**Unit 11**

**Exponents, Roots and Scientific Notation**

**Topic A: Exponents**

* Basic exponent properties review
* Degree of a polynomial

**Topic B: Properties of exponents**

* Properties of exponents
* Properties of exponents examples
* Simplifying exponential expressions

**Topic C: Scientific notation and square roots**

* Scientific notation
* Square roots
* Simplifying square roots

**Unit 11: Summary**

**Unit 11: Self-test**

**Topic A: Exponents**

**Basic Exponent Properties Review**

**Exponent review:** *an* or BaseExponent

|  |
| --- |
| **Exponential notation Example** |
| Power  **Exponent**    ***a*n** = *a ∙ a ∙ a ∙ a … a* 24 = 2 ∙ 2 ∙ 2 ∙ 2 = 16  Base  Read “*a* to the *n*th” Read “2 to the 4th.”  or “the *n*th power of *a*.” |

**Exponents - basic properties:**

|  |  |
| --- | --- |
| **Name Property** | |
| Zero Exponent *a*0 | *a*0 = 1 (00 is undefined) |
| One Exponent *a*1 | *a*1 = *a* |
| 1*n* = 1 |

**Example**: Write the following exponential expressions in expanded form.

Exponential expressionsExpanded form

1. **43**  4 ∙ 4 ∙ 4*an = a ∙ a ∙ a* …
2. **(-*u*)3** (-*u*) (-*u*) (-*u*)
3. **-*u*3** *-u u u*
4. **(2*x*3*y*0)2**  (2*x*3*y*0) (2*x*3*y*0)

**Example**: Write each of the following in the exponential form.

Expanded formExponential notation

* + - 1. **(0.3) (0.3) (0.3)**  (0.3)3
      2. **(4*t*) (4*t*) (4*t*) (4*t*)** (4*t*)4
      3. **(3*x*) (2*y*) (*x*) (2*y*)** 12*x*2 *y*2  12*(x ∙ x*) (*y ∙ y*)

**Example**: Evaluate.

1. **2*x*3 + *y*, for *x* = 2, *y* = 3**

2*x*3 + *y* = 2 *∙* 23 + 3 Substitute *x* for 2 and *y* for 3.

= 2 (8) + 3 = 19

1. **(2*a*)4 – *b*, for *a* = 1, *b* = 4** Substitute *a* for 1 and *b* for 4.

(2*a*)4 – *b* = (2∙ 1)4 – 4

= 24 – 4 = 12

**Degree of a Polynomial**

**The degree of a *term* with *one* variable:** the exponent of its variable.

**Example:** **9*x*3** degree: 3

**-7*u*5**  degree: 5

**2*a*** degree: 1 2*a* = 2*a*1 , *a*1 = *a*

**The degree of a *term* with *more* variables:** the sum of the exponents of its variables.

**Example:** **-8*x*2 *y*4 *z*3** degree: 2 + 4 + 3 = 9

**The degree of a *polynomial* with more variables:** the highest degree of any individual term.

**Example**: **9*t*2*u* + 4*t*3*u*2*v*5 – 6*t* + 5**  degree: 10 3 + 2 + 5 = 10

3 10 1 *a*1 = *a*

**Examples of degree of a polynomial:**

|  |  |  |
| --- | --- | --- |
| **Polynomial** | **5*x*3**– ***x*2  *+* 21** | **2*x*2*y*** – **5*z* + 7*x*4y2z** |
| **Term** | 5*x*3, -*x*2*,* 21 | 2*x*2*y*, -5*z ,*  7*x*4*y*2*z* |
| **Degree of the term** | 3 , 2, 0 | 3, 1 , 7 |
| **Degree of the polynomial** | 3 | 7 |

**Example**: What is the degree of the following term / polynomial?

1. **3*xy*3** degree: 4
2. degree: 9

9 2 3 0

**Descending order:** the exponent of a variable ***decreases*** for each succeeding term.

**Example**: *a*1 = *a*

The descending order of exponent *v*.

**Ascending order:** the exponent of a variable ***increases*** for each succeeding term.

**Example**: *a*1 = *a*

The ascending order of power *z*.

