**Unit 15**

**Graphing Linear Equations**

**Topic A: Cartesian graphing**

* The Coordinate plane
* Graphing linear equations

**Topic B: The slope of a straight line**

* Slope
* Vertical and horizontal lines

Topic C: Graphing a linear equation

* Slope-intercept equation of a line
* Graphing using the slope and the *y*-intercept
* Graphing linear equations

– Intercept method

Topic D: Writing equations of lines

* Finding an equation of a line

**Unit 15 Summary**

**Unit 15 Self-test**

**Topic A: Cartesian Graphing**

**The Coordinate Plane**

**The coordinate plane** (or Cartesian / rectangular coordinate system): a powerful tool to mark a point and solution of linear equations on a graph.

*y*

* Coordinate axes:

*x* axis - the horizontal line.

*x*

∙ (0, 0)

*y* axis - the vertical line.

* The origin: the intersection of the *x* and *y* axes (both lines are 0 at the origin).

**Ordered pair (*x*, *y*):** a pair of numbers (each point on the plane corresponds to an ordered pair).

*A*

∙ (2, 1)

*y*

*B*

∙ (-3, 2)

(*x , y*) **Example**: Point *A*: (2, 1)

*x*

1st-coordinator (abscissa) 2nd-coordinator (ordinate) Point *B*: (-3, 2)

**Example**: (coke, $0.90) , (juice, $1.25)

**Coordinate:** the numbers in an ordered pair (the *x*-distance and the *y*-distance from a given origin).

**Example**: the coordinate of the point *A* is (2, 1) and the point *B* is (-3, 2).

***y***

**Four Quadrants:**

∙ (0, 0)

***x***

III

IV

II

I

|  |  |  |
| --- | --- | --- |
| **Quadrant** | **(*x*, *y*)** | **Example** |
| The 1st quadrant I | (+*x*, +*y*) | (+2, +3) |
| The 2nd quadrant II | (-*x*, +*y*) | (-2, +3) |
| The 3rd quadrant III | (-*x*, -*y*) | (-2, -3) |
| The 4thquadrant IV | (+*x*, -*y*) | (+2, -3) |

∙ (1, 3)

**Example**: Plot the points and name the quadrants.

∙ (-3, 2)

**

*y*

(1, 3) (-3, 2) (-2, -2) (2, -1)

∙ (2, -1)

*x*

∙ (-2, -2)

**(1, 3):**  I , **(-3, 2):**  II , **(-2, -2):** III, **(2, -1):** IV

***x* - intercept (*x*, 0):** the point at which the graph crosses the *x* - axis.

*y*

**Example**: (*x, y*) = (3, 0)

∙ (0, 2)

***y* - intercept (0, *y*):** the point at which the graph crosses the *y* - axis.

***x***

∙ (3, 0)

**Example**: (*x, y*) = (0, 2)

Points are on the axes.

**Graphing Linear Equations**

**A linear (first-degree) equation:**  an equation whose graph is a straight line.

**A linear equation in two variables:** a linear equation that contains two variables, such as 2*x* + *y* = 3.

**The standard form of linear equation in two variables:**  *Ax* + *By* = *C*

|  |  |
| --- | --- |
| **Standard Form** | **Example** |
| *Ax* + *By* = *C* | 5*x* **–** 7*y* = 4 |

**Solutions of equations:** solutions for a linear equation in two variables are an ordered pair. They are the particular values of the variables in the equation that makes the equation true.

**Example:** Find the ordered pair solution of the given equation.

**2*x* – 3*y* = 7**, **when *x* = 2.** Replace *x* with 2.

2(2) – 3*y* = 7 4 – 3*y* = 7 Subtract 4 from both sides.

- 3*y* = 3 *y* = -1 Divide -3 both sides.

? (*x, y*)

Check: 2 ∙ 2 – 3(-1) = 7 , 7 = 7 .The ordered pair solution is (2, -1) .

**The graph of an equation** is the diagram obtained by plotting the set of points where the equation is true (or satisfies the equation).

**Procedure to graph a linear equation**

**Steps Example:** Graph **2*x* – *y* = 3**

Isolate *y*.

