**Unit 2**

**Introduction to Algebra**

**Topic A: Algebraic expressions**

* Basic algebraic terms
* Evaluating algebraic expressions

**Topic B: Translating words into algebraic expressions**

* Key words in word problems
* Translating phrases into algebraic expressions
* Writing algebraic expressions
* Steps for solving word problems

**Topic C: Exponents and order of operations**

* Introduction to exponents
* Read and write exponential expressions
* Order of operations

**Unit 2 Summary**

**Unit 2 Self-test**

**Topic A: Algebraic Expressions**

**Basic Algebraic Terms**

**Algebra:** a branch of mathematics containing numbers, letters and arithmetic operators (+, – , ×, ÷ , etc.) with the letters used to represent unknown quantities (variables).

**Example:**  3 + 2 = 5 in algebra may look like *x* + 2 = 5 *x* represents 3.

**Constant:** a ***number*** stands for a fixed value that does not change.

**Example:**  ***2*** in *x* + ***2*** is a constant.

**Variable:** a ***letter*** that can be assigned different values (it represents an unknown quantity).

**Example:**  ***x*** + 2 when *x* = 0, ***x*** + 2 = 0 + 2 = **2**

when *x* = 3, ***x*** + 2 = 3 + 2 = **5**

**Coefficient:** the ***number*** that in front of a letter (variable).

**Example: 9** *x* coefficient:9

**-***x* coefficient:-

*x* coefficient: 1 *x* = 1 *x*

**Algebraic expression:** a mathematical phrase that contains numbers, letters, grouping symbols (parentheses) and arithmetic operations (+, – , ×, ÷ , etc.)

**Example:** 5*x* + 2,  , (3*x* – 4*y*2) + 6

**Term:** a term can be a number, letter, or the product (multiplication) of a number and letter. (Terms are separated by addition or subtraction signs.)

**Example:** **a)** 3*x* – 4 + + *y* hasfourterms: 3*x*, - 4 , and *y*.

**b)** 7*xyz* + 12 – *z*2 has threeterms: 7*xyz*, 12and – *z*2.

**Like terms:** the terms that have the same variables and exponents.

**Example:** 2*x* –3*y*2 + 5*x* + 9 + 4*y*2

Like terms: 2*x* and 5*x* The same variable: *x*

–3*y*2 and 4*y*2 The same variable raised to the same power:  *y*2

and 9 All constants are like terms.

**Evaluating Algebraic Expressions**

**Evaluating an algebraic expression:** substitute a specific value for a variable and perform the mathematical operations (+,, , ÷, etc.).

**Note:**

* + In algebra, a multiplication sign “×” is usually omitted to avoid confusing it with

the letter *x*.

* + If there is no symbol or sign between a number and letter, it means multiplication,

such as 3*x =* 3 ∙ *x .*

**Steps to evaluate an algebraic expression:**

* + Replace the variable(s) with number(s).
  + Calculate.

**Example:** Evaluate the following algebraic expressions.

1. 3*x* – 4, given *x* = 5.

3*x* – 4 = 3**5** – 4 Substitute *x* for 5.

= 15 – 4  Calculate.

= 11

1. + 8 given *x* = -9 and *y* = 3. Substitute *x* for -9 and *y* for 3.

+ 8 = + 8

= 5

1. 3*a* – 4 + 2, given *a* = 5.

3*a* – 4 + 2 = 3**5** – 4 + 2 Substitute *a*  for 5.

= 15 – 4 + 2 Calculate.

= 13

1. , given *x* = 1 and *y* = 9.

Substitute *x* for 1 and *y* for 9.

= Calculate.

= 6

**Topic B: Translating Words into Algebraic Expressions**

**Key Words in Word Problems**

**Identifying keywords:**

* When trying to figure out the correct operation(+, , , ÷, etc.) in the word problem it is important to pay attention to keywords (clues to what the problem is asking).
* Identifying keywords and pulling out relevant information that appear in the word problem are effective ways for solving mathematical word problems.