**Topic B: Properties of Exponents**

**Properties of Exponents**

**Properties of exponents:**

|  |  |  |
| --- | --- | --- |
| **Name Rule Example** | | |
| Product rule | | *am* *an*= *am* + *n* 23 22 = 23 + 2 = 25 =32 |
| Quotient rule  (the same base) | | (*a* ≠ 0) |
| Power of a power | | (*am*)*n = amn* (*x*3)2 *= x*3*∙*2  *= x*6 |
| Power of a product  (different bases) | | (*a* *b*)*n*= *an**bn*  (2 )2 = 22 32 =4 ∙9 = 36  *(a*m *b**n*)*p*= *amp**bnp* (*t*3 *s* 4)2 = *t*3∙2 *s*4∙2 = *t*6 *s*8 |
| Power of a quotient  (different bases) | | (*b* ≠ 0)  (*b* ≠ 0) |
| Negative exponent | | (*a* ≠ 0) |
| Zero exponent *a*0 | *a*0 = 1 150 = 1 |
| One exponent *a*1 | *a*1 = *a* (But 1n = 1)71 = 7 , 113 = 1 |

**Properties of exponents explained:**

* **Product rule** (multiplying the same base):when multiplying two powers with the same base, keep the base and add the exponents. *am* *a*n = *am* + *n* *an* or BaseExponent

**Example**: **23 22**= (2 22)(22)=25 = 32

Or **23 22** = 23 + 2 =25 = 32 A short cut, *am* *a*n = *am* + *n*

* **Quotient rule** (dividing the same base):when dividing two powers with the same base, keep the base and subtract the exponents.

**Example**: 4

Or = 4A short cut,

This law can also show that why *a*0 = 1 (zero exponent *a*0): = 1

* **Power rule:**
* Power of a power: when raise a power to a power, just multiply the exponents.

(*am*)*n = amn*

**Example**: **(43)2** *=* (43) (43)=(4 44) (4 44) *=* 46=4096

Or **(43)2** *=* 43 *∙* 2 *=* 46 =4096 A short cut, (*am*)*n = amn*

* Power of a product: when raise a power to different bases, distribute the exponent to each base. (*a* *b*)*n*= *an**bn*

**Example**: **(2 )2**= (2 =6 ∙ 6 = 36

Or**(2 )2** = 22 32 =4 ∙9 = 36 A short cut , (*a* *b*)*n*= *an**bn*

* Power of a product (different bases): when raise a power to a power with different bases, multiply each exponent inside the parentheses by the power outside the parentheses. (*am* *b n*) *p*= *amp**bnp*

**Example**: **(22 32)2**= (22 32) (22 32) =(22 22) (32 32) =16 ∙81 = 1296

Or**(22 32)2**= 22∙2 32∙2 =24 34 =16 ∙81 = 1296 A short cut , (*a* *b*)*n*= *an**bn*

* **Power of a quotient** (different bases):
* When raise a fraction to a power, distribute the exponent to the numerator and denominator of the fraction.

**Example**:

Or A short cut,

* When raise a fraction with powers to a power, multiply each exponent in the numerator and denominator by the power outside the parentheses.

**Example**:

Or A short cut,

* **Negative exponent:** a negative exponent is the reciprocal of the number with a positive exponent. , *a−n* is the reciprocal of *an*.

**Example**:

**Example**:81

**Properties of Exponents Examples**

**Example**: Simplify (do not leave negative exponents in the answer).

1. **(-4)1** = -4 *a*1 *= a*
2. **(-2345)0** = 1 *a*0 *=* 1
3. **(-0.3)3** = -0.027*an = a ∙ a ∙ a* …
4. **-52** = -(52) = -25
5. ***x*2 *x*3**= *x*2 + 3 = *x*5 *am an = am+ n*
7. **(*x*4) -3** = *x*4 (-3)= *x* -12  (*an*)*m = an m ,*
8. **7*b* -1** =  = , *a*1 *= a*
9. **[(-4) )]2** = (-4)2 0.72 = (16) (0.49) = 7.84 (*a b*)*n = an bn*
10. **(2*t*3  *w* 2)4**= 24 *t*3∙4  *w* 2∙4 = 16 *t*12 *w* 8  (*a*m *b*n)p = *amp* *b*np
12. ,
13. ,
14. 1
15. *a*0 *=* 1,,
16. ,

**Using a calculator:** 42 = ? 4 *x*2  = (The display reads 16)

34 = ? 3 *xy* 4 = (The display reads 81)

(Or *yx* or ^ on some calculators.)

**Simplifying Exponential Expressions**

**Steps for simplifying exponential expressions:**

* Remove parentheses using “power rule” if necessary. (*a*m *b*n)p = *amp* *b*np
* Regroup coefficients and variables.
* Use “product rule” and “quotient rule”.  *am an = am + n ,*
* Simplify.
* Use “negative exponent” rule to make all exponents positive if necessary.

**Example**: Simplify.



Remove brackets. (*a*m*b*n)p = *amp* *b*np*, a*0 *=* 1

Regroup coefficients and variables.

= Simplify. *am an = am + n*

Make exponent positive.  , *a*1 *= a*



Remove brackets. (*a b*)*n = an bn*

Regroup coefficients and variables.

Simplify.

**Example**: Evaluate for *a* = 2, *b* = 1, *c* = -1.

1. = 1*a*0 *=* 1
2. Substitute 2 for *a* and1 for *b,*

, ,

1. = 16Substitute 2 for *a*,1 for *b,* and -1 for *c*.