**Unit 8**

**Formulas**

**Topic A: Substitution into formulas**

* Geometry formulas
* Substituting into formulas

**Topic B: Solving formulas**

* Solving for a specific variable
* More examples for solving formulas

**Topic C: Pythagorean theorem**

* Pythagorean theorem
* Applications of the Pythagorean theorem

**Unit 8 Summary**

**Unit 8 Self-test**

**Topic A: Substitution into Formulas**

**Geometry Formulas**

**Formula:** an equation that contains more than one variable and is used to solve practical problems in everyday life.

*h*

**Geometry formulas review:**

*s* – side, *P* – perimeter, *C* – circumference, *A* – area, *V* – volume

|  |  |  |
| --- | --- | --- |
| **Name of the figure** | **Formula** | **Figure** |

|  |  |  |
| --- | --- | --- |
| **Rectangle** | *P* = 2 *w* + 2 *l*  *A* = *w l*  (*w* = width, *l* = length) | *w*    *l* |
| **Parallelogram** | *P* = 2 *a* + 2 *b*  *A* = *b h*  (*a* and *b* = sides,  *h* = height) | *a*  *h*  *b* |
| **Circle** | (*r* = radius, *d* = diameter) | r ***r******d*** |
| **Triangle** | *< A + < B + < C* = 1800    (*b* = base,  *h* = height) | *B*    *h*  *A b C* |
| **Trapezoid** | (*b* = top base, *B* = bottom base, *h* = height) | *b*  *h*    *B* |
| **Cube** | *V* = **s3**  (*s* = side) | *s* |
| **Rectangular solid** | *V* = *w* *l h*  (*w* = width, *l* = length, *h* = height) | *h*  *w*  *l* |
| **Cylinder** | *V* = *π r*2 *h*  (*r* = radius,  *h* = height) | *r*  *h* |
| **Sphere** | (*r* = radius) | *r* |
| **Cone** | (*r* = radius,  *h* = height) | *h*  *r* |

**Substituting into Formulas**

* **Examples of formula:**

|  |  |  |
| --- | --- | --- |
| **Application** | **Formula** | **Components** |
| Distance | *d = v t* | *d* – distance  *v* – velocity  *t* – time |
| Simple interest | *I = P r t* | *I* –interest  *P* – principle  *r* – interest rate (%)  *t* – time (years) |
| Compound interest | *B = P* (100% *+ r*) *t* | *B* – *balance*  *P* – principle  *r* – interest rate (%)  *t* – time (years) |
| Percent increase |  | *N* – new value  *O* – original value |
| Percent decrease |  | *N* – new value  *O* – original value |
| Sale price and  discount | *S = P d P*  *D = d P* | *S –* sale price  *P* – price (original or regular price)  *d* – discount rate  *D* – discount |
| Original price and  markup | *P = C + m C*  *M = m C* | *P –* price (original or selling price)  *C* – cost  *m* – markup rate  M – markup |
| Intelligence quotient (I.Q.) |  | *I* – I.Q.  *m* – mental age  *c* – chronological age |
| Cost of running  electrical appliances |  | *C –* Cost (in cents)  *W –* power in watts (watts used)  *t –* time (hours)  *r –* rate (per kilowatt-hour) |

**Substitution into formula:** "substitution" means replacing numbers with variables (letters).

**Example**: Find the IQ of a 10-year-old girl with a mental age of 12.

* Formula:
* Facts: *m**=* 12 years, *c* = 10 years
* Substituting:

= Substitute *m* for 12 y and *c* for 10 y.

*I* = 120

The 10-year-old girl has an IQ of 120.

**Example**: Find the distance travelled by a train which has a velocity of 83 km per hour

for 3 hours.

* Formula: *d = v t*
* Facts: *v**=* 83 km/h, *t* = 3 h
* Substituting: *d = v t* = (83 km/h) (3 h) Substitute *v* for 83 km/h and *t* for 3h.

*d* = 249 km = 249 km

The distance is 249 km.

