**Topic B: Trigonometric Functions**

Sides and Angles

**Trigonometry: the study of the relationships between sides and angles of right triangles and trigonometric functions.**

**Right triangle review:** a **triangle** that has a 90° angle (right-angled triangle).

**Sides and angles:**

* < *C* is a right angle (900).
* Sides are labeled with lower case letters (or two capital letters).

**Example**: The side *a* or *BC,* the side *b* or *AB,* the side *c* or *AC.*

* Angles are labeled with uppercase letters.

**Example**: < *A*, < *B*, < *C*

* Side *a* will be the side opposite angle *A*; side *b* will be the side opposite

angle *B*; and side *c* will be the side opposite angle *C*.

**Hypotenuses, adjacent, and opposite:**

* The longest side of the triangle is the hypotenuses (the side opposite the 90° angle).
* “Opposite” and “adjacent” refer to sides that are opposite or adjacent to the two acute

angles (< *A* and < *B*) of the triangle.

hypotenuse

hypotenuse

opposite < *A*

adjacent < *B*

* Adjacent side: the side next to the acute angle.

opposite < *B*

adjacent < *A*

* Opposite side: the side opposite the acute angle.

(An acute angle < 900.)

**Example:** Fill in the blanks in each of the following

1. Side *EG* (or *f*) is angle *F*. opposite

*E*

*G*

*F*

1. Side *FG* (or *e*) is angle *F*. adjacent

*g*

1. Side *EF* (or *g*) is the . hypotenuse

*f*

1. Side *EG* (or *f*) is angle *E*. adjacent

*e*

1. Side *FG* (or *e*) is angle *E*. opposite
2. Side *EG* is opposite to angle .*F*

Trigonometric Functions

**Trigonometric functions** (of right triangles):

* There are six trigonometric functions (or ratios): sine (sin), cosine (cos), tangent (tan),

secant (sec), cosecant (csc), and cotangent (cot).

* The lengths of the sides are used to define the trigonometric functions (or ratios).

**Sine, cosine, and tangent** (three main trigonometric functions):

* The sine of the angle =

hypotenuse

sin =

opposite

adjacent

* The cosine of the angle =
* cos = **θ** is a Greek letter that uses for an angle.
* The tangent of the angle =

tan =

**Secant, cosecant, and cotangent:** the inverse trigonometric functions.

* Secant is the inverse of cosine: sec =
* Cosecant is the inverse of sine: csc =
* Cotangent is the inverse of tangent: cot =

**Six trigonometric functions:**

|  |  |  |  |
| --- | --- | --- | --- |
| Trigonometric function | **Read** | **Diagram** | **Memory aid** |
| sin = | Sine of | hypotenuse  adjacent  opposite | Soh |
| cos = | Cosine of | Cah |
| tan = | Tangent of | Toa |
| sec = | Secant of | Inverse of cosine |
| csc= | Cosecant of | Inverse of sine |
| cot = | Cotangent of | Inverse of tangent |

Sine, Cosine, and Tangent

**Example:** Find the sine, cosine, and tangent for each of the following.

|  |
| --- |
| sin *X* = = 0.8949 Soh |
| cos *X* = 0.4474 Cah |
| tan *X* = = 2 Toa |
| sin *Z* = = 0.4474 |
| cos *Z* == 0.8949 |
| tan *Z* = 0.5 |

*Y*

*Z*

*X*

4 cm cm

4.47 cm

2 cm

The sine of one angle in the right triangle is equal to the cosine of the other angle in that same right triangle.

**Example:** Find the sine, cosine, and tangent for each of the following.

|  |
| --- |
| sin *F* = = 0.8302 Soh |
| cos *F* = 05587 Cah |
| tan *F* = = 1.4858 Toa |
| sin *E* = = 0.5587 |
| cos *E* == 0.8302 |
| tan *E* = 0.673 |

*E*

*G*

*F*

3.52 ft cm

6.3 ft

5.23 ft

**Memory Aid:**

|  |  |  |  |
| --- | --- | --- | --- |
| Sine, cosine, and tangent | Trigonometric function | **Memory aid** | **Diagram** |
| Sine | sin = | Soh | hyp  adj  opp |
| Cosine | cos = | Cah |
| Tangent | tan = | Toa |

**Topic C: Solving Right Triangles**

Trigonometry Using a Calculator

**Find the trigonometric functions of an angle:**

**Example:** Find each of the following using a scientific calculator.

1. sin 1320 =?

Type in: sin 132 Display: **0.7431**… sin 1320 ≈0.7431

Or 132 sin with some calculators.