* Choose two values of *x,* calculate the  ***x* *y* = 2*x* – 3 (*x*, *y*)** corresponding *y,* and make a table. 0 2∙0 – 3 = -3 (0, -3) *y*-intercept
* Plot these two points on the coordinate plane. 1 2∙1 – 3 = -1 (1, -1)

3rd point

Select *x* Calculate *y*  Ordered pair

*y*

* Connect the points with a straight line.

∙ (2, 1)

(Any two points determine a straight line.)

∙ (0, -3)

*x*

∙ (1, -1)

* Check with the third point.

Is third point (2, 1) on the line? Yes. Correct!  ***x* *y* = 2*x* – 3 (*x*, *y*)**

Another solution

2 2∙2 – 3 = 1 (2, 1)

**Example:** Graph ***y* =**  and determine another point.

∙ (8, 1)

*y*

(0, -3)∙

∙ (2, -2)

*x*

|  |  |  |
| --- | --- | --- |
| ***x*** | ***y*** | **(*x*, *y*)** |
| 0 | - 3 | (0, -3) |
| 2 | - 2 | (2, -2)  Solutions |

**Topic B: The Slope of a Straight Line**

**Slope**

**Recall: the graph of a linear equation is a straight line**.

**Slope (*m*)** (grade or pitch): the slope of a straight line is the rate of change. It is a measure of the “steepness” or “incline” of the line and indicates whether the line rises or falls.

**A line with a positive slope rises from left to right and a line with a negative slope falls.**

**The slope formula:**

*y*

(*x2, y*2)

|  |  |
| --- | --- |
| **The slope formula** | |
|  | The slope of the straight line that passes through two points (*x*1, *y*1) and (*x*2, *y*2):  *m* = *m* = *x*1 ≠ *x*2 |

Change in *y*

∙

Change in *x*

rise

run

(*x*1*, y*1)

∙

*x*

**Example**: Determine the slope containing points (3, -2) and (4, 1).

*m* = = 3 (*x*1*,* y1) = (3, -2), (*x*2*,* y2) = (4, 1)

or  *m* = = 3

Solve for *y* from 6*x* – *y* – 5 = 0

6*x* – 5 = *y* (add *y* both sides.)

**Example**: Determine the slope of **6*x* – *y* *–* 5 = 0.**

|  |  |  |
| --- | --- | --- |
| ***x*** | ***y =* 6*x –* 5** | **(*x*, *y*)** |
| 0 | 6 ∙ 0 ***–*** 5 = -5 | (*x*1, *y*1) = (0, -5) |
| 1 | 6 ∙ 1 ***–*** 5 = 1 | (*x*2, *y*2) = (1, 1) |

Choose Calculate

*m* = = 6 or  *m* = = 6

Other points on the line will obtain the same slope *m*.

|  |  |  |
| --- | --- | --- |
| ***x*** | ***y =* 6*x –* 5** | **(*x*, *y*)** |
| 2 | 7 | (2, 7) |
| -1 | -11 | (-1, -11) |

Choose Calculate

*m* = 6(*x*1*,* y1) = (2, 7), (*x*2*,* y2) = (-1, 11)

**Vertical and Horizontal Lines**

**Horizontal line**: a line that is parallel to the *x*-axis.

* It has a zero slope (*m* = 0).
* With a *y*-intercept *y* = *b* or (0, *b*). Where *b* is any constant.

**Example**: *y* = -4

∙ (0, 0)

*x*

*y*

|  |  |
| --- | --- |
| ***x*** | ***y* (*x, y*)** |
| 1 | -4 (1, -4) |
| 2 | -4 (2, -4) |

*y* = -4

Choose Given

*m* = = 0 The horizontal line *y* = -4 has a zero slope.

**Vertical line**: a line that is parallel to the *y*-axis.

* It has an infinite slope (*m* =
* With a *x*-intercept *x* = *a* or (*a*, 0).

*y*

**Example**: *x* = -3  *x* = -3

∙ (0, 0)

*x*

|  |  |
| --- | --- |
| ***x*** | ***y* (*x, y*)** |
| -3 | 3 (-3, 3) |
| -3 | -1 (-3, -1) |

*m* = = The vertical line *x* = -3 has an infinite slope.

