**Unit 3**

**Introduction to Geometry**

**Topic A: Perimeter, area, and volume**

* Perimeter of plane figures
* Circle
* Perimeter
* Perimeters of irregular / composite shapes

**Topic B: Area**

* Areas of quadrilaterals and circles
* Arears of irregular / composite shapes

**Topic C: Volume**

* Volume of solids

**Topic D: Surface and lateral area**

* Surface and lateral area – rectangular solids
* Surface and lateral area – cylinders, cones and spheres

**Unit 3 Summary**

**Unit 3 Self - test**

**Topic A: Perimeter, Area, and Volume**

**Perimeter of Plane Figures**

|  |  |
| --- | --- |
| **Polygon:** a closed figure made up of three or more line segments.  **Regular polygon:** a polygon that has all angles equal and all sides equal. |  |
| **Classify regular polygons):**   |  |  |  | | --- | --- | --- | | **Number of sides** | **Name of polygon** | **Figure** | | **3** | Triangle |  | | **4** | Quadrilateral |  | | **5** | Pentagon |  | | **6** | Hexagon |  | | **8** | Octagon |  | | **10** | Decagon |  | |  |

**Quadrilateral:** a four-sided polygon.

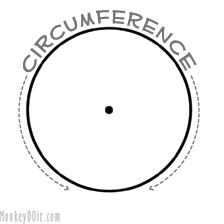
**Classify quadrilaterals:**

|  |  |  |
| --- | --- | --- |
| **Name of quadrilateral** | **Definition** | **Figure** |
| **Rectangle** | A four-sided figure that has four right angles (900). |  |
| **Square** | A four-sided figure that has four equal sides and four right angles. |  |
| **Parallelogram** | A four-sided figure that has opposite sides parallel (//) and equal. (*a // b, c // d; a = b, c = d* ) | *c*  *a b*  *d* |
| **Rhombus**  (diamond) | A four-sided figure that has four equal sides, but no right angle. |  |
| **Trapezoid** | A four-sided figure that has one pair of parallel sides. |  |

**Circle**

**Circle:** a round shape bounded by a curved line that is always the same distance from the center.

·



**Circumference (*C*):** the line bounding the edge of a circle.

**Diameter (*d*):** a straight line between any two points on the circle through the center

of the circle.

*d*

**Radius (*r*):** a straight line between any point on the circle to the center of the circle (half of the diameter, *r =*  (or  *d* = 2*r*).

*r*

·

**Example:** Identify the parts of a circle (what is a, b and c?).

c

b 

a

a. Circumference b. Radius c. Diameter

**Example:**

1. Find the radius of a circle with a diameter of 12 meters.

*d* = 12 m , *r* = *d* = 12 m *=* 6 m

1. If the radius of a circle is 15 meters, what is the diameter of this circle?

*d* = 2*r* = 2 30 m

**Perimeter**

**Perimeter (*P*):** the total length of the outer boundary of a figure.

**Find the perimeter:** add together the length of each side.

**Example**: To find the perimeter (*P*) of the following figure, add the lengths of all 4 sides. *P* = 3 in + 1 in + 4 in + 1.5 in 1.5 in

=9.5 in3 in 4 in

1 in

**The perimeter of any regular (equal sided) polygon:** the number of sides (*n*) times the length of any side (*s*) of that polygon. *P = ns*

**Example:** Theperimeter (*P*) of a square is *P* = 4 *s* *s* *s*

4 sides

**Units of perimeter:** the meter (m), centimeter (cm), foot (ft), inch (in), yard (yd), etc. (The same units as length.)

**The perimeter of regular polygons:** *s* – the length of the side

|  |  |  |
| --- | --- | --- |
| **Name of the figure** | **Perimeter (*P = ns*)** | **Figure** |
| **Equilateral triangle**  (A triangle with three equal sides.) | *P* = 3 *s* | *s* |
| **Square** | *P* = 4 *s* | *s* |
| **Pentagon** | *P* = 5 *s* | *s* |
| **Hexagon** | *P* = 6 *s* | *s* |
| **Octagon** | *P* = 8 *s* | *s* |
| **Decagon** | *P* = 10 *s* | *s* |

**Example:**

1. What is the perimeter (*P*) of the following triangle?

*s* = 3.5 m *P* = 3 *s* = (3) (3.5 m) = 10.5 m

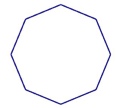
1. What is the perimeter (*P*) of the following square?