1. cos 250 =?

Type in: cos 25 = Display: **0.9063 …** cos 250 ≈0.9063

Or 25 cos with some calculators.

1. tan 480 =?

Type in: tan 48 Display: **1.11061**… tan 480 ≈1.1106

Or 48 tan with some calculators.

**Find an angle when given the trigonometric function (ratio):**

**Example:** Find each of the following using a scientific calculator.

1. sin *A* = 0.5446, < *A* = ?

Type in: 2ndF sin-1  0.5446 = Display: **32.997333**… < *A* ≈ 330

Or INV with some calculators.

1. tan *B* = 0.57736, < *B* = ?

Type in: 2ndF tan-1  0.57736 = Display: **30.000418**… < *B* ≈ 300

Or INV with some calculators.

Solving Triangles

**Angles in a triangle**: the sum of the three internal angles in a triangle is always 1800.

*< A + < B + < C* = 1800

**Pythagorean theorem review**: a relationship between the three sides of a right triangle.

a c **

**There are six elements (or parts) in a triangle**, that is, three sides and three internal angles.

**Solving a triangle:** to solve a triangle means to know all three sides and all three angles.

**Example: 1)** Solve for the variable.

tan = , *x* = ? *x* tan = *x*  Multiply both sides by *x*.

= Divide both sides by tan 320.

***x*** = 11.2Use a calculator.

*B*

**2)** Find side *c* if *b* = 10m and < *B* = 360.

*c*

*a*

*b* = 10 m

sin = sin=

*C*

*A*

sin 360*c* = *c*  Multiply both sides by *c.*

*c* = Divide both sides by sin360.

*c* = ≈17.01 m

**Example**: Solve the triangle (< *A* = ? *b* = ? *c* = ?). Find all unknown sides and angles.

* **< *A*** = 1800 – (< *C* + < *B*) < *A* + < *B* + < *C* = 1800

*c*

*a =* 10 mm

*B*

= 1800 – (900 + 37.40)

*b*

= 52.60

*C*

* tan = tan =

*b* = *a* (tan *B*) = 10 (tan37.4) 7.65 mm Multiply both sides by *a* and reverse the sides.

* 12.59 mmPythagorean theorem.

**Example:** Find the missing part of each triangle.

500

3m

**1)**

*b* ?

400

2

cos 500 = cos =

cos 500 = 3 Multiply both sides by 3.

3 (cos 500) = *b*

***b*** = 3 (cos 500) ≈ 1.928 m Reverse the sides of the equation.

66.90

5.1 cm

**2)**

2 cm

4.69 cm cm

?

sin *A*= sin =

**< *A*** = sin -1 ≈ 23.10 2nd F sin-1

**Example**: Solve the right triangle. Find all unknown sides and angles.

1. **< B: < *B*** = 1800 – (< *C* + < *A*)(< *B* = ? *b* = ? *c* = ?)

*=*1800 – (900 + 350) = 550

350

*b*

*c*

***b***: tan 350 = tan =

2 m

*b* tan 350 = *b* Multiply both sides by *b*.

Divide both sides by tan 350.

2.856 m

***c*: *c*** 3.487 mPythagorean theorem.

1. ***a*:** 3.464 m(*a* = ? < *B* = ? < *A* = ?)

**<*A*:** cos *A* = cos =

*a*

4 cm

**< *A*** = cos-1*A* = cos-1 0.5 = 600 2nd F cos -1

2 cm

**<*B*: *<B*** = 1800 – (900 + 600) = 300

?

Check: < *A* + < *B* + < *C* = 1800 , 600 + 300 + 900 = 1800 Correct!

Angles of Depression and Elevation

**Angle of depression**: the angle between a horizontal line and the line of sight for an object below the horizontal. The word "depression" means "fall" or "drop".

**Angle of elevation**: the angle between a horizontal line and the line of sight for an object above the horizontal.   The word "elevation" means "rise" or "move up".

Horizontal line

Angle of depression



Angle of elevation

Horizontal line



Angle of elevation

**Example: 1)** Find the angle of elevation.

tan *B* = tan =

3cm

**< *B*** = tan-1*B* = tan-1  ≈ 56.30 2nd F tan-1

2 cm

**2)** Find *y* if the angle of depression is 360.

Angle of depression

360

5 mm

sin 360 = sin=

*y*

***y*** = 8.507 mm

(Divide both sides by sin 360 and multiply both sides by *y*.)

Angle of depression

**Example**: From the top of a rock wall, the angle of depression to a swimmer is 560. If the

wall is 20m high, how far from the base of the wall is the swimmer?

