t$

    i) $h = \frac{35w}{\pi h^2}$

    j) $w = \frac{y-x}{2z+3}$, 0.091
11. 20.86 cm
12. 0.946 m
13. 14.91 ft
14. 283.65 km
15. 68.35 ft

Unit 9

1. a) $\frac{1}{3}$

   b) $\frac{3}{11}$

   c) $\frac{\text{7 people}}{\text{30 tickets}}$

   d) $\frac{11}{31}$

   e) $\frac{\text{8 km}}{\text{37 min}}$
2. 0.14%
3. 1.25%

4. 76.5 km/h

5. 4 L

6. 8-lb.

7. a) $\frac{5}{110} = \frac{15}{330}$

   b) $\frac{24}{1970} = \frac{12}{985}$

8. $3.69

9. 16 ft

10. $18,000

11. 117

12. 300

13. 40%

14. 20.1%

15. a) 3 cm

    b) 11.2 m

    c) 5.25 cm

Unit 10

1. a) Acute angles

   b) Obtuse angles

   c) Obtuse angle

   d) Reflex angle

2. 48^0

3. 34^0

4. 44^0

5. a) Supplementary

   b) $< A = 147^0$, $< B = 33^0$

6. $< C = 40^0$

7. $< C = 72^0$, $< D = 108^0$, $b = 5$ cm

8. a) vi b) i c) iii d) ii

9. a) $< \theta = 60^0$, $x = 23$ cm It is an equilateral triangle (an acute triangle).

   b) $< B = 102^0$, $a = 43$ ft It is an isosceles triangle (an obtuse triangle).

   c) $< B = < C = 28^0$, It is an isosceles triangle (an obtuse triangle).

   d) $< Z = 72$ opposite ^{0} , $x = 32$ cm It is an isosceles triangle (an acute triangle).

10. a) opposite

    b) adjacent

    c) hypotenuse

d) adjacent

e) opposite

f) Y

11. $\sin X = \frac{5\text{ cm}}{7.81\text{ cm}} \approx 0.6402$, $\sin Z = \frac{6\text{ cm}}{7.81\text{ cm}} \approx 0.7682$

$\cos X = \frac{6\text{ cm}}{7.81\text{ cm}} \approx 0.7682$, $\cos Z = \frac{5\text{ cm}}{7.81\text{ cm}} \approx 0.6402$

$\tan X = \frac{5\text{ cm}}{6\text{ cm}} \approx 0.8333$, $\tan Z = \frac{6\text{ cm}}{5\text{ cm}} = 1.2$

12. $\sin O = \frac{4.25\text{ ft}}{7.62\text{ ft}} \approx 0.5577$, $\sin Q = \frac{6.32\text{ ft}}{7.62\text{ ft}} \approx 0.8294$

$\cos O = \frac{6.32\text{ ft}}{7.62\text{ ft}} \approx 0.8294$, $\cos Q = \frac{4.25\text{ ft}}{7.62\text{ ft}} \approx 0.5577$

$\tan O = \frac{4.25\text{ ft}}{6.32\text{ ft}} \approx 0.6725$, $\tan Q = \frac{6.32\text{ ft}}{4.25\text{ ft}} \approx 1.4871$

13. a) 0.8387

b) 0.8090

c) 19.0811

d) 12.5^0

e) 62.83^0

f) 51.02^0

14. $x \approx 7.793$

15. $c = 36.58$ cm

16. $< A = 51^0$, $b \approx 4.86$ m , $c \approx 7.72$m

17. a) $b \approx 6.25$ cm

b) $<A = 41^0$

18. a) $< B = 45^0$, $b = 6$ m , $c \approx 8.458$ m

b) $a = 4$ ft , $< A \approx 53.13^0$, $< B = 36.87^0$

19. a) $< B \approx 32^0$

b) $y \approx 16.04$ m

20. $x \approx 25.74$ m

21. $< \theta \approx 41.21^0$

22. $x \approx 22.67$ m

23. $< \theta \approx 49.09^0$

24. $x \approx 47.34$ cm

Unit 11

1. a) $7 \cdot 7 \cdot 7 \cdot 7$

b) $(-t)\,(-t)\,(-t)$

c) $(5a^4b^0)\,(5a^4b^0)$

d) $\left(\frac{-7}{11}x\right)\left(\frac{-7}{11}x\right)\left(\frac{-7}{11}x\right)$

