**Unit 12**

**Solving Word Problems**

**Topic A: Value mixture problems**

Solving value mixture problems

**Topic B: Concentration mixture problems**

Solving mixture problems

**Topic C: Motion and business problems**

* Distance, speed and time problems
* Business problems

**Topic D: Mixed problems**

Solving mixed problems

**Unit 12: Summary**

**Unit 12: Self-test**

**Topic A: Value Mixture Problems**

**Solving Value Mixture Problems**

**Steps for solving word problems:**

|  |
| --- |
| **Steps for Solving Word Problems** |
| * Organize the ***facts*** given from the problem (make a table). * Identify and labelthe unknown quantity (***let x = unknown***). * Draw a ***diagram*** if it will make the problem clearer. * Convert words into a mathematical ***equation***. * ***Solve*** the equation and find the solution(s). * ***Check*** and state the ***answer***. |

**Table for value mixture problems**:

|  |  |
| --- | --- |
| **Item** | **Value of the item Number of items Total value** |
| Item A | value of A # of A (value of A) (# of A) = amount of A |
| Item B | value of B # of B (value of B) (# of B) = amount of B |
| Item C | value of C # of C (value of C) (# of C) = amount of C |
| … | … … … |
| Total or mixture | total value |

Let *x* = unknown

Value of item A + Value of item B + Value of item C + … = Total value of the mixture

**Example**: Susan has ***$5.95*** in nickels, dimes and quarters. If she has two ***less*** than ***three* *times quarters of dimes***, and ***three more nickels than quarter*s**. How many of each coin does she have?

(value of 25) (# of 25)

* Let *x* = number of quarters

(value of 10) (# of 10)

* Organize the facts:

(value of 5 ) (# of 5)

Convert $ to

|  |  |
| --- | --- |
| **Coin** | **Value of the coin Number of coins Total value (in cents)** |
| Quarter | 25 C *x* 25 *x* |
| Dime | 10 C 3*x* – 2 10 (3*x* – 2) |
| Nickel | 5 C  *x +* 3 5 (*x* + 3) |
| Total | $5.95 = 595 |

* Equation: **25*x* + 10(3*x* – 2) + 5(*x* + 3) = 595** value of quarters + value of dimes + value of nickels = 595
* Solve for *x*: 25*x* + 30*x* – 20 + 5*x* + 15 = 595 Remove parentheses.

60*x* – 5 = 595 Combine like terms.

60*x* = 600 Solve for *x.*

***x***= 10

* Check:

|  |  |
| --- | --- |
| Number of quarters | ***x*** *=* **10** |
| Number of dimes | **3*x* – 2** = 3(10) – 2 = **28** |
| Number of nickels | ***x* + 3** = 10 + 3 = **13** |

25*x* + 10(3*x* – 2) + 5(*x* + 3) = 595Equation

?

2510 + 10(3 10 – 2) + 5(10 + 3) = 595 Substitute *x* for 10.

? ?

250 + 280 + 65 = 595 Check LS = RS

595 = 595 Correct! LS = RS

* State the answer:

|  |  |
| --- | --- |
| Number of quarters | 10 |
| Number of dimes | 28 |
| Number of nickels | 13 |

**Example:** Damon purchased $1.00, $1.19, and $1.20 Canadian stamps with a ***total value of* *$23.72.*** If the number of $1.19 stamps is ***7 more than the number of $1.00*** stamps, and thenumber of ***$1.20*** stamps is ***8 more than three times*** of ***$1.00*** stamps. How many of each did Damon receive?