*s* = 2.3cm *P* = 4 s = 4 (2.3 cm) = 9.2 cm

1. What is the perimeter (*P*) of the following hexagon?

*s* = 5ft *P* = 6 *s* = (6) (5 ft) = 30 ft

1. What is the perimeter (*P*) of the following octagon?

  *s* = yd *P* = 8 s = 8 yd = 6 yd

**The perimeter of some basic geometric shapes:**

|  |  |  |
| --- | --- | --- |
| **Name of the figure** | **Perimeter formula** | **Figure** |
| **Rectangle** | *P* = 2 *w* + 2 *l*  (*w* – width, *l* – length) | *w*  *l* |
| **Parallelogram** | *P* = 2 *a* + 2 *b*  (*a* and *b* – the length of the sides) | *a*  *b* |
| **Trapezoid** | *P = a + b + c + d* | Image result for trapezoid definition *b*  *a*  *c*    *d* |
| **Circle**  (The perimeter of the circle is its circumference *C*) | *π* (pi) is the ratio of circle’s circumference *C* to its diameter *d,* that is approximately 3.14.  () | *r d* |

**Example:** What is the perimeter (*P*) of the following polygons?

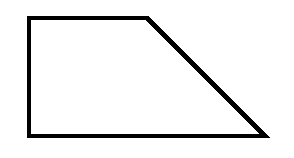
1. *w* = 5ft *P* = 2 *w* + 2 *l =* 2 (5 ft) + 2 (7 ft) = 24 ft

*l* = 7 ft

1. *a* = 3.4cm *P* = 2*a* + 2*b* = 2 (3.4 cm) + 2 (5.2 cm) = 17.2 cm

*b* = 5.2cm

1.8.m



4.3m

2.4m

1. *P = a + b + c + d*

5.8m

= 2.4 m + 1.8m + 4.3 m + 5.8m = 14.3m

**Example:** What are the circumferences (*C*) of the circles shown below?

**1)**  *d=*5cm*C* = 15.7 cm

**2)**

· *C* = 17.58 cm

*r* =2.8cm

**Perimeters of Irregular / Composite Shapes**

**Example:** What are the perimeters (*P*) of the following figures?

2m

**1)** 3m 1m

2m

*P* = 2m + 3m + 1m + 2m + (3m + 2m) + (1m + 2m) = 16 m

**2)**

2cm

*P* is equal to of the circumference of the

circle (*C* = and two sides with 2m.

*=* 4 cm + *. r* = 2cm

13.42 cm

**3)** 3ft  *P* is equal to of the circumference

3ft of the circle and two sides with 3ft.

*C =*

= 6 ft + *d* = 3ft (An equilateral triangle.)

10.71 ft

2 yd

**4)**

*P* is the circumference of 4 half circles.

= 4  *C =*, *d* = 2 yd

12.57 yd

**Example:** Damon is renovating his living room that is the shape indicated in the diagram below. He wishes to put molding around the base of the walls of the living room. How much molding does he need?

4.5m

2.2 m

4.3 m

= 20 m

**Topic B: Area**

**Areas of Quadrilaterals and Circles**

**Area (*A*):** the size of the outermost surface of a shape (space within its boundaries).

**Units of area:** the units of measurement of area are always expressed as square units.

Such as square meter (m2), square centimeter (cm2), square foot (ft2), square inch (in2), square yard (yd2), etc.

**Areas of some basic geometric shapes:**

|  |  |  |
| --- | --- | --- |
| **Name of the figure** | **Area formula (*A*)** | **Figure** |
| **Rectangle** | *A* = *wl* (*w* – width,  *h* – height) | *w*  *l* |
| **Square** | *A* = *s*2 (*s* – the length of the side) | *s* |
| **Triangle** | (*b* - base, *h* – height) | *h*  *b* |
| **Parallelogram** | *A* = *bh*  (*b* - base, *h* - height) | *h*  *b* |
| **Trapezoid** | (*b*-upper base, *B*-lower base, *h*- height) | *b*  *h*  *B* |
| **Circle** | (*r* - radius, *π* ≈ 3.14) | *r* |