**Summary of horizontal and vertical lines:**

*y*

|  |  |
| --- | --- |
| **Line** | **Equation Slope (*m*) Example Graph** |
| Horizontal line | *y* = *b* *m* = 0 *y* = 2 |
| Vertical line | *x* = *a* *m* = ∞ *x* = 1 |

**Example**: **1)** Graph *y* = -0.5  *x* = 4

*x*

**2)** Graph *x* = 4

*y* = - 0.5

**3)** Graph *x* = 0 *x* = 0

**Topic C: Graphing a Linear Equation**

**Slope-Intercept Equation of a Line**

**Slope - intercept form of a linear equation**

|  |
| --- |
| **Slope - intercept equation of a line** |
| *y* = ***m*** *x +* ***b*** of the line  *y*-intercept |

*y*

∙

*b*

*x*

**Recall: *y -* intercept:** the point at which the line crosses the *y* axis. *b* = (0, *y*)

**Example**: Identify the slope and *y*-intercept of the following equations.

1. *y* **= -3*x* – 5***y* = ***m*** *x +* ***b***

The slope: ***m* =** -3 *y* = -3*x* – 5

*y*-intercept: ***b* =** -5 or (0, -5)

**2)** **3*y* – 2*x* = 1**

3*y* = 2*x* + 1Add 2*x* on both sides.

Divide both sides by 3.

The slope: *y* = ***m*** *x +* ***b***

*y*-intercept: ***b =***  or (0, )

**3)**

∙ 3 + *y* ∙ 3 = 5 ∙ 3 Multiply 3 by each term.

12*x* + *y* = 15 Subtract 12*x* from both sides.

*y* = -12*x* + 15 *y* = ***m*** *x +* ***b***

The slope: ***m* =** -12

*y*-intercept: ***b* =** 15 or (0, 15)

**Graphing Using the Slope and the *y* - Intercept**

**Slope-intercept equation:**  *y* =*x* + *b*

**The slope and a point can determine a straight line.**

**Example**: Graph the equation using the slope and the *y*-intercept. ***y* = *x* + 5**

* Plot the *y*-intercept (0, 5). The change in *y*: the rise (move 2 units down, ∵ *y* is negative).
* Determine the rise and run: *m* = The change in *x*: the run (move 3 units to the right, ∵ *x* is positive).
* Plot another point by moving 2 units down and 3 units to the right (3, 3).
* Connect the two points with a straight line.

Starting point

*y*

Starting point: y-intercept

∙ (0, 5)

5

Ending point

∙ (3, 3)

3

3

0

*x*

**Example**: Graph the equation using the slope and the *y*-intercept. **-** 9*x* + 12 **=** **–** 3***y***

* Convert to the slope - intercept form. 3*y* = 9*x* **–** 12 Divide each term by (-1).

*y* = 3*x* **–** 4 Divide both sides by 3.

* *y*-intercept: (0, -4) *y* = *x* + *b*
* Slope:

*y*

Move 3 units up and 1 unit to the right (both *x* & *y* are positive).

Ending point (1, -1)

0

*x*

∙

Starting point

- 4

∙

*+y*

**Tip:**  *m* =

*- y*

*-x*

*+x*

**Graphing Linear Equations**

**- Intercept Method**

**Recall:** The *x* interceptis the point at which the line crosses the *x* axis. (*a*, 0)

The *y* interceptis the point at which the line crosses the *y* axis. (0, *b*)

**Find the intercepts:**

**Example:** Determine the intercepts of the line 5*x* – *y* = 6.

* The *x*-intercept: let *y* = 0, and solve for *x.* 5*x* – 0 = 6, 5*x* = 6

*x* = = 1.2 Divide both sides by 5.

(1.2, 0)

* The *y*-intercept: let *x* = 0, and solve for *y.* 5 0 – *y* = 6, - *y* = 6

*y* = - Divide both sides by -1.

(0, -6)

**Procedure to graph a linear equation using the intercept method**

**Steps Example: 5*x* – *y* = 6**

* Choose *x* = 0 and calculate the corresponding *y*. ***x*  *y* = 5*x* –6 (*x*, *y*) Intercept**

0 -6 (0, -6) *y*-intercept

* Choose *y* = 0 and calculate the corresponding *x.* 1.2 0 (1.2, 0) *x*-intercept

*y*

* Plot these two points on the coordinate plane.

∙ (1, -1)

∙ (1.2, 0)

*x*

* Connect the points with a straight line.
* Check with the third point.