900 560 = 340

tan 340 **=**  tan =

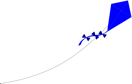
***x*** = 20 (tan 340) ≈ 13.49 m

?

(Multiply both sides by 20 and reverse the sides of the equation.)

.

**Example**: Mike has let 25 m of string out on his kite. He is flying it 11.5 m above his

eye level. Find the angle of elevation of the kite.

sin = ≈ 0.46sin=

= sin-1 0.46 27.4.0

Applications of Trigonometry

**Example:** When Brandon stands 37 m from the base of a building and sights the top of

the building, he is looking up at an angle of 43°. How high is the building?

 tan 430 **=**  tan =

(37) tan 430 **=**  Multiply both sides by 37.

*x*

*x* = (37) tan 43034.5 m

37 m

43°

The building is approximately **34.5 m** high.

**Example:** Tom tries to swim straight across a river. He can swim at 1.6 m/sec, but the river is

flowing at 1.2 m/sec. At what angle to his intended direction will Tom actually travel?

tan  **=**  0.75 tan =

**<**  = tan-1 0.75 36.8702nd F tan-1

Tom will travel about **36.87°** off course.



1.6 m/sec

1.2 m/sec

**Example**: An equilateral triangle has a height of 12 mm. Find the length of each side.

sin 600 **=**  Each angle = 600 (an equilateral triangle.)

?

?

***x* =** 13.86 mm

(Multiply both sides by *x* and divide both sides by sin 600.)

?

The length of each side is about **13.86 mm**.

**Unit 10: Summary**

**Trigonometry**

**An angle can vary from 0 to 360 degrees (3600).**

**Classifying angles:**

|  |  |  |
| --- | --- | --- |
| **Angle** | **Definition** | **Figure** |
| **Straight angle** | An angle of exactly 1800. | 1800 |
| **Right angle** | An angle of exactly 900. | 900 |
| **Acute angle** | An angle between 0 and 900. | *A A <* 900 |
| **Obtuse angle** | An angle between 90 and 1800. | *A* 900 *< A <* 1800 |
| **Reflex angle** | An angle between 180 and 3600. | *A* 1800 *< A <* 3600 |
| **Complementary angles** | Two angles whose sum is exactly 900. | *A*  *B < A + < B =* 900 |
| **Supplementary angles** | Two angles whose sum is exactly 1800. | A B <A + <B = 1800 |
| **Vertical angles** | Two angles formed by the [intersection](http://www.mathopenref.com/intersection.html) of two straight lines.  ＜*A* and < *B* are vertical angels. | *A 　　　 B* |

**Classify triangles:**

|  |  |  |
| --- | --- | --- |
| **Name of triangle** | **Definition** | **Figure** |
| **Equilateral triangle** | A [triangle](http://www.mathopenref.com/triangle.html) that has three equal sides and  three equal angles.  *a* = *b* = *c*, < *A* = < *B* = < *C =* 600 | *C*  *b a*  *A c B* |
| **Isosceles triangle** | A triangle that has two equal sides and two equal angles.  *a* = *b*, < *A* = < *B* | *b a*  *A B* |
| **Acute triangle** | A triangle that has three acute angles (< 900). |  |
| **Right triangle** | A triangle that has a right angle (= 900). |  |
| **Obtuse triangle** | A triangle that has an obtuse angle (> 900). |  |
| **Scalene triangle** | A triangle that has three unequal sides. |  |

**Angles in a triangle:** the sum of the three angles in a triangle is always 1800.

*< X + < Y + < Z* = 1800

**How to use a protractor:**

* + Place the protractor so that the center hole is over the angle’s vertex.
  + Line up the base line of the protractor with one of the sides of the angle.
  + Read the angle over the second side of the angle.

**Sides and angles:**

* Sides are labeled with lower case letters (or two capital letters).
* Angles are labeled with uppercase letters.
* Side *a* will be the side opposite angle *A*; side *b* will be the side opposite angle *B*; and side

*c* will be the side opposite angle *C*.

**Hypotenuses, adjacent, and opposite:**

* The longest side of the triangle is the hypotenuses (the side opposite the 90° angle).
* “Opposite” and “adjacent” refer to sides that are opposite or adjacent to the two acute

adjacent < *B*

hypotenuse

angles (< *A* and < *B*) of the triangle.

opposite < *A*

hypotenuse

* Adjacent side: the side next to the acute angle.

adjacent < *A*

opposite < *B*

* Opposite side: the side opposite the acute angle.