2. a) $(0.5)^4$

b) $(6w)^3$

c) $42\, u^2\, v^2$

3. a) 24

b) 982

4. a) 5

b) 7

5. a) $9x^4 - 7x^3 + x^2 - x + 2$

b) $21uv^3 - uv^2 + 4v - 67$

6. a) $43 - 5x + 26x^2 - 17x^3$

b) $-9 + \frac{4}{7}tw + 4.3t^2w^2 - 8w^3 + w^4$

7. a) - 92

b) 1

c) - 0.064

d) - 64

e) y^7

f) x^3

g) $\frac{1}{t^{20}}$

h) $\frac{13}{a}$

i) - 0.512

j) $81a^8\, b^{12}$

k) 64

l) $\frac{u^3}{w^3}$

m) a^6b^8

n) 1

o) $\frac{5x^2}{y^{10}}$

p) $\frac{u^6}{w^{12}\, v^9}$

q) $72\, x^4\, y^5$

r) $\frac{27}{64}x^3y^3$

8. a) 1

b) $\frac{8}{27}$

c) 9

9. a) 4.56×10^7

b) 5.23×10^{-6}

10. a) 3578
    b) 0.000043
11. a) 2.37396×10^6
    b) 3.75×10^{-6}
12. a) 14
    b) $\frac{11}{15}$
    c) $8\sqrt{5}$
    d) $\frac{\sqrt{13}}{3}$

Unit 12

1. 3, 17, 1
2. 2, 7, 14
3. 234.55 g , 625.45g
4. 6.3 L
5. 13 km/h
6. 0.2 h, 0.286 h
7. 5%
8. 22.5%
9. $61.11
10. $27,960
11. $29.85, $169.15
12. $23,450, $445,550
13. $20,000, $120,000
14. $ 2662.56
15. $ 41.05
16. $33170.73
17. $3500, $2000
18. 3, 9, 10, 30
19. 1.2 L

Unit 13

1. a) 12
   b) 9
   c) 8
2. a) -8*y*

b) $\frac{5}{8}x$

c) $-9xy^2 + 4x^2 - y^3$

3. $9x^4 - x^3 - 8x + 10$

4. $4x^2 + 7x - 18$

5. a) $11a^3 - 4a^2 + 9a + 2$

b) $5x^2 - 4x + 11$

6. a) $24\,x^7\,y^5$

b) $12a^6 - 24a^3$

c) $14x^2y^6 + 7x^4y^3 - 21xy^3$

d) $12x^2 - 31x + 20$

e) $-2a^4 + a^3 + 13a^2 - 15a$

7. a) $8t^7 - 20t^4$

b) $3x^2 - 17x + 10$

c) $36a^2 - 25$

d) $9w^2 - 6w + 1$

e) $25u^2 + 5u + \frac{1}{4}$

f) $36x^2 - 4xy + \frac{1}{9}y^2$

g) $\frac{1}{25}z^2 - \frac{1}{16}$

8. a) $56\,x^3$

b) $-9\left(\frac{a^2}{b^3}\right)$

c) $4y + 1 - \frac{3}{7y}$

d) $3\,(2a + 1)$

9. a) $3x + 2$, Remainder = 2

b) $2x^2 - 7x + 14$, Remainder = 2

Unit 14

1. $2 \cdot 2 \cdot 3 \cdot 5$

2. a) $5x$

b) $3ab$

c) $y + 4$

d) $\frac{1}{4}x$

e) $-4y$

3. a) $(5x - 1)\,(5x + 4)$

b) $(6b - a)\,(8ab + 1)$

c) $25uv - 6vw$

d) $(x + y)(x - y)^2$

e) $5(y + 2)(y - 2)$

f) $(1 + 7w)(1 - 7w)$

g) $(9u + 11)(9u - 11)$

h) $(5a + 6b)(5a - 6b)$

i) $(2y^3 + 0.3)(2y^3 - 0.3)$

4. a) $(x + 4)(x + 5)$

b) $(x - 4)(x - 6)$

c) $(x + 3)(x - 6)$

d) $2(x - 2)(x + 7)$

e) $(x - 3)(4x + 5)$

f) $(5y - 6)(y + 3)$

g) $(6b - a)(4ab + 1)$

h) $17uv - 6vs$

5. a) $3(2x + 5)(x - 4) = 0$

b) $2(3x - 4)(x + 2)$

6. a) $(3x + 5)^2$

b) $3(2y - 3)^2$

c) $2(3t^4 - 2)^2$

7. a) 0 , $-\frac{7}{23}$

b) $\pm\frac{7}{9}$

c) - 9, 17

8. a) -6, 7

b) $-\frac{4}{7}$, 5

c) $\frac{1}{4}$, $\frac{3}{4}$

9. -9,

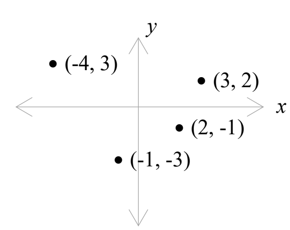
10. 4, -6

11. 7m, 9m

12. 6m , 8m

Unit 15

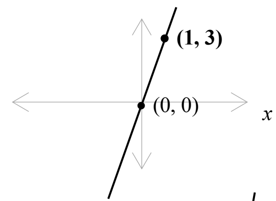
1. (2, -1): IV, (-4, 3): II, (-1, -3): III, (3, 2): I



2.