* Let *x* = number of $1.00 stamps

(value of $1.19) (# of $1.19)

(value of $1.00) (# of $1.00)

* Organize the facts:

(value of $1.20) (# of $1.20)

Value of $1.00 + value of $1.19 + value of $1.20 = $20.68

|  |  |
| --- | --- |
| **Stamps** | **Value of the stamps Number of stamps Total value** |
| $1.00 | $1.00 *x* 1.00 *x* |
| $1.19 | $1.19 7 + *x* 1.19 (7 + *x*) |
| $1.20 | $1.20 8 + 3*x* 1.20 (8 + 3*x*) |
| Total | $23.72 |

* Equation: **1.00*x* + 1.19 (7 + *x*) + 1.20 (*x*) = 23.72**
* Solve for *x*: 1*x* + 8.33 + 1.19*x* + 9.6 + 3.6*x* = 23.72 Remove parentheses.

5.79*x* + 17.93 = 23.72 Combine like terms.

579*x* + 1793 = 2372 Remove decimals (

579*x* = 579 Divide both sides by 579.

***x*** = 1

* State the answer:

|  |  |
| --- | --- |
| Number of $1.00 | ***x*** *=* 1 |
| Number of $1.19 | **7** **+ *x*** = 7 + 1 = 8 |
| Number of $1.20 | **8 + 3*x***= 8 + 3 1= 11 |

**Topic B: Concentration Mixture Problems**

**Solving Mixture Problems**

**Table of concentration mixture**:

|  |  |
| --- | --- |
| **Iterm** | **Concentration Volume Amount** |
| Item A | concentration of A volume of A (concentration of A) (volume of A) = amount of A |
| Item B | concentration of B volume of B (concentration of B) (volume of B) = amount of B |
| … | … … … |
| Mixture | concentration of mixture volume of mixture (concentration of mixture) (volume of mixture) = amount of mixture |

Let *x* = unknown

Amount of item A + Amount of item B + … = Amount of the mixture

**Example:** A **shrimp meal** is **35%** protein and a **fish meal** is **25%** protein. Susan wants a **750 grams mixture** that is **30%** protein. How many grams of protein each meal should she have?

* - Let *x* = the protein volume of the shrimp meal

(concentration of shrimp meal) (volume of shrimp meal)

- The protein volume of fish meal = 750 – *x*

(concentration of fish meal) (volume of fish meal)

The protein volume of ***mixture*** – The protein volume of***shrimp*** meal = The protein volume of ***fish*** meal

(concentration of mixture) (volume of mixture)

(If there is a total mixture protein volume of 750 g, then 750 – *x* must be the protein volume of fish meal.)

* Organize the facts:

|  |  |
| --- | --- |
| **Meal** | **Concentration Protein volume Amount** |
| Shrimp meal | 35% = 0.35 *x*  0.35 *x* |
| Fish meal | 25% = 0.25 750 – *x*  0.25(750 – *x*) |
| Mixture | 30% = 0.30 750 0.3(750) |

* Equation: **0.35 *x* + 0.25 (750 – *x*) = (0.3) (750)** Remove parentheses.

Amount of shrimp meal + Amount of fish meal = Amount of mixture

* Solve for *x*: 0.35 *x* + 187.5 – 0.25*x* = 225 Combine like terms.

0.1 *x* = 37.5 Divide both sides by 0.1.

* State the answer: - Shrimp meal: ***x*** = 375 g
* Fish meal: **750 – *x*** = 750 – 375

= 375 g

**Example**: How much ***8% sugar******solution*** must be added to ***15 liters of 27% solution*** to make a ***20% solution***?

* - Let *x* = volume of 8% solution

- Volume of 20% = *x* + 15

Volume of 20% = Volume of 8% + Volume of 27%

(concentration of 8%) (volume of 8%)

Mixture

(concentration of 27%) (volume of 27%)

(concentration of 20%) (volume of 20%)

* Organize the facts:

|  |  |
| --- | --- |
| **Solution** | **Concentration Volume Amount** |
| 8% | 0.08 *x*  0.08 *x* |
| 27% | 0.27 15 (0.27) (15) |
| 20% | 0.2 *x +* 15 0.2 (*x +* 15) |

* Equation: **0.08 *x* + (0.27) (15) = 0.2 (*x* + 15)** Amount of 8% + Amount of 27% = Amount of 20%
* Solve for *x*: 0.08 *x* + 4.05 = 0.2 *x* + 3 Combine like terms.