**Six trigonometric functions:**

|  |  |  |
| --- | --- | --- |
| Trigonometric function | **Diagram** | **Memory aid** |
| sin = | hypotenuse  opposite  adjacent | Soh |
| cos = | Cah |
| tan = | Toa |
| csc= | Inverse of sine |
| sec = | Inverse of cosine |
| cot = | Inverse of tangent |

**Pythagorean theorem review**: a relationship between the three sides of a right triangle.

*a c* 

*b*

**Solving a triangle:** to solve a triangle means to know all three sides and all three angles.

Angle of depression

**Angle of depression**: the angle between a horizontal line and

Angle of elevation

the line of sight for an object below the horizontal.

**Angle of elevation**: the angle between a horizontal line and the

line of sight for an object above the horizontal.

**Unit 10: Self-Test**

**Trigonometry**

**Topic A**

1. Label each of the following angles.

1. 490 650
2. 1340

1150

1. **d)**

1. What is the complementary angle to 42 degrees?
2. What is the supplementary angle to 146 degrees?
3. What is the size of the angle *x*?

1200

*x* 760

1. **a)** Two angles *A* and 330 that add together to measure 1800 are said to be\_\_\_\_\_\_?

**b)** What is the size of angle *A* and *B*?

*A*

1. What is the size of angle *C* in the following figure?

*C* ?

1130

270

360

1. What is the size of angle *C, D* and the side *b* in the following figure?

*b*

5m

*D*

720

1. Match the following triangles to the letter with the best definition.
2. Equilateral triangle i. has two equal sides
3. Isosceles triangle ii. has three unequal sides
4. Supplementary angles iii. Two angles whose sum is exactly 1800.
5. Scalene triangle iv. has three equal sides
6. Find the missing measurement and then name the kind of triangle.

*x =* ?

600



23 cm

= ?

600



*B =* ?

43 ft *a* = ?

390

390

21 m

1240

21 m

*C* = ?

*B* = ?



*<Z* = ?

*x* = ?

*y* 32cm

540

540

**Topic B**

1. Fill in the blanks in each of the following
2. Side *ZY* (or *x)* is angle *X*.

*X*

*Z*

*Y*

1. Side *XZ* (or *y*) is angle *X*.

*z*

1. Side *XY* (or z) is the .

*y*

1. Side *ZY* (or *x*) is  angle *Y*.

*x*

1. Side *XZ* (or *y*) is angle *Y*.
2. Side *XZ* (or *y*) is opposite to angle .

*Y*

*Z*

*X*

5 cm cm

7.81 cm

6 cm

1. Find the sine, cosine, and tangent of each acute angle.

1. Find the sine, cosine, and tangent of each acute angle.

6.32 ft

*Q*

*P*

*Oq*

7.62 ft

.

4.25 ft

**Topic C**

1. Use a calculator to find the trigonometric value of each angle.
2. sin 57° = ?
3. cos 360 = ?
4. tan 870 = ?
5. sin ( ) = 0.2165
6. cos ( ) = 0.4567
7. tan ( ) = 1.2356
8. Solve for the variable.

tan = , *x* = ?

*B*

1. Find side *c* if *b* = 24 cm and < *B* = 410.

*a*

*c*

*b* = 24 cm

*A*

*C*

1. Solve the triangle (< *A* = ? *b* = ? *c* = ?).

*c*

*B*

*b*

*a =* 6 m

1. Find the missing part of each triangle.

*C*

460

9cm



350

*b ?*

7cm

490

7.8 m



3.6m cm

?

6.92m cm

1. Solve the right triangle. (< *B* = ? *b* = ? *c* = ?)

*b*

*c*

450

6 m

1. (*a* = ? < *B* = ? < *A* = ?)

*a*

5ft

3ft

5 mm

1. **a)** Find the angle of elevation.

8 mm

**b)** Find *y* if the angle of depression is 320.

320

8.5m

*y*

1. From the top of a wall, the angle of depression to a boy is 430. If the wall is 24 m high,

how far from the base of the wall is the boy?

1. Todd has let 34 m of string out on his kite. He is flying it 22.4 m above his eye level. Find

the angle of elevation of the kite.

1. When Ann stands 28 m from the base of a building and sights the top of the building, she is

looking up at an angle of 39°. How high is the building?

1. Damon tries to swim straight across a river. He can paddle at 1.3 m/sec, but the river is

flowing at 1.5 m/sec. At what angle to his intended direction will Damon actually travel?

1. An equilateral triangle has a height of 41 cm. Find the length of each side.