**Key or clue words in word problems**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Addition (+)** | **Subtraction ()** | **Multiplication (×)** | **Division (÷)** | **Equals to (=)** |
| add | subtract | times | divided by | equals |
| sum (of) | difference | product | quotient | is |
| plus | take away | multiplied by | over | was |
| total (of) | minus | double | split up | are |
| altogether | less (than) | twice | fit into | were |
| increased by | decreased by | triple | per | amounts to |
| gain (of) | loss (of) | of | each | totals |
| combined | (amount) left | how much (total) | goes into | results in |
| in all | savings | how many | as much as | the same as |
| greater than | withdraw |  | out of | gives |
| complete | reduced by |  | ratio /rate | yields |
| together | fewer (than) |  | percent |  |
| more (than) | how much more |  | share |  |
| additional | how long |  | average |  |

**Examples:**

1. Edward drove from Prince George to Williams Lake (235 km), then to Cache Creek (203 km) and finally to Vancouver (390 km). How many kilometers in ***total*** did Edward drive? 235km **+** 203 km **+** 390 km = 828 kmThe key word: total (+)
2. Emma had $150 in her purse on Friday. She bought a pizza for $15, and a pair of shoes for $35. How much money does she have ***left***?

$150 – 15 **–** 35 = $100 The key word: left (–)

1. Lucy received $950 per month of rent from Mark for the months September to November. ***How much*** rent in ***total*** did she receive?

$950 3 = $2850The key word: how much total (×)

1. Julia is going to buy a $7500 used car from her uncle. She promises to pay $500 ***per*** month, in how many months she can pay it off?

$7500 **÷** $500 = 15 monthThe key word: per (÷)

**Translating Phrases into Algebraic Expressions**

**Method to translate words into algebraic expression:**

* Look for basic key words for translating word problems from English into algebraic expressions.
* Translate English words into mathematical symbols (the language of mathematics).

**Translate words into algebraic expression:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Algebraic**  **expression** | **Word phrases** |  | **Algebraic expression** | **Word phrases** |
| **7 + *y*** | the **sum** of 7 and *y* | ***t* – 8** | 8 **less than** *t* |
| 7 **more than** *y* | *t* **decreased** (or reduced) by 8 |
| *y* **increased by** 7 | **subtract** 8 from *t* |
| 7 **plus** *y* | the **difference** between *t* and 8 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Algebraic**  **expression** | **Word phrases** |  | **Algebraic**  **expression** | **Word phrases** |
| **2*x* or 2 *x*** | the **product** of 2 and *x* | ***z* ÷ 3 or** | The **quotient** of *z* and 3 |
| 2 **multiplied** by *x* | *z* **divided** by 3 |
| **double** (or twice) of *x* | **One third** of *z* |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Algebraic**  **expression** | **Word phrases** |  | **Algebraic expression** | **Word phrases** |
| ***y*3** | The **third power** of *y* | **4*y* – 9** | 9 **less than** 4 **times** *y* |
| *y* **cubed** | **2(*t* – 5)** | **Twice** the **difference** of *t* and 5 |
| *y* raised to the **third power** |  | 1. **more than** the **quotient** of 2*x* by 3 |

**Note:**

* + The order of the subtraction and division is important when translate words into algebraic expression.
  + Place the numbers in the correct order for subtraction and division.

**Example**:

**1)** The difference between *t* and 7 means *t* **–** 7 not 7 – *t. t* appears first.

**2)** 8 less than t means *t* **–** 8 not 8 – *t.* 8 less than *t* not *t* lees than 8.

**3)** The quotient of z and 3 means not  **.** *z* appears first.

**Writing Algebraic Expressions**

**Example**: Write a mathematical equation for each of the following:

1. Five ***greater than*** four ***divided*** by a number ***is*** seventeen. **Equation**

5 + 4 **÷ *x* = 17** 5 + = 17

**(**Let *x* = a number)

1. A ***number is***7 ***times*** the number *y* ***added*** to 23.

*x* = 7  y + 23 *x* = 7*y* + 23

(Let *x* = a number)

**Example**: Write an algebraic expression for each of the following: **Expression**

1. The difference of *y* and 3.45. *y* – 3.45
2. The difference of and *w*. *w*
3. *z* less than the number 67. 67 – *z*
4. 27 minus the product of 18 and ***a number***. 27 – 18*x*

