**Unit 6**

**Polynomials**

**Topic A: Introduction to polynomials**

* Polynomials
* Degree of a polynomial
* Combine like terms
* Removing parentheses

**Topic B: Multiplying and dividing polynomials**

* Multiplying and dividing monomials
* Multiplying / dividing polynomials by monomials
* FOIL method to multiply binomials

**Unit 6 Summary**

**Unit R Self-test**

**Topic A: Introduction to Polynomials**

**Polynomials**

**Basic algebraic terms:**

|  |  |  |
| --- | --- | --- |
| **Algebraic term** | **Description** | **Example** |
| **Algebraic expression** | A mathematical phrase that contains numbers, variables (letters), and arithmetic operations (+, *–* , etc.). | 3*x* – 4  5*a*2 *– b +* 3  12*y*3 + 7*y*2 *–* 5*y* + |
| **Constant** | A number on its own. | 2*y* + 5 constant: 5 |
| **Coefficient** | The number in front of a variable. | -9*x2* coefficient: -9  *x* coefficient: 1 (*x* = 1*x*) |
| **Term** | A term can be a constant, a variable, or the product of a number and variable.  (Terms are separated by a plus or minus sign.) | 2*x*3 + 7*x*2 *–* 9*y –* 8  Terms: 2*x*3, 7*x*2, - 9*y, -*8 |
| **Like terms** | The terms that have the same variables and exponents (differ only in their coefficients). | 2*x* and -7*x*  -4*y*2 and 9*y*2  0.5*pq*2 and *pq*2 |

**Polynomial**: an algebraic expression that contains one or more terms. The prefix “poly-” means many.

**Example:** 7*x ,* 5*ax –* 9*b*, 6*x*2 *–* 5*x* + , 7*a*2 *+* 8*b + ab –* 5

**There are special names for polynomials that have one, two, or three terms:**

* **Mono**mial: an algebraic expression that contains only one term.

**Example**: 9*x ,* 4*xy*2 ,  0.8*mn*2  , *a*2*b* The prefix “mono” means one.

* **Bino**mial: an algebraic expression that contains two terms. The prefix “bino-” means two.

**Example:** 7*x* + 9 , 9*t*2 –2*t ,*  0.3*y +*

* **Tri**nomial:an algebraic expression that contains three terms.

**Example:** *ax*2 *+ bx + c*,*–* 4*qp*2 +3*q* +5 The prefix “tri-” means three.

**Polynomials in ascending or descending order:** a polynomial can be arranged in ascending or descending order.

* Descending order: the exponents of variables are arranged from largest to smallest number.

**Example**: The exponents of *a* decrease from left to right.

The exponents of *y* decrease from left to right.

* Ascending order: the exponents of variables are arranged from smallest to largest number.

**Example**: The exponents of *x* increase from left to right.

The exponents of *w* increase from left to right.

**Degree of a Polynomial**

**Classification of polynomial:** polynomials are classified according to their number of terms and degrees.

**Degree of a term:**

* The degree of a term with one variable: the exponent of its variable.

**Example:** 9*x***3** The degree of the term: 3

-7*y***5** The degree of the term: 5

* The degree of a term with more variables: the sum of the exponents of its variables.

**Example:** -8*a***2** *b***3** *c***6** The degree of the term: 11(2 + 3 + 6 = 11)

* **More examples:**

|  |  |
| --- | --- |
| **Monomial** | **Degree Reason** |
| 4*x* | 1*x = x*1 (*x* has an exponent of 1.) |
| 7*xy*3 | 41 + 3 = 4 |
| *x*2*y*4*z* | 7 2 + 4 + 1 = 7 (*z = z*1) |
| 13 | 013 = 13 ∙ 1 = 13 ∙ *x*0 = 13 (*x*0 = 1) |

**Degree of a polynomial:** the highest degree of any individual term in it.

**Examples**:

|  |  |  |
| --- | --- | --- |
| **Polynomial** | **Degree** | **Reason** |
| 7*x***8** + 5*x*5 + 8 | 8 | The highest exponent of the term is 7*x***8** . |
| 3*a***2** + 4*a***2***b***3** + 7*a***4***b***5***c***2**  2 2 + 3 = 5 4 + 5 + 2 = 11 | 11 | The highest degree of the term is 7*a***4***b***5***c***2**. |

**Example**: Arrange polynomials in descending order and identify the degrees and

coefficients.