- 0.12 *x* = -1.05 Divide both sides by -0.12.

***x* =** 8.75

* State the answer: 8.75 liters of 8% sugar solution must be added to 15 liters of 27% solution.

**Topic C: Motion and Business Problems**

**Distance, Speed and Time Problems**

**Formulas of motion:**

* Distance = Speed ∙ Time *d* = *r t*
* Speed =  *r =*
* Time = *t =*

**Example**: Adam walks for ***4.4 hours*** at a rate of ***2 km per hour****.* ***How far*** does he walk?

Equation: ***d* = *r t*** *t* = 4.4 h, *r* = 2 km/h, *d* = ?

=(2 km/h) (4.4 h) **=** 8.8 km km/h: km per hour

**Table of motions**:

|  |  |  |  |
| --- | --- | --- | --- |
| **Condition** | **Speed (*r*)** | **Time (*t*)** | **Distance (*d*)** |
| **Condition A** | *r* | *t* | *d = r t* |
| **Condition B** | *r* | *t* | *d = r t* |
| **…** | *…* | *…* | *…* |
| **Total** |  |  |  |

**Example:** Two cyclists are ***60 km apart*** and are travelling towards each other. Their ***speeds differ*** by ***1.5 km*** per hour. What is the ***speed*** of ***each*** cyclist if they meet after ***2 hours***?

|  |  |
| --- | --- |
| **Condition** | **Speed (*r*) Time (*t*) Distance (*d*****=** *r t***)** |
| **Bike A** | *r* 2 2 *r* |
| **Bike B** | *r* – 1.5 2 2 (*r* – 1.5) |
| **Total** | 60 km |

* Equation: **2*r +* 2(*r* – 1.5) = 60** Distance of A + Distance of B = 60km

2*r +* 2*r* – 3 = 60 Remove parentheses.

4*r* = 63 Combine like terms.

* Bike A:  ***r =***15.75 km/hDivide both sides by 4.
* Bike B: ***r* – 1.5 =** 15.75 – 1.5 = 14.25 km/h

**Example**: Mike ***boats*** at a ***speed*** of ***28 km*** per hour in still water. The **river** flows at a **speed**of ***5 km*** per hour. ***How long*** will it take Mike to boat ***3 km downstream***? ***3 km upstream*?**

|  |  |
| --- | --- |
| **Condition** | **Speed(*r*)Distance (*d*)Time ( *t =* )** |
| **Downstream** | *r* = 28 + 5 = 33 km/h *d* = 3 km *t* = |
| **Upstream** | *r* = 28 – 5 = 23 km /h *d* = 3 km *t* = |

Downstream (fast): speed of boat + speed of river

Upstream (slower): speed of boat - speed of river

* Downstream: ***t* =** 0.091 h
* Upstream: ***t* =** = 0.13 h

**Business Problems**

**Business math formulas:**

|  |  |
| --- | --- |
| **Business problems** | **Formulas** |
| **Percent increase** | , |
| **Percent decrease** | , |
| **Sales tax** | Sales tax = Sales × Tax rate |
| **Commission** | Commission = Sales × Commission rate |
| **Discount** | Discount = Original price × Discount rate  Sale price = Original price – Discount |
| **Markup** | Markup = Selling price × Markup rate  Original price = Selling price – Markup |
| **Simple interest** | Interest = PrincipleInterest rateTime , *I* = *P r t*  Balance = Principle + Interest |
| **Compound interest** | Balance = Principle (100% + Interest rate) *t*  Balance = *P* (100% + *r*) *t* |

**Example:** A product increased production from ***230 last month*** to ***250 this month***.  Find the ***percent increase.***

* New value (*N*): 250 This month.
* Original value (*O*): 230 Last month.
* Percent increase: ***x*** *=* 0.087 = 8.7% About 8.7% increase.

