**Unit 9**

**Ratio, Proportion, and Percent**

**Topic A: Ratio and rate**

* Ratio
* Rate

**Topic B: Proportion**

* Solving proportion

**Topic C: Percent**

* Percent review
* Solving percent problems

**Topic D: Similar triangles**

* Similar triangles
* Solving similar triangles

**Unit 9 Summary**

**Unit 9 Self-test**

**Topic A: Ratio and Rate**

**Ratio**

**Ratio**

* Ratio: a relationship between two numbers, expressed as the quotient with the ***same unit*** in the denominator and the numerator.
* Express a ratio: there are three ways to write a ratio.

The ratio of *a* and *b* is: *a* to *b* or *a* : *b* or 

**Example:**  Write the ratio of 5 cents to 9 cents.

5 to 9 or 5 : 9 or 

* Write a ratio in lowest terms (simplify):
* Write the ratio in a fractional form.
* Simplify and drop the units if given (as they cancel each other out).

÷4

**Example:**  

÷4

×100 ÷25

**Example**: 0.75 meters to 0.25 meters 

×100 ÷25

**Grade and pitch**

* Grade (or slope, pitch, incline etc.): the slope of a straight line is the rate of change in height over a distance. It is a measure of the “steepness” “or incline” of a line.
* **The grade or slope formula:**

Horizontal distance

Vertical distance

Rise

Run

|  |
| --- |
| **Formula** |
| Grade or |

**Example**: Determine the grade (%) of a road that has a length of 75 m and a vertical height of 3 m.

**Grade** = = = 4%

**Rate**

**Rate**

* Rate: a ratio of two quantities with different units.

**Example:** teachers to students; money to time; distance to time, etc.

 ,  , 

* Write a rate in lowest terms (simplify the rate):

÷80

**Example:**  80 kilometres per 320 minutes: 

÷80

**Unit rate:** a rate in which the number in the ***second term (denominator***) is 1.

**Example:** 15 dollars per hours:  = $ 15 per h

* Some unit rates:
* Miles (or kilometres) per hour (or minute).
* Cost (dollars/cents) per item or quantity.
* Earnings (dollars) per hour (or week).
* Unit price and the best buy.

**Example:**  Find the best buy.

12 eggs for $ 3.19; 18 eggs for $4.91; 30 eggs for $7.13.

$ 0.266 per egg

$ 0.273 per egg

$ 0.238 per egg

So the best buy is 30 eggs for $7.13(the lowest price). 0.238 < 0.266 < 0.273

**Topic B: Proportion**

**Solving Proportion**

**Proportion**: an equation with a ratio (or rate) on two sides ( ), in which the two ratios are equal.

**Example:**  Write the following sentence as a proportion.

3 printers is to 18 computers as 2 printers is to 12 computers.

**Review ratio, rate and proportion:**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Representation** |  | **Example** |
| **Ratio** | *a* to *b* or *a*:*b* or | with the same unit. | 5 to 9 or 5:9 or |
| **Rate** | *a* to *b* or *a*:*b* or | with different units. | 3 to 7 or 3:7 or |
| **Proportion** |  | an equation with a ratio/rate on each side. | , |
| **Note:** the units for both numerators must match and the units for both denominators must match.  **Example**: , | | | |

**Solving a proportion: Example**

* Cross multiply: multiply along two diagonals.  
* Solve for the unknown. 6 · *x* = 2 · 9

3

**Application**

**Example**: 4 liters of milk cost $4.38, how much does a 2 liters of milk cost?

* Facts and unknown:

|  |  |
| --- | --- |
| 4 L milk | 2 L milk |
| $4.38 | $ *x =* ? |

The same units.

* Proportion: 

The same units.

* Cross multiply: 

(4) (*x*) = (2) (4.38)

* Solve for *x*:  Divide both sides by 4.



2 liters of milk cost $2.19.

* Check:  Replace *x* with 2.19.

?

(4) (2.19) = (2) (4.38)

**√**

8.76 = 8.76 Correct! (LS = RS)

**Example:**  Tom’s height is 1.75 meters, and his shadow is 1.09 meters long. A building’s shadow is 10 meters long at the same time. How high is the building?

* Facts and unknown:

|  |  |
| --- | --- |
| Tom’s height = 1.75 m | Building’s height (*x*) = ? |
| Tom’s shadow = 1.09 m | Building’s shadow = 10m |

* Proportion: 
* Cross multiply: 

(1.75) (10) = (1.09) (*x*)

* Solve for *x*:  Divide both sides by 1.09.

 The building’s height is 16.055m.

* Check:  Replace *x* with 16.055.

?

(1.75) (10) = (16.055) (1.09)

**√**

17.5 = 17.5 Correct! (LS = RS)

**Example:** If 15 mL of medicine must be mixed with 180 mL of water, how

many mL of medicine must be mixed in 230 mL of water?

* Proportion:
* Cross multiply:
* Solve for *x*: 19.17 mL

19.17 mL of medicine must be mixed in 230 mL of water.

**Topic C: Percent**

**Percent Review**

**Percent (%):** one part per hundred, or per one hundred.

**Review - converting between percent, decimal and fraction:**

|  |  |  |
| --- | --- | --- |
| **Conversion** | **Steps** | **Example** |
| **Percent Decimal** | Move the decimal point two places to the left, then remove %. | 31% = 31.% = 0.31 |
| **Decimal Percent** | Move the decimal point two places to the right, then insert %. | 0.317 = 0. 317 = 31.7 % |
| **Percent Fraction** | Remove %, divide by 100, then simplify. | 15% = |
| **Fraction percent** | Divide, move the decimal point two places to the right, then insert %. |  |
| **Decimal Fraction** | Convert the decimal to a percent, then convert the percent to a fraction. | % = per one hundred |

**Two methods to solve percent problems**

* Percent proportion method
* Translation (translate the words into mathematical symbols.)

**Percent proportion method:**

With the word “***is***”

or

With the word “***of****”*

**Step Example**

***8*** percent ***of*** what number **is** ***4*** ?

* Identify the part, whole, and percent. Percent Whole (*x*) Part
* Set up the proportion equation. 
* Solve for unknown (*x*).  50 ,*x* = 50

**Solving Percent Problems**

**Translation method (**translate the words into mathematical symbols):

Translation:

* What *x* : the word “what” represents an unknown quantity *x*.
* Is = : the word “is” represents an equal sign.
* of ×: the word “of” represents a multiplication sign.
* % to decimal: always change the percent to a decimal.

**Example:**

**1)**  What is 15% of 80?

*x* = 0.15 • 80*x* = (0.15)(80) = 12

**2)**  What percent of 90 is 45?

*x* % • 90 = 45 *x*% = 50%Divide both sides by 90.

**3)**  12 is 8% of what number?

12 = 0.08 • *x*  *x* = 150 Divide both sides by 0.08.

* + **Percent increase or decrease:**

|  |  |
| --- | --- |
| **Application** | **Formula** |
| **Percent increase** | , |
| **Percent decrease** | , |

**Example**: A product increased production from ***1500 last month*** to ***1650 this month***.  Find the ***percent increase*.**

* + New value (N): 1650 This month.
  + Original value (O): 1500 Last month.
  + Percent increase: *x*= A 10% increase.

**Example:** A product was ***reduced*** from ***$33*** to ***$29*.**  What percent ***reduction*** is this?

Percent decrease: *x*= A 12 % decrease.