1. 5 + 2*a* 4*a*2 + *a*3

Descending order: *a*3 4*a*2 + 2*a* + 5

Coefficients: 1 -4 2

Degree of the polynomial: 3

1. – 2*xy* + 9*x*3 + 5*x*5y+ + 7*x*2 – *x*4

Descending order:  5*x*5*y*– *x*4 + 9*x*3 + 7*x*2 – 2*xy* +

Coefficients: 5 – 9 7 -2

Degree of the polynomial: 6 *y = y*1

**Combine Like Terms**

**Like terms:** terms that have the same variables and exponents (the coefficients can be different).

**Examples:**

|  |  |
| --- | --- |
| **Example Like or unlike terms** | |
| 7*y* and -9*y* | Like terms |
| 6*a*2, -32*a*2 ,and-*a*2 | Like terms |
|  | Like terms |
| *u*2*v*3 and *u*2*v*3 | Like terms |
| -8*y* and 78*x* | Unlike terms |
| 6*m*3 and -9*m*2 | Unlike terms |
| -9*u*3*w*2 and -9*w*3*u*2 | Unlike terms |

**Combine like terms:** add or subtract their coefficients and keep the same variables and exponents.

**Note**: unlike terms cannot be combined.

**Example:** Combine like terms.

1. 3*a* + 7*b* – 9*a* + 15*b* = (3*a* – 9*a*) + (7*b* + 15*b*) Regroup like terms.

= -6*a* + 22*b* Combine like terms.

1. 2*y*² *–* 4*x* + 3*x* – 5y² = (2*y*² – 5y²) + (-4*x* + 3*x*) Regroup like terms.

= -3*y*² – 1*x* Combine like terms.

= -3*y*² – *x*

1. 8*xy*² – *x*²*y* + 4*x*²*y* – 6*xy*2

=8*xy*² **– *x*²*y*** + **4*x*²*y***– 6*xy*2 Or underline like terms and without regrouping.

=2*xy*² + 3*x*2*y* Combine like terms.

1. 2(2*m* + 3*n*) + 3(*m* – 4*n*) = 4*m* + **6*n*** + 3*m* **– 12*n*** Distributive property.

= 7*m* – 6*n*Combine like terms.

1. 8*v* + 4(2*v* – *u*2) + 3(*u*2 + *v*) = 8*v* + 8*v* **–** **4*u*2** + **3*u*2** + 3*v*   Distributive property.

= –*u*2 + 19*v* Combine like terms.

**Removing Parentheses**

**If the sign preceding the parentheses is positive (+),** do not change the sign of terms inside the parentheses, just remove the parentheses.

**Example**: (*x* *–* 5) = *x* *–* 5

**If the sign preceding the parentheses is negative (-),** remove the parentheses and the negative sign (in front of parentheses), and change the sign of each term inside the parentheses.

**Example**: - (*x* *–* 7) = *-x* + 7

**Remove parentheses:**

|  |  |
| --- | --- |
| **Algebraic expression Remove parentheses Example** | |
| (*ax* + *b*) | *ax* + *b* (5*x* + 2) =5*x* + 2 |
| (*ax* *–* *b*) | *ax* *–* *b* ( = |
| - (*ax* + *b*) | *-ax* *–* *b* - (*x* + 7) = *-x* *–* 7 |
| - (*ax* *–* *b*) | *-ax* + *b* - ( = -0.52.4 |

**Example:** Simplify.

1. 9*x*² + 7 – (2*x*² – 2) = 9*x*² + **7** – 2*x*² + **2** Remove parentheses.

= 7*x*² + 9 Combine like terms.

1. (-8*y* + 5*z*) – 4(*y* – 7*z*) = -8*y* + **5*z*** – 4*y* +**28*z***  Remove parentheses.

= -12*y* + 33*z* Combine like terms.

1. - (3*a*² + 4*a* – 4) + 3(4*a*² – 6*a* + 7) Remove parentheses.

= **- 3*a*²** – 4*a* + 4 + **12*a*²** – 18*a* + 21 Distributive property.