**Example**: Find the volume of a cylinder with a radius of 2.3 cm and a height of 4.2 cm.

* Formula: *V* = *π r*2 *h*
* Facts: *r**=* 2.3 cm, *h* = 4.2 cm
* Substituting: = (2.3 cm)2 (4.2 cm) Substitute *r* for 2.3cm and *h* for 4.2 cm.

*V*  69.8 cm3 (cm2) (cm) = cm3

The volume of the cylinder is 69.8 cm3.

**Example**: Find the area of a triangle with a base of 12 ft and a height of 34 ft.

* Formula: 
* Facts: *b**=* 12 ft , *h* = 34 ft
* Substituting:  = (12 ft) (34 ft) Substitute *b* for 12 ft and *h* for 34 ft.

*A* = 204 ft2 (ft) (ft) = ft2

The area of the triangle is 204 ft2.

**Example**: An electric stove top burner runs for 2 hours and uses 750 watts of electricity at a cost of 10 cents per kilowatt-hour. What is the total cost of running the stove top burner?

* Formula:
* Facts: *t**=* 2 h , *W* = 750 w , *r* = 10 / kwh
* Substituting: = Substitute *W*, *t* and *r*.

=15

The cost of running the stove top burner is 15 cents.

**Topic B: Solving Formulas**

**Solving for a Specific Variable**

**To solve for a variable in a formula:** isolate the unknown or desired variable so that it is by itself on one side of the equals sign and all the other terms are on the other side.

* Use the same process as you would for regular linear equations, the only difference is that you will be working with more variables.
* Remember to always do the same thing to both sides of the formula (add, subtract, multiply or divide the same variable or number to both sides of a formula).

**Rearrange the formula so that the unknown or desired variable is by itself** on one side of the equals sign. You can reverse the sides of the formula if you want.

**Example**: Solve each formula for the given variable.

1. Solve ***d = r t*  for *t*** *.*  Isolate *t* (*t* is the desired variable).

= Divide both sides by *r*.

= *t*  or *t =*  Reverse the sides of the formula.

**Tip:** solve a formulafor agiven letterbyisolatingthegiven letteron one side of theformula.

1. Solve ***I = P r t* for *r* and *P***. Isolate *r* (*r* is the desired variable).

***r*** : = Divide both sides by *Pt.*

= *r* or*r =*  Reverse the sides of the formula.

***P*** : = Divide both sides by *r t.*

= *P* or*P =*  Reverse the sides of the formula.

1. Solve ***P =* 2 *w +* 2 *l* for *w*** *.*  Isolate 2*w* (*w* is the desired variable).

*P* **2 *l*** *=* 2 *w +* 2 *l* **2 *l*** Subtract 2*l* from both sides.

*P*  2 *l* *=* Divide both sides by 2*.*

or Reverse the sides of the formula.

**More Examples for Solving Formulas**

**Example**: Solve each formula for the given variable.

1. **a)** Solve**for *C .*****b)**If ***F =* 68 *, C* = ?**

Solution:

1. Subtract 32 from both sides.

Multiply both sides by  *.*

or Reverse the sides of the formula.

**b)**If *F =* 68*,*Substitute 68 for *F* in the formula.

*C =* 20

1. Solve***P = C* + *m C* for *C*** *.*

*P = C* (1 + *m*) Factor out *C.*

Divide both sides by (1 + *m*)*.*

= *C* or *C* = Reverse the sides.

1. Solve ***p =* 35 *q*2 *+ s q***  *for* ***s*** .

*p* 35 *q*2 *=* 35 *q*2+ *s q* 35 *q*2Subtract 35*q*2 from both sides.

*p* 35 *q*2 *= s q*

= Divide both sides by *q*.

*s =* Reverse the sides.

1. Solve***x =*  for *y*** *.*  Multiply both sides by *t*.

*x t =*  *t*

*x t + z = y z + z* Add *z*to both sides.

*y = x t + z* Reverse the sides.