∙ (0, -6)

Is third point (1, -1) on the line? Yes. Correct!

***x* *y* = 5*x* –6 (*x*, *y*)**

1. -1 (1, -1)

( 5 1 – *y* = 6, - *y* = 6 – 5, y = -1 )

**Topic D: Writing Equations of Lines**

**Finding an Equation of a Line**

**Equation of a straight line:**

|  |  |  |
| --- | --- | --- |
| **Straight-line equation** | **Equation** | **Example** |
| Point-slope form | *y* – *y*1 = *m* (*x* – *x*1) | *y* – 3 = (*x* + 2) *m* = -4 *y*1 *=* 3*, x*1= -2 |
| Slope-intercept form | *y* = *mx* + *b* | *y* = 3*x* –  *m* = 3 , *b* = - |

**Finding an equation of a line from the graph:**

**Example**: Write the slope intercept equation of the given line. *y* = *m x* + *b*

(0, 5)

*y*

*y*

∙

(1, 2)

0

∙

*x*

*y*

* Choose two points on the given line, such as (0, 5) and (1, 2).
* The slope: *m* = = (*x*1 *, y*1) *=* (0, 5), (*x*2 *, y*2) *=* (1, 2)
* *y*-intercept: *b* = 5 The line crosses the y-axis at (0, 5).
* Equation of the line: *y* = -3*x* + 5 *y* = *m* *x + b*: *m* = -3, *b* = 5

**Finding an equation of a line when the slope and a point are given:**

**Example**: Write an equation for a line passing the point (5, 3) with slope *m* = -4.

* Start with: *y = mx + b* Replace (*x**, y*)by(5, 3) & *m* by *-*4.
* Solve for *b*: 3 = -4 ∙ 5 + *b* Add 20 on both sides.
* *y*-intercept: *b* = 23
* Equation of the line: *y* = -4*x* + 23*y* = *m* *x + b*: *m* = -4, *b* = 23

**Finding an equation of a line when two points are given:**

**Example**: Write an equation for a line that passes through the points (2, 1) and (3, -5).

* The slope: *m* = = (*x*1 *, y*1) *=* (2, 1) , (*x*2 *, y*2) *=* (3, -5).

Substitute values into point-slope equation: *y* – *y*1 = *m* (*x* – *x*1)

* Point-slope equation: *y* – 1 = -6(*x* – 2) Replace (*x*1 *, y*1)with(2, 1) & *m* with *–* 6.
* Slope-intercept form: *y* – 1 = -6*x* + 12 Remove parentheses.

*y* = -6*x* + 13Add 1 on both sides, *y* = *m x* + *b*.

**Unit 15: Summary**

**Graphing Linear Equations**

**The coordinate plane**: a powerful tool to mark a point and solution of linear equation on a graph.

* Coordinate axes: *x* axis and *y* axis.
* The origin: the intersection of the *x* and *y* axes (both lines are 0 at the origin).

**Ordered pair (*x*, *y*):** a pair of numbers (each point on the plane corresponds to an ordered pair).

**Coordinate**: the numbers in an ordered pair (the *x*-distance and the *y*-distance from a given origin).

***y***

**Four quadrants:**

IV

III

***x***

II

I

|  |  |
| --- | --- |
| **Quadrant** | **(*x*, *y*)** |
| The 1st quadrant I | (+*x*, +*y*) |
| The 2nd quadrant II | (-*x*, +*y*) |
| The 3rd quadrant III | (-*x*, -*y*) |
| The 4thquadrant IV | (+*x*, -*y*) |

***x* – intercept (*x*, 0):**  the point at which the graph crosses the *x* - axis.

***y* – intercept (0, *y*):** the point at which the graph crosses the *y* - axis.

**A linear (first-degree) equation:** an equation whose graph is a straight line.

**A linear equation in two variables:**  a linear equation that contains two variables, such as 5*x* + 2*y* = 7.

**The standard form of linear equation in two variables:**  *Ax* + *By* = *C*

|  |  |
| --- | --- |
| **Standard Form** | **Example** |
| *Ax* + *By* = *C* | 4*x* **–** 9*y* = 11 |

**Solutions of equations:** solutions for a linear equation in two variables are an ordered pair. They are the particular values of the variables in the equation that makes the equation true.