**Example:** What are the areas (*A*) of the following figures?

**1**) 3.8m *A* = *s*2 = (3.8 m) (3.8 m)= 14.44 m2m  m = m2

**2)** cm *A* = *w l*= (cm) (cm)= cm2cm cm = cm2

cm

**3**) 4.2yd 11.13 yd2

5.3yd

**4**) *h* = in *A* = *bh* = ( in) ( in)= in2

in

2ft

**5**) *h* = 5ft

6ft

**6**) *r* = 0.25cm *A* = (3.14) (0.25cm)2  0.2 cm2

**Areas of Irregular / Composite Shapes**

**Example:**  Find the areas (*A*) of the following figures.

4m

**1)**  *h=* 3m 2m

1m

Total area = Area of parallelogram + Area of triangle



**2)**

1ft 3ft 2.5ft

Total area = Area of trapezoid + Area of () circle

(*d* = 1ft, *r* = 0.5 ft)



**Example:** Damon is renovating his living room that is the shape indicated in the diagram below. He wishes to purchase new flooring. How much does he need to order to cover the entire living room floor?

3.6 mm

12 mm

4.5m

2.2 m

Total area = Area of square + Area of triangle

= 25.2 m2

**Example:**  William built a wooden deck at the back of his home. It is shown in the following diagram. He decides to insert a circular hot tub that has a diameter of 2.4 m. Calculate the area of the remaining exposed wooded floor of the deck.

5m *d* = 2.4m  **=**

7m

Shaded area = Area of rectangle Area of circle

(*d* = 2.4m, *r* = 1.2m)

*A* = (*wl*) – (*r*2) = (5m) (7m) – (3.14) (1.2m)2 30.48 m2

**Topic C: Volume**

**Volume of Solids**

**Volume (*V*):** the amount of space a solid object (three-dimensional) occupies.

1m  1m

**Example**: the volume of a can of food is the amount of food inside. 1m

**Units of volume:** the units of measurement of volume are always expressed as cubic units.

Such as the cubic meter (m3), cubic centimeter (cm3), cubic foot (ft3), cubic inch (in3), cubic yard (yd3), etc.

**Volumes of basic geometric shapes:**

|  |  |  |
| --- | --- | --- |
| **Name** | **Figure** | **Volume formula (V)** |
| **Cube** | *s* | *V* = **s3**  (*s* – the length of the side) |
| **Rectangular solid** | *h*  *w*    *l* | *V* = *w* *l h*  (*w –* width, *l –* length, *h* - height) |
| **Cylinder** | *r*  *h* | (*r -* radius*, h* – height, ) |
| **Sphere** | *r* | *V* =  (*r-* radius) |
| **Cone** | *h*  *r* | (*r -* radius*, h* - height) |
| **Pyramid** | *h*  *l w* | (*w –* width, *l –* length, *h* - height) |

**Example:** Find the volumes (*V*) of the following figures.

**1)** 1.4 m *V* = *s***3** = (1.4 m)(1.4 m) (1.4 m)

= (1.4 m)**3** = 2.744 m3 m  m  m = m3

**2)**2.4in *V* = *w* *l h* = (4.2in) (1.3in) (2.4in) 13.1 in3in  in in = in3

4.2in 1.3in

**3)** *r=*3m  226.2 m3



*h* = 8m

**4)** *d=* 4cm *V* = = 33.51 cm3

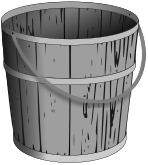
(*d* = 4 cm, *r* = 2 cm)

**5)**5ft *V* = = 47.1 ft3

*r* =3ft

**6)** *h=*4m *w=*2m *V* = = 8 m3

*l* = 3m

 **7)** Determinethe amount of water that will fill the following bucket.

*V* = = 1963.5 cm3

(*d* = 10 cm, *r* = 5 cm) *d* = 10cm 25cm

**Volume of composite shapes**

**Example:** Find the volume (*V*) of the following figure.

4.5 in

*r* =2.5 in

4 in

Total volume= Volume of the cylinder + Volume of the cone

*V* = =

*=* 107.99 in3

**Example:** Find the volumes (*V*) of the following figure (a rectangular solid with a cylinder removed from inside).

4m (Cylinder: *h* = 4m, *r*= 1m)

2m

5m

Unknown volume= Volume oftherectangular solid Volumeof the cylinder

*V= (wlh) – (*27.43 m3

**Topic D: Surface and Lateral Area**

**Surface and Lateral Area**

**Rectangular Solids**

**Surface area (SA):** the total area on the surface of a solid object (a three-dimensional object).

**Lateral area (LA):** the surface area of a solid object excluding its top and bottom.

**Lateral area (LA) of a rectangular solid:** the sum of the surface areas of the four sides excluding its top and bottom.