= 9*a*² – 22*a* + 25 Combine like terms.

1. -5(*u*² *–* 3*u*) + 3(2*u* *–* 4) *–* (5 *–* 3*u* *+* 4*u*²) Distributive property.

= **-5*u*²** + 15*u* + 6*u* – 12– 5 + 3*u* – **4*u*²** Remove parentheses. = -9*u*² + 24*u* – 17 Combine like terms.

1. 8(*pq* – 4*cd*) – 3(-*pq* + 5*cd*) = 8*pq* – **32*cd***+ 3*pq* **– 15*cd*** Distributive property. = 11*pq* – 47*cd* Combine like terms.

**Topic B: Multiplying and Dividing Polynomials**

**Multiplying and Dividing Monomials**

**Basic rules of exponents:**

|  |  |
| --- | --- |
| **Name Rule Example** | |
| Product of like bases  (The same base) | *am* *a*n = *am* + *n* 23 22 *=*3 + 2 = 25  Since 23 22 = (2 2 = 25 |
| Quotient of like bases  (The same base) | Since |

|  |  |
| --- | --- |
| Negative exponent | Since 1 |

**Example**: Simplify the following.

1. *x*4 *x*3 = *x* 4 + 3 = *x*7  *am an = a m+ n*
2. ,

**Multiplying monomials (one term):**

* Regroup coefficients and variables.
* Multiply coefficients (the numbers in front of the variable).
* Multiply variables (add exponents with the same base, apply *am an* = *am+n*).

**Example**: **1)** (-4*x*4 *y*3) (7*x*3 *y*2) = (-4 ∙ 7) (*x*4*∙ x*3) (*y*3 *∙ y*2) Regroup the coefficients & the variables.

= -28 *x*4+3 *y*3+2 = -28 *x*7*y*5 Multiply the coefficients & add the exponents. *am an* = *am+n*

( ∙ ) () Regroup.

*a* = *a*1 , *am an* = *am+n*

**Dividing monomials:**

* Divide coefficients.
* Divide variables (subtract exponents with the same base, apply  *= am-n*).

**Example**: **1)** = Regroup the coefficients & the variables.

= **=**  Divide the coefficients & subtract the exponents.

**2)** = ,

**3)** Regroup.

,

**Multiplying / Dividing Polynomials by Monomials**

**Multiplying a monomial and a polynomial:**

* Use the distributive property: *a* (*b* + *c*) = *ab* + *ac*
* Multiply coefficients and add exponents with the same base. Apply *am* *an* = *am+n*

**Examples**:

**1)** **3*x*3 (5*x*2 – 2*x*)** = (3*x*3) (5*x*2) **–** (3*x*3) (2*x*) Distributive property: *a* (*b* + *c*) = *ab* + *ac*

= (3∙5) (*x*3 *x*2) **–** (3∙2) (*x*3 *x*1) Regroup *x* = *x*1

= 15 (*x*3+2) **–** 6 (*x*3+1) Multiply the coefficients & add the exponents.

= 15*x*5 *–* 6*x*4 *am* ∙ *an* = *am+n*

**2)** **5*ab*2 (2*a*2*b +* *ab*2 – *a*)** Distribute.

= (5*ab*2) (2*a*2*b*) *+* (5*ab*2) (*ab*2) + (5*ab*2) (-*a*) Multiply the coefficients and add exponents.

= (5∙2) (*a*1+2 *b*2+1) *+* (5*a*1+1 *b*2+2) **–** (5*a*1+1*b*2) *b* = *b*1 , *a* = *a*1

= 10*a*3*b*3 + 5*a*2*b*4 – 5*a*2*b*2 *am* ∙ *an* = *am+n*

**Dividing a polynomial by a monomial**

* Split the polynomial into several parts.
* Divide a monomial by a monomial. Apply  *= am-n* .