**Procedure to graph a linear equation:**

* Choose two values of *x,* calculate the corresponding *y,* and make a table.
* Plot these two points on the coordinate plane.
* Connect the points with a straight line.
* Check with the third point – is third point on the line?

**Slope (*m*)** (grade or pitch): the slope of a straight line is the rate of change. It is a measure

of the “steepness” or “incline” of the line and indicates whether the line rises or falls.

* A line with a positive slope rises from left to right and a line with a negative slope falls.
* **The slope formula:**

|  |  |
| --- | --- |
| **The slope formula** | |
|  | The slope of the straight line that passes through two points (*x*1, *y*1) and (*x*2, *y*2):  *m* = *m* = *x*1 ≠ *x*2 |

**Horizontal and vertical lines:**

|  |  |  |
| --- | --- | --- |
| **Line** | **Equation** | **Slope (*m*)** |
| Horizontal line | *y* = *b* | *m* = 0 |
| Vertical line | *x* = *a* | *m* = ∞ |

**The slope and a point can determine a straight line.**

**Procedure to graph a linear equation using the intercept method:**

* Choose *x* = 0 and calculate the corresponding *y*.
* Choose *y* = 0 and calculate the corresponding *x.*
* Plot these two points on the coordinate plane.
* Connect the points with a straight line.
* Check with the third point - is third point on the line?

**Equation of a straight line:**

|  |  |
| --- | --- |
| **Straight-line equation** | **Equation** |
| Point-slope form | *y* – *y*1 = *m* (*x* – *x*1) |
| Slope-intercept form | *y* = *mx* + *b* |

**Finding an equation of a line from the graph:**

* Choose two points on the given line.
* Calculate the slope: *m* =
* Determine the *y*-intercept on the line: *b* or (0, *y*) The line crosses the *y*-axis.
* Equation of the line: *y* = *m* *x + b*Replace *m* and *b* with values.

**Finding an equation of a line when the slope and a point are given:**

* Start with: *y = mx + b* Replace (*x**, y*)& *m* withgivenvalues.
* Solve for *b*.
* Equation of the line: *y = mx + b* Replace *m* and *b* with values.

**Finding an equation of a line when two points are given:**

* Calculate the slope: *m* =
* Point-slope equation: *y* – *y*1 = *m* (*x* – *x*1) Replace (*x*1 *, y*1)& *m* with values.
* Slope-intercept equation: *y = mx + b* Solve for *y*.

**Unit 15: Self-Test**

**Graphing Linear Equations**

**Topic A**

1. Plot the points and name the quadrants.

(2, -1) (-4, 3) (-1, -3) (3, 2)

1. Graph the following.
2. *y* = 3*x*
3. 7*x* – *y* = 3
4. *x* + 3*y* = 6
5. Graph *y* = and determine another point.
6. Find the ordered pair solution of the given equation.
7. 3*x* – 5*y* = 11, when *x* = 2.
8. *x* – 0.6*y* = -3, when *x* = -6.
9. *x* – 4*y* = 5, when *x* = -4.

**Topic B**

1. Determine the slope containing points (4, -1) and (3, 5).
2. Determine the slope of 8*x* – *y* *–* 3 = 0.
3. Graph the following.
   1. *y* = -0.9

**b)** *x* = 3

**c)** *y* = 0

**Topic C**

1. Identify the slope and *y*-intercept of the following equations.
2. *y* = -7*x* – 11
3. 5*y* –3*x* = 2
5. Graph the equation using the slope and the *y*-intercept.
   1. ***y* = *x* + 5**
   2. **-** 6*x* + 9 **=** **–** 3***y***
6. Determine the intercepts of the line 3*x* – *y* = 9.
7. Graph the equation using the intercept method.
8. 4*x* – *y* = 8
9. *y* = *x* + 3

**Topic D**

1. Write the slope intercept equation of the given line.

*y*

**a)**

∙ 6

∙

-3

*x*

1

**b)**

∙ 3

*x*

∙

4

1

1. Write the equation for the following lines:
2. The line with a slope of -4 passing the point (-2, 5).
3. The line with a slope of passing the point (5, -7).
4. Write an equation of the line that passes through each pair of points.
5. (3, 2) and (4, -7).
6. (-3, 0) and (0, 6).
7. (0, 5) and (5, 3).