LA of a rectangular solid = front side + back side + 2 sides *h*

*w*

= 2 (*l h*) + 2 (*wh*) *l* The front and back sides. The left and right sides.

(*w –* width, *l –* length, *h* - height)

**Example**: Determine the lateral area (LA) of the rectangular solid.

LA = 2(5ft ∙ 2ft) + 2(1ft ∙ 2ft)

2 ft

1 ft

5 ft

= 20 ft2 + 4 ft2

= 24 ft2

**Surface area (SA) of a rectangular solid:**  the sum of the areas of the top, bottom and the four sides.

*h*

SA of a rectangular solid = top area + bottom area + 4 sides

*w*

*l*

= (*l w*) + (*l w*) + 2(*lh*) + 2(*wh*)

= 2 (*l w*) + 2(*lh*) + 2(*wh*) (*w –* width, *l –* length, *h* - height)

The top & bottom.

The front and back sides.

The left and right sides.

**Example**: Determine the SA of the rectangular solid.

SA = 2 (3m ∙ 1m) + 2(3m ∙ 2m) + 2(1m ∙ 2m)

2m

1m

= 6m2 +12m2 + 4m2 = 22m2

3m

**Example**: How many square centimeters of glass are needed to make a fish tank which is 15 cm long by 10 cm wide by 12 cm high if the top is left open?

10cm

12cm

*A* = 2 (15cm ∙ 12m) + 2(12cm ∙ 10cm) + (15cm ∙ 10cm) = 750 cm2

15cm

The left and right sides.

The bottom part.

The front and back sides.

**Surface and Lateral Area**

**Cylinders，Cones and Spheres**

**Cylinders**

* Lateral area (LA) of a cylinder: the area of the the rectangular side that wraps around the cylinder's side (the rectangular side folded around).

LA of a cylinder = π*dh* or 2π*rh*  *h h*

*πd* or 2*πr*

- Imagine a fruit can that is cut down the side and rolled flat.

- Recall: the circumference of a circle *C = πd* or 2*πr*  (*r* – radius, *d* – diameter)

* Surface area (SA) of a cylinder: the sum of the surface areas of the top, bottom and the side (the lateral area).

SA of a cylinder = top area + bottom area + LA of a cylinder

*d*

SA of a cylinder = 2(πr2) + *πdh* *h*

Recall: the area of a circle: *A = πr*2

(*r –* radius, *d –* diameter, *h* - height)

**Example**: Determine the lateral area and surface area of the following cylinder.

LA = π*dh* = π (3m)(3.5m) 32.99 m2  *d* =3m

SA = 2(π*r*2) + π*dh* 3.5m

= 2[π (m)2] + 32.99 m2 *d =* 3m*, r* = 1*.*5m

14.137 m2 + 32.99 m2

47.13 m2

**Cones**

* Lateral area of a cone: Vertex

LA of a cone = (*π*) (radius) (slant height) = *πrs* Slant height *s*

Slant height (*s*): the height from the vertex to a point on the circle base.  *r* *r* – radius

(*r –* radius, *s* – slant height)

* Surface area (SA) of a cone:

SA of a cone = LA of a cone + area of the circular base (a circle)

SA of a cone = *πrs* + *πr*2 *s* - slant height, *r* - radius

**Example**: Determine the lateral area and total area of a cone whose diameter is 2m and slant height is 4m.

LA = π*rs* = π ()(4m) 12.57 m2*d* = 2m, *r* = *d* = 1m

SA = *πrs* + *πr*2 = 12.57 m2 *+* π)2 15.71 m2

**Spheres**

Surface area (SA) of a sphere:

*r*

SA of a sphere = 4*πr*2 *r* - radius

**Example**: Determine the surface area of a sphere whose radius is 4.5cm.

SA = 4π*r*2 = 4π (4.5cm)2  254.47 cm2

**Example**: Mary wishes to paint 5 balls with green paint. The diameter of each ball is 18 cm. What area should Mary tell the paint store she needs to cover?