**Example:** A product was ***reduced*** from ***$59*** to ***$39.***  What was the percent ***reduction***?

Percent decrease: ***x*** =0.339 = 33.9% 33.9 % decrease.

**Example:** Find the ***sales tax*** for a ***$999*** laptop with a ***tax rate*** of ***7%***.

**Sales tax** = Seles × Tax rate

= ($999) (7%) = ($999) (0.07) = $69.93

**Example**: Find the ***commission*** for a ***$950,000*** house with a ***commission rate*** of ***5%***.

**Commission** = Sales × Commission rate

= ($950,000) (5%) = ($950,000) (0.05) = $47,500

**Example:** A men’s coat was ***originally*** priced at ***$159*,** and is on sale at a ***25% discount***. Find the ***discount*** and ***sale price***.

* **Discount** = Original price × Discount rate

= ($159) (25%)

= ($159) (0.25)

= $39.75

* **Sale price** = Original price – Discount

= $159 – $39.75

= $119.25

**Example:**  A condo was sold at ***$399,000***, with a ***markup*** ***rate*** of ***8%***. What was the ***markup*** and ***original price***?

* **Markup** = Selling price × Markup rate

= ($399,000) (8%)

= ($399,000) (0.08)

= $31,920

* **Original price** = Selling price – Markup

= $399,000 – $31,920

= $367,080

**Example:** Jo borrowed ***$150,000*** mortgage from a bank. Find the interest at ***3%*** per year for ***3.5 years***, and also find the ***total*** amount that Jo paid the bank.

* Interest = Principle Interest rate Time

**I** *= P r t* = ($150,000) (3%) (3.5)

= ($150,000) (0.03) (3.5)

= $15,750

* **Balance** = Principle + Interest

= $150,000 + $15,750

= $165,750

**Example:**  David deposited ***$3,000*** in an account at ***4.5% interest compounded*** per year for ***5 years***. How much was in the account at the end of ***5 years***?

**Balance** = Principle (100% + Interest rate) *t* Compound interest

= *P* (100% + *r*) *t*

= $3,000 (100% + 4.5%)5

= $3,000 (1+ 0.045)5

≈ $ 3738.55

**Topic D: Mixed Problems**

**Solving Mixed Problems**

**Example**: After a ***ten percent reduction***, a toy is on sale for ***twenty-nine dollars***. What was the ***original price***?

* Let *x* = original price
* Equation**: *x* – 10% *x =* 29** Original price – Reduction = Sale price

1 *x* – 0.1 *x =* 29 *x* = 1 *x*

0.9*x* *=* 29

* Answer: $ 32.22 The original price was $32.22.

**Example**: William receives a ***1.5%*** ***raises*** bring his salary ***to*** ***$39,000***. What was his salary ***before*** the ***raise***?

* Let *x* = Tom’s salary before the raise Raise = (1.5% )(Previous salary) = 1.5% *x*
* Equation: ***x* + 1.5% *x =* 39,000** Previous salary + Raise = Current salary

1 *x* + 0.015 *x =* 39,000

1.015 *x =* 39,000

* Answer: ***x*** = $38423.65 Tom’s salary before the raise was $38423.65.

**Example**: Bob deposits a certain amount of money in a ***chequing*** ***account*** that earns ***2.5%*** in annual interest, and deposits ***$2000*** ***less than*** that in a ***saving account*** that pays ***1.5%*** in annual interest. If the total interest from ***both*** accounts at the end of the year is ***$95***, how much is deposited in each account?

* Let *x* = money deposited in the saving account

|  |  |
| --- | --- |
| **Account** | **Deposit Interest rate Interest** |
| Chequing account | *x*  2.5% 0.025*x* |
| Saving account | *x* – 2000 1.5% 0.015 (*x* – 2000) |

Total interest = $95

* Equation: **0.025*x* + 0.015 (*x* – 2000) = 95** 2.5% of saving + 1.5% of checking = $95

0.025*x* + 0.015*x* – 30 = 95 Combine like terms.