**Example**:

**Steps Solution**

* Split the polynomial into three parts:
* Divide a monomial by a monomial: = 3*x +* 1  *= am-n*

**FOIL Method to Multiply Binomials**

**The FOIL method:** an easy way to find the product of two binomials (two terms).

|  |  |  |
| --- | --- | --- |
| **(*a* + *b*) (*c* + *d*) = *ac* + *ad* + *bc* + *bd***  **F O I** **L** | | **Example** |
| **F** - First terms | first term × first term (*a* + *b*) (*c* + *d*) | (*x* + 5) (*x* + 4) |
| **O** - Outer terms | outside term × outside term (*a* + *b*) (*c* + *d*) | (*x* + 5) (*x* + 4) |
| **I** - Inner terms | inside term × inside term (*a* + *b*) (*c* + *d*) | (*x* + 5) (*x* + 4) |
| **L** - Last terms | last term × last term (*a* + *b*) (*c* + *d*) | (*x* + 5) (*x* + 4) |

|  |  |
| --- | --- |
| **FOIL method** | **Example** |
| (*a* + *b*) (*c* + *d*) = *ac* + *ad* + *bc* + *bd*  **F O I L** | (*x* + 5) (*x* + 4) = *x∙x* + *x∙*4 + 5*x* + 5∙4 = *x*2 + 9*x* + 20  **F O I** **L** |

**Multiplying binomials (2 terms × 2 terms)**

**Example**: Multiply.

1. (2*x* + 3) (5*x* – 6) = 2*x* ∙ 5*x* + 2*x* (- 6)+ 3 ∙ 5*x* + 3 (-6) The FOIL method.

**F O I L**

= 10*x*2 *–* 12*x* *+* 15*x* – 18 *an am = a n+ m*

= 10*x*2 + 3*x* – 18 Combine like terms.

1. (3*r* – *t*) (5*r* + *t*2) = 3*r* ∙ 5*r* + 3*r* ∙ *t*2– *t* ∙ 5*r* – *t* ∙ *t*2 FOIL

= 15*r*2 + 3*rt*2 – 5*rt* – *t*3 *an am = an+m*

1. (*xy*2 + *y*) (2*x2y* + *x*) = *xy*2 ∙ 2*x*2*y* *+ xy*2∙ *x + y* ∙ 2*x*2 *y + y x* FOIL

= 2*x*3 *y*3 *+ x*2*y*2 *+* 2*x*2 *y*2 *+ x y an am = a n+ m*

= 2*x*3 *y*3 + 3*x*2*y*2 + *x y* Combine like terms.

1. (*a* – ) (*a* – ) = *a*2  FOIL

= *a*2 *+* . Combine like terms.

**Unit 6: Summary**

**Polynomials**

**Basic algebraic terms:**

|  |  |  |
| --- | --- | --- |
| **Algebraic term** | **Description** | **Example** |
| **Algebraic expression** | A mathematical phrase that contains numbers, variables (letters), and arithmetic operations (+, *–* , etc.). | 3*x* – 4 , 5*a*2 *– b +* 3 |
| **Constant** | A number on its own. | 2*y* + 5 constant: 5 |
| **Coefficient** | The number in front of a variable. | -9*x2* coefficient: -9  *x* coefficient: 1 |
| **Term** | A term can be a constant, a variable, or the product of a number and variable.  (Terms are separated by a plus or minus sign.) | 7*a*2 *–* 6*b +* 8  Terms: 7*a2*, - 6*b,* 8 |
| **Like terms** | The terms that have the same variables and exponents (differ only in their coefficients). | 2*x* and -7*x*  -4*y*2 and 9*y*2 |

|  |  |
| --- | --- |
| **Polynomial** | **Example** |
| **Mo**nomial **(one term)** | 0.67*x* |
| **Bi**nomial **(two terms)** | 4*x* *–* |
| **Tri**nomial **(three terms)** | 2*a*2 *–* *ab +* 5 |
| Polynomial (one or more terms) | 2*xy,*  4*x*3 + 11 , *–* – 5*y* + 4 |

**Descending order:** the exponents of variables are arranged from largest to smallest number.

**Ascending order:** the exponents of variables are arranged from smallest to largest number.

**Degree of a term/polynomial:**

* The degree of a term with one variable: the exponent of its variable.
* The degree of a term with more variables: the sum of the exponents of its variables.
* Degree of a polynomial: the highest degree of any individual term in it.