SA = 4π*r*2 = 4π (9cm)2  1017.88 cm2(The surface area of one ball)

*d =* 18 cm*, r* = 9 cm

5 (SA) = 5 (1017.88 cm2)=5089.4 cm2(The surface area of 5 balls)

**Surface and lateral area summary:**

|  |  |  |
| --- | --- | --- |
| **Figure** | **Lateral area (LA)** | **Surface area (SA)** |
| **Rectangular Solid** | Front side + back side + 2 sides  2(*l h*) + 2(*wh*) | Top area + bottom area + 4 sides  (*l w*) + (*l w*) + 2(*lh*) + 2(*wh*) |
| **Cylinder** | π*dh* or 2π*rh* | 2(π*r*2) + π*dh* |
| **Cone** | π*rs* | π*rs +* π*r*2 |
| **Sphere** |  | 4*πr*2 |

There is no difference between lateral area and surface area in a sphere.

**Unit 3: Summary**

**Introduction to Geometry**

**Classify quadrilaterals (four-sided shapes):**

|  |  |  |
| --- | --- | --- |
| **Name of quadrilateral** | **Definition** | **Figure** |
| **Rectangle** | A four-sided figure that has four right angles (900). |  |
| **Square** | A four-sided figure that has four equal sides and four right angles. |  |
| **Parallelogram** | A four-sided figure that has opposite sides parallel (//) and equal. (*a // b, c // d; a = b, c = d* ) | *c*  *a b*  *d* |
| **Rhombus**  (diamond) | A four-sided figure that has four equal sides, but no right angle. |  |
| **Trapezoid** | A four-sided figure that has one pair of parallel sides. |  |

**Terms of geometry:**

|  |  |
| --- | --- |
| **Term** | **Definition** |
| **Perimeter (*P*)** | The total length of the outer boundary of a shape. |
| **Circumference (*C*)** | The line bounding the edge of a circle. |
| **Diameter (*d*)** | A straight line between any two points on the circle through the center of the circle. |
| **Radius (*r*)** | A straight line between any point on the circle to the center of the circle (half of the diameter, *r =* or  *d* = 2*r*). |
| **Area (*A*)** | The size of the outermost surface of a shape. |
| **Volume (*V*)** | The amount of space a solid object (3D) occupied. |
| **Surface area (SA)** | The total area on the surface of a solid object (a 3D object). |
| **Lateral area (LA)** | The surface area of a solid object excluding its top and bottom. |

**Units of perimeter:** the meter (m), centimeter (cm), foot (ft or’), inch (in or”), yard (yd), etc.The same units as length.

**Units of area:** the units of measurement of area are always expressed as square units.

**Units of volume:** the units of measurement of volume are always expressed as cubic units.

**Surface and lateral area summary:**

|  |  |  |
| --- | --- | --- |
| **Figure** | **Lateral area (LA)** | **Surface area (SA)** |
| **Rectangular Solid** | Front side + back side + 2 sides  2(*l h*) + 2(*wh*) | Top area + bottom area + 4 sides  (*l w*) + (*l w*) + 2(*lh*) + 2(*wh*) |
| **Cylinder** | π*dh* or 2π*rh* | 2(π*r*2) + π*dh* |
| **Cone** | π*rs* | π*rs +* π*r*2 |
| **Sphere** |  | 4*πr*2 |

**Geometry formulas:** *s* – side, *P* – perimeter, *C* – Circumference, *A* – area, *V* – volume

|  |  |  |
| --- | --- | --- |
| **Name of the figure** | **Formula** | **Figure** |
| **Equilateral triangle** | *P* = 3*s* | *s* |
| **Pentagon** | *P* = 5*s* | *s* |
| **Hexagon** | *P* = 6*s* | *s* |
| **Octagon** | *P* = 8*s* | *s* |
| **Decagon** | *P* = 10*s* | *s* |
| **Square** | *P = 4s*  *A* = *s*2 | *s* |
| **Rectangle** | *P* = 2 *w* + 2 *l*  *A* = *wl* | *l*  *w* |
| **Parallelogram** | *P* = 2 *a* + 2 *b*  *A* = *bh* | *h* *a*  *b* |
| **Circle** |  | *r d* |
| **Triangle** | *< X + < Y + < Z* = 1800 | *X*  *h*  *Y b Z* |
| **Trapezoid** |  | *b*  *h*  *B* |
| **Cube** | *V* = **s3** | *s* |
| **Rectangular solid** | *V* = *w* *l h* | *h*  *l*  *w* |
| **Cylinder** |  | *r*  *h* |
| **Sphere** |  | *r* |
| **Cone** |  | *h*  *r* |
| **Pyramid** |  | *h*  *l w* |