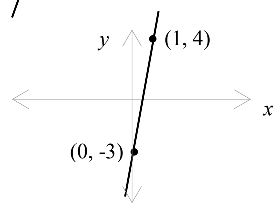
a)

x	$y = 3x$	$(x,\ y)$
0	0	(0, 0)
1	3	(1, 3)



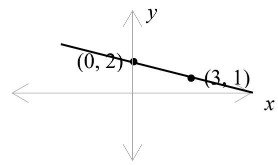
b)

x	$y = 7x - 3$	$(x,\ y)$
0	-3	(0, -3)
1	4	(1, 4)



c)

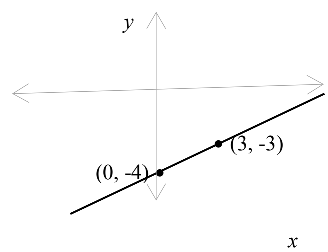
x	$y = -\frac{1}{3}x + 2$	$(x,\ y)$
0	2	(0, 2)
3	1	(3, 1)



3.

x	$y = \frac{1}{3}x - 4$	$(x,\ y)$
0	-4	(0, -4)
3	-3	(3, -3)

Third point may vary.



4. a) $y = -1$

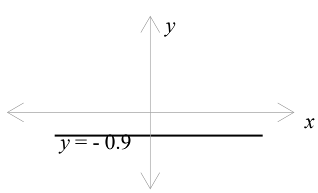
   b) $y = -5$

   c) $y = -2$

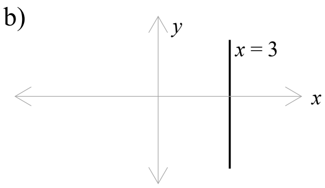
5. $m = -6$

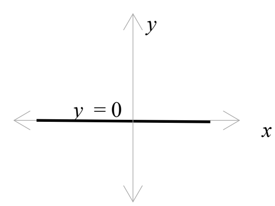
6. $m = 8$

7. a)



b)





8. a) $m = -7$

   $b = -11$ or $(0, -11)$

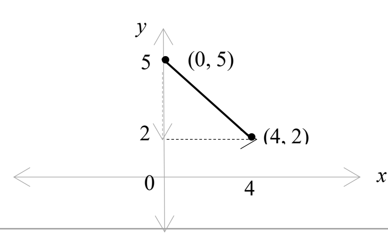
   b) $m = \frac{3}{5}$

   $b = \frac{2}{5}$ or $(0,\ \frac{2}{5})$

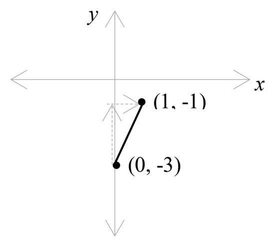
   c) $m = -35$

   $b = 10$ or $(0, 10)$

9. a)



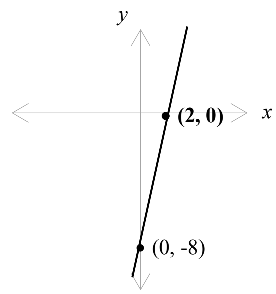
b)



10. (3, 0), (0, -9).

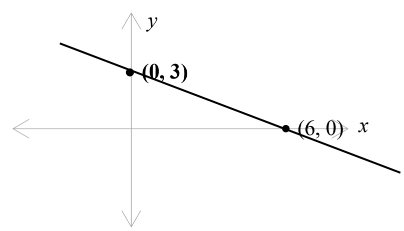
11. a)

x	$y = 4x - 8$	(x, y)
0	-8	(0, -8)
2	0	(2, 0)

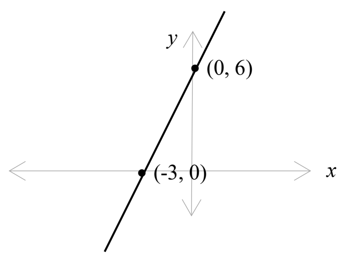


b)

x	$y = \frac{-1}{2}x + 3$	(x, y)
0	3	(0, 3)
6	0	(6, 0)

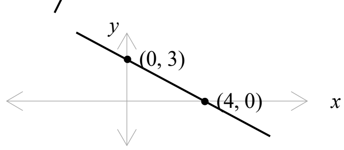


12. a)



$y = 2x + 6$

b)



$y = \frac{-3}{4}x + 3$

13. a) $y = -4x - 3$

b) $y = \frac{3}{5}x - 10$

14. a) $y = -9x + 29$

b) $y = 2x + 6$

c) $y = -\frac{2}{5}x + 5$

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