0.04*x* = 125

* Answer: Chequing account: ***x*** = = $3125 $3125 in the chequing account.

Saving account:***x* – 2000** = 3125 – 2000 = $1125 $1125 in the saving account.

**Example**: A string ***103 meters*** long is cut into ***four pieces***. The ***second*** is ***four*** ***times*** as long ***as*** the ***first***. The ***third*** piece is ***five*** meters ***longer*** than the ***first***. ***The*** ***fourth*** piece is ***twice*** as long ***as*** the ***third***. How long is each piece of string?

* Let *x* = the length of the first piece.

|  |  |
| --- | --- |
| **1st piece** | *x* |
| **2nd piece** | 4*x* |
| **3rd piece** | *x +* 5 |
| **4th piece** | 2(*x +* 5) |

* Equation:***x* + 4*x* + (*x* + 5) + 2(*x* + 5) = 103** 1st + 2nd + 3rd + 4th = 103

*x* + 4*x* + *x* + 5 + 2*x* + 10 = 103 Combine like terms.

8*x* + 15 = 103

8*x* = 88

***x*** = 11 m

* Answer:

|  |  |
| --- | --- |
| **1st piece** | *x =* 11 m |
| **2nd piece** | 4*x =* 4 (11) = 44 m |
| **3rd piece** | *x +* 5 = 11 + 5 = 16m |
| **4th piece** | 2(*x +* 5) = 2(11 + 5) = 32 m |

**Example:** A fruit punch that contains ***25% fruit juice***. ***How much*** ***water*** would you have to add to ***1 liter*** of punch to get a new drink that contains ***10% fruit juice***?

* Let *x* = water to add to 1 L of punch to get a 10% fruit juice.

|  |  |
| --- | --- |
|  | **Concentration Volume Amount** |
| Fruit punch | 25 % 1 (L) 0.25(1) |
| New drink | 10 % *x +* 1 0.1(*x +* 1) |

* Equation: **0.25(1) = 0.1(*x +* 1)** Amount of 25% = Amount of 10%

0.25 = 0.1*x +* 0*.*1 Multiply 100 for each term.

25 = 10*x +* 10 Combine like terms.

15 = 10 *x* Divide both sides by 10.

* Answer: *x* = 1.5 L

It needs to add 1.5 L of water to get a new drink that contains 10% fruit juice.

**Unit 12: Summary**

**Solving Word Problems**

**Steps for solving word problems:**

|  |
| --- |
| **Steps for solving word problems** |
| * Organize the ***facts*** given from the problem (make a table). * Identify and labelthe unknown quantity (***let x = unknown***). * Draw a ***diagram*** if it will make the problem clearer. * Convert words into a mathematical ***equation***. * ***Solve*** the equation and find the solution(s). * ***Check*** and state the ***answer***. |

**Table for value mixture problems**: Let *x* = unknown

|  |  |
| --- | --- |
| **Item** | **Value of the item Number of items Total value** |
| Item A | value of A # of A (value of A) (# of A) = amount of A |
| Item B | value of B # of B (value of B) (# of B) = amount of B |
| Item C | value of C # of C (value of C) (# of C) = amount of C |
| … | … … … |
| Total or mixture | total value |

Value of item A + Value of item B + Value of item C + … = Total value of the mixture

**Formulas of motion:**

Distance = Speed ∙ Time *d* = *r t t = r* =

**Table of motions**:

|  |  |  |  |
| --- | --- | --- | --- |
| **Condition** | **Speed (*r*)** | **Time (*t*)** | **Distance (*d*)** |
| **Condition A** | *r* | *t* | *d = r t* |
| **Condition B** | *r* | *t* | *d = r t* |
| **…** | *…* | *…* | *…* |
| **Total** |  |  |  |