**Unit 3: Self - Test**

**Introduction to Geometry**

**Topic A**

1. Find the radius of a circle with a diameter of 42 centimeters.
2. What is the perimeter (*P*) of the following triangle?

*s* = 4.7 cm

1. What is the perimeter (*P*) of the following polygons?
2. *s* = 1.4 in
3. *w* = 2.3 ft

*l* = 3.2 ft

1. *a* = 7.2 cm

*b* = 10.4 cm

1. *s* = yd
2. What is the circumferences (*C*) of the circle shown below?

*d =* 2.5 in

1. What are the perimeters (*P*) of the following figures?

5cm

1. 6 cm 1.5 cm

4 cm



4 cm

**c)** 5 in

5 in

3.6 yd

**d)**

1. A flower bed in the shape of a parallelogram has sides of 5.5 inches and

3.4 inches. What is its perimeter?

1. The floor of a rectangular room measures 5.2 m by 4.3 m. The doorway

is 1 m wide. Baseboard is to be installed around the perimeter of the room,

except in the doorway. What length of baseboard needs to be purchased?

1. Tom’s rectangular yard is 10 meters wide and 15 meters long.

a. If Tom wants to fence the whole lot, how many meters of fencing would

Tom has to buy?

b. If the fencing cost $15 per meter, estimate the cost of fencing the yard.

1. A rectangular swimming pool is 8 m long and 4 m wide. It is surrounded

by concrete deck 1.5 m wide on all sides. Find the outside perimeter of

the deck.

**Topic B**

1. Find the areas of the following figures.

5cm

**a)**  3cm

1.5 cm

2in 5in 4in

**b)**

3.6 mm

12 mm

4.5 m

1.6 m

**c)**

1. Find the area (A) of the shaded area in the following figure.

7.3 m

4.2m

8.4 m

1. A rectangular lawn measuring 24 m by 18 m has 3 circular flowerbeds

cut from it. If the circular flowerbeds each have a diameter of 8 m, find

the area of the grass remaining.

**Topic C**

1. Find the volumes (*V*) of the following figures.

**a)** 3.7 cm

**b)**3.3mm

5.7mm 2.4 mm

**c)**

*r=* 6.3cm

*h* = 28.8cm

**d)** *h =* 7cm

*r* =3.5cm

**e)**

5.3cm

*r* =3.2 cm

5cm

1. A snowman is made of three balls of snow. One has a diameter of 28 cm,

one of 18 cm, and one of 8 cm. What volume of snow does the snowman

contain?

1. A conveyor belt unloading salt from a ship makes a conical pile 18 m high

with a base diameter of 8 m. What is the volume of the salt in the pile?

1. A spherical balloon is filled with water and has a diameter of 30 cm. If the

water was poured out into an empty tin can measuring 24 cm across and

28 cm high, would the water all fit?

1. The height of a cylindrical pail is 26 cm and the radius of the base is 10 cm.

A ball with radius 6 cm is dropped in the pail. Find the volume of the

region inside the pail but outside of the ball.

**Topic D**

1. Determine the LA of the rectangular solid.

5.2cm

2.1 cm

7.4cm

1. Determine the SA of the rectangular solid.

2.5 in

1.4 in

3.3 in

1. Determine the lateral area and surface area of the following cylinder.

5.7 yd

*d* = 5.2 yd

1. Determine the lateral area and total area of a cone whose diameter

is 6.4 cm and slant height is 7.3 cm.

1. Determine the SA of a sphere whose diameter is 1.8 m.
2. A toy box measures 0.7 m long by 0.6 m wide and is 0.5 m high. What

is the total area of plywood needed to build the box if it has no top?

1. A greenhouse is semi-cylindrical in shape.

If a clear vinyl is used to cover the greenhouse

(including the ends), how much vinyl is needed?

13 m

6 m