**Topic C: Pythagorean Theorem**

**Pythagorean Theorem**

**Right triangle:** atriangle containing a 90° angle.

**Pythagorean theorem**: a relation among the three sides of a right triangle which states that the square of the hypotenuse is equal to the sum of the squares of the other two sides (legs).

leg hypotenuses hypotenuse2 = leg2 + leg2

leg hypotenuse = , leg =

**Using the Pythagorean theorem can find the length of the missing side in a right triangle.**

*a* *c*  

*b*

* *c* is the longest side of the triangle (hypotenuses).
* Other two sides (legs) of the triangle *a* and *b* can be exchanged.

**Example:** Find the missing side of the following triangles.

*c* ?

*a* = 3cm

*b =* 4cm

*c* = = = 5 cmhypotenuse =

3 ft

***x* = ?** 5 ft

*x* = 4 ftarm =

**Applications of the Pythagorean Theorem**

**Example:**  Find the distance of the diagonal across the rectangle.

m

*x*

m

*x* = 1.067 m*c* =

*x* 1.067 m

The distance of the diagonal is 1.067 m.

**Example:**  What is the length of one leg of a right triangle whose hypotenuse measures 5.36

cm and the other leg measures 3.24 cm?

*x* = 4.27 cm*a* =

*x* 4.27 cm

The length of one leg is 4.27 cm.

**Example:**  A plane leaves the Vancouver airport and flies 245 km west, then 350 km north.

How far is the plane from the airport?

*x* = ?

*x* = 427.23 km 350 km

*x* 427.23 km 245 km

The distance of the plane from the airport is 427.23 km.

**Example:**  A kite at the end of a 55 m line is 26 m behind the runner. How high is the kite?



*x*

55 m

*x* = 48.47 m

*x* 48.47 m26 m

Theheight of the kite is 48.47 m.

**Unit 8: Summary**

**Formulas**

**Formula:** an equation that contains more than one variable and is used to solve practical problems in everyday life.

**Geometry formulas review:**

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*a* *c* 

*b*

**Using the Pythagorean theorem can find the length of the missing side in a right triangle**.

* *c* is the longest side of the triangle (hypotenuses).
* Other two sides (legs) of the triangle *a* and *b* can be exchanged.

**Unit 8: Self-Test**

**Formulas**

**Topic A**

1. Find the IQ of a 70-year-old man with a mental age of 85.
2. Find the distance travelled by a train which has a velocity

of 78 km per hour for 2.5 hours.

1. Steve rides his bicycle at a speed of 11 miles per hour. He goes

on a 22-mile bike ride.  How many hours does this ride take?

1. Find the volume of a cone with a radius of 4.6 cm and a height

of 8.4 cm.

1. Find the area and perimeter of a rectangle with a width of 11 cm

and a length of 35 cm.

1. Find the area of a triangle with a base of 24 ft and a height of 58 ft.
2. The diameter of a circle is 4.8 ft. Find the circumference and area

of the circle.

1. Ann invests $15,000 at an annual interest rate of 0.75%. How much

simple interest will she earn by the end of 3 years?

1. An electric stove top burner runs for 2.5 hours and uses 800 watts of

electricity per hour at a cost of 9 cents per kilowatt-hour. What is the

total cost of running the stove top burner?

**Topic B**

1. Solve each formula for the given variable.
2. Solve *d = r t*  for *r.*
3. Solve *I = P r t* for *t*.
4. Solve *P =* 2 *w +* 2 *l* for *l.*
5. Solve for *F .*

If *C =* 24, *F* = ?

1. Solve*P = C* + *m C* for *m .*
2. Solve *x =* 35 *y*2 *+ z y* for  *z* .
3. Solve*A =*  for *b .*
4. Solve*x =*  for *z .*
5. Solve*w =*  for *h .*
6. Solve *x = y* (2*z* + 3) *w* for *w,*

if *x* = 2, *y* = 3, *z* = 4

**Topic C**

1. Find the missing side of the following triangles.

7cm 22cm

*x =* ?

1. Find the distance of the diagonal across the rectangle.

m

*x* =?

m

1. What is the length of one leg of a right triangle whose hypotenuse

measures 21.34 ft and the other leg measures 15.27 ft?

1. A plane leaves the Calgary airport and flies 134 km east, then 250

km south. How far is the plane from the airport?

1. A kite at the end of a 89 ft line is 57 ft behind the runner. How high is the kite?