* Downstream (fast): speed of boat + speed of river
* Upstream (slower): speed of boat - speed of river

**Table of concentration mixture:** Let *x* = unknown

|  |  |
| --- | --- |
| **Iterm** | **Concentration Volume Amount** |
| Item A | concentration of A volume of A (concentration of A) (volume of A) = amount of A |
| Item B | concentration of B volume of B (concentration of B) (volume of B) = amount of B |
| … | … … … |
| Mixture | concentration of mixture volume of mixture (concentration of mixture) (volume of mixture) = amount of mixture |

Amount of item A + Amount of item B + … = Amount of the mixture

**Business math formulas:**

|  |  |
| --- | --- |
| **Business problems** | **Formulas** |
| **Percent increase** | , |
| **Percent decrease** | , |
| **Sales tax** | Sales tax = Sales × Tax rate |
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| **Discount** | Discount = Original price × Discount rate  Sale price = Original price – Discount |
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| **Simple interest** | Interest = PrincipleInterest rateTime , *I* = *P r t*  Balance = Principle + Interest |
| **Compound interest** | Balance = Principle (100% + Interest rate) *t*  Balance = *P* (100% + *r*) *t* |

**Unit 12: Self-Test**

**Solving Word Problems**

**Topic A**

1. Robert has $2.50 in nickels, dimes and quarters. If he has two more than five times quarters of dimes, and two less nickels than quarters. How many of each coin does he have?
2. William purchased $1.00, $1.19, and $1.20 Canadian stamps with a total value of $27.13. If the number of $1.19 stamps is 5 more than the number of $1.00 stamps, and the number of $1.20 stamps is 6 more than four times of $1.00 stamps. How many of each did Damon receive?

**Topic B**

1. A lamb meal is 36% protein and a pork meal is 25% protein. Peter wants an 860 grams mixture that is 28% protein. How many grams of protein each meal should he have?
2. How much 5% salt solution must be added to 18 liters of 32% solution to make a 25% solution?

**Topic C**

1. Two cyclists are 72 km apart and are travelling towards each other. Their speeds differ by 2 km per hour. What is the speed of each cyclist if they meet after 3 hours?
2. Linda boats at a speed of 17 km per hour in still water. The river flows at a speed of 3 km per hour. How long will it take Linda to boat 4 km downstream? 4 km upstream?
3. A product increased production from 400 last month to 420 this month.  Find the percent increase.
4. A product was reduced from $80 to $62.  What was the percent reduction?
5. Find the sales tax for a $ 679 laptop with a tax rate of 9%.
6. Find the commission for a $699,000 townhouse with a commission rate of 4%.
7. A women’s dress was originally priced at $199, and is on sale at a 15% discount. Find the discount and sale price.
8. A condo was sold at $469,000, with a markup rate of 5%. What was the markup and original price?
9. Smith borrowed $100,000 mortgage from a bank. Find the interest at 4% per year for 5 years, and also find the total amount that Smith paid the bank.
10. Susan deposited $2,500 in an account at 3.2% interest compounded per year for 2 years. How much was in the account at the end of 2 years?

**Topic D**

1. After a five percent reduction, a toy is on sale for thirty-nine dollars. What was the original price?
2. Ruth receives a 2.5% raises bring her salary to $34,000. What was her salary before the raise?
3. Amy deposits a certain amount of money in a chequing account that earns 1.5% in annual interest, and deposits $1500 less than that in a saving account that pays 1.2% in annual interest. If the total interest from both accounts at the end of the year is $76.50, how much is deposited in each account?
4. A string that is 52 meters long is cut into four pieces. The second is three times as long as the first. The third piece is seven meters longer than the first. The fourth piece is three times as long as the third. How long is each piece of string?
5. A fruit punch is 45% fruit juice. How much water would you have to add to 1.5 liter of punch to get a new drink that is 25% fruit juice?