**Like terms:** terms that have the same variables and exponents (the coefficients can be different.)

**Combine like terms:** add or subtract their numerical coefficients and keep the same variables and exponents.

**Remove parentheses:**

* If the sign preceding the parentheses is positive (+), do not change the sign of terms inside the parentheses, just remove the parentheses.
* If the sign preceding the parentheses is negative (-), remove the parentheses and the negative sign (in front of parentheses), and change the sign of terms inside the parentheses.

**Basic rules of exponents:**

|  |  |
| --- | --- |
| **Name Rule Example** | |
| Product of like bases  (The same base) | *am* *a*n = *am* + *n* (*a* ≠ 0) 23 22 = 23 + 2 = 25 =32 |
| Quotient of like bases  (The same base) | (*a* ≠ 0) |

|  |  |
| --- | --- |
| Negative exponent | (*a* ≠ 0) |

**Multiply monomials (one term):**

* Multiply coefficients.
* Multiply variables (add exponents with the same base, apply *am an* = *am+n*).

**Dividing monomials:**

* Divide coefficients.
* Divide variables (subtract exponents with the same base, apply  *= am-n*).

**Multiplying a monomial and a polynomial:**

* Use the distributive property: *a* (*b* + *c*) = *ab* + *ac*
* Multiply coefficients and add exponents with the same base. Apply *am* *an* = *am+n*

**Dividing a polynomial by a monomial**

* Split the polynomial into several parts.
* Divide a monomial by a monomial. Apply  *= am-n*

**The FOIL method:**

|  |  |
| --- | --- |
| **(*a* + *b*) (*c* + *d*) = *ac* + *ad* + *bc* + *bd***  **F O I** **L** | |
| **F** - First terms | first term × first term (*a* + *b*) (*c* + *d*) |
| **O** - Outer terms | outside term × outside term (*a* + *b*) (*c* + *d*) |
| **I** - Inner terms | inside term × inside term (*a* + *b*) (*c* + *d*) |
| **L** - Last terms | last term × last term (*a* + *b*) (*c* + *d*) |

**Unit 6: Self-Test**

**Polynomials**

**Topic A**

1. Identify the terms of each polynomial.
2. 5*x*3 8*x*2 + 2*x*
3. – *y*4 + 9*a*2 + *a* – 1
4. Identify the coefficients and the degree of the polynomials.
5. 2*a*3 7*a*2*b*3 +9*b* + 11
6. -8*xy*5 – *y*4 + 11*x*2*y*3 + 4*y*2 – 23*y* +

1. Identify each polynomial as a monomial, binomial, or trinomial.
   1. 3*x*2 7*x*
   2. -29*xy*3
   3. 8*mn*2 +7*m –* 45
2. Arrange polynomials in descending order.
   1. 3 + 8*x* 23*x*2 + 15*x*3
   2. –3*y*3 – 45*y*2 + 4*y**y*4
3. Combine like terms.
   1. 7*x* + 10*y* – 8*x* + 9*y*
   2. 12*a*² *–* 33*b* + 2*b* – 6*a*²
   3. 12*uv*² –5*u*²*v* + 15*u*²*v* – 8*uv*2
   4. 5(4*t* – 6*r*) + 3(*t* + 7*r*)
   5. 13*n* + 5(6*n* – *m*2) **+** 7(2*m*2 + 3*n*)
4. Simplify.
   1. 15*a*² + 9 – (5*a*² – 4)
   2. (-13*x* + 9*y*) – 6(*x* – 5*y*)

* 1. - (7*z*² + 6*z* – 15) + 2(7*z*² – 5*z* + 8)
  2. -11(*y*² *–* 3*y*) + 4(2*y* *–* 5) *–* (13 *–* 6*y +* 9*y*²)
  3. 5(*ab* – 2*xy*) – 6(-2*ab* + 3*xy*)

**Topic B**

1. Simplify the following.
   1. *a*3 *a*6

   4. (-6*a*3 *b*5) (7*a*4 *b*6)
2. Perform the indicated operation.
   1. -4*x*3 (3*x*4 – 7*x*)
   2. 9*a*3*b* (3*ab*2 *+* 2*a*2 *b*2 – *a*)
   4. (5*y* – 7) (8*y* + 9)
   5. (7*r* – 2*t*) (3*r* + 4*t*2)
   6. (2*ab*2 + 3*b*) (5*a2b* + 3*a*)
   7. (*x* – ) (*x* – )

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