(Let *x* = a number)

1. The sum of ***a******number*** and 7 divided by 2.
2. Steve has $200 in his saving account. If he makes a deposit of

*x* dollars, how much in total will he have in his account? 200 + *x*

1. Ann weighs 150 pounds. If she loses *y* pounds, how much will

she weigh? 150 – *y*

1. A piece of wire 30 centimeters long was cut in two pieces and one piece

is *z* centimeters long. How long is the other piece? 30 – *z*

1. Alice made 3 dozen cupcakes. If it costs her *y* dollars, what was

her cost per dozen cupcakes? What was her cost per cupcake? ,

(1 dozen = 12 , 3

**Steps for Solving Word Problems**

**Steps for solving word problems:**

|  |
| --- |
| **Steps for solving word problems** |
| * **Organize** the **facts** given from the problem (create a **table** or **diagram** if it will make the problem clearer). * Identify and label the unknown quantity (**let *x* = unknown**). * Convert words into mathematical symbols, and **determine the operation** – write an **equation**(looking for ‘key’ or ‘clue’ words). * **Estimate**and**solve** the equation and find the solution(s). * **Check** and state the **answer**.   (Check the solution to the equation and check it back into the problem – is it logical?) |

**Example to illustrate the steps involved**

**Example:** William bought 5 pairs of socks for $4.35 each. The cashier charged him an additional $2.15 in sales tax. He left the store with a measly $5.15. How much money did William start with?

* Organize the facts (make a table):

|  |  |
| --- | --- |
| 5 socks | $4.35 each |
| Sales tax | $2.15 |
| Money left | $5.15 |

* Determine the unknown:How much did William start with? (***x* = ?)**
* Convert words into math symbols, and determine the operation (find ***key words***):
* The ***total*** cost without the sales tax: $4.35 **×** 5
* With an ***additional*** $2.15 sales tax: ($4.35 × 5) **+** $2.15
* William started with: *x* = [($4.35 × 5) + $2.15] **+ $5.15**
* Estimate and solve the unknown:
  + Estimate: *x* = [($4 × 5) + $2] + $5

= $27

* + Actual solution: *x* = [($4.35 × 5) + $2.15] + $5.15

= $29.05

* Check: If William started with $29.05, and subtract 5 socks for $4.35 each and sales tax in $2.15 to see if it equals $5.15.

$29.05 [($4.35 × 5) + $2.15] = $5.15

**√**

$29.05 – $23.9 = $5.15 Correct!

* State the answer: William started with $29.05.

**More examples:**

**Example:**  James had 96 toys. He sold 13 on first day, 32 on second day, 21 on third day, 14 on fourth day and 7 on the last day. What percentage of the toys were not sold?

* Organize the facts:

|  |  |
| --- | --- |
| James had | 96 toys |
| The total number of toys sold | 13 + 32 + 21 + 14 + 7 |
| The toys not sold | 96 – the total number of toys sold |

* Determine the unknown: Let *x* = percentage of the toys were not sold
* The total number of toys sold: 13 + 32 + 21 + 14 + 7 = 87
* The toys not sold: 96 – 87 = 9
* Percentage of the toys were not sold: *x* = = 9.4%
* State the answer: percentage of the toys were not sold.

**Example:** The 60-liter gas tank in Robert’s car is 1/2 full. Kelowna is about 390 km from Vancouver and his car averages 7 liters per 100 km. Can Robert make his trip to Vancouver?

* Let *x* = liters of fuel are required to get to Vancouver.
* The 60-liter gas tank in Robert’s car is 1/2 full:

60 L × = 30 L Robert has 30 liters gas in his car.

* Robert’s car averages 7 liters per 100 km, and Vancouver is about 390 km from Kelowna.

= Proportion:

(*x*)(100km) = (7 L) (390 km) Cross multiply and solve for *x*.

= = 27.3 LRobert needs 27.3 liters gas to get to Vancouver.

* State the answer: 30L > 27.3L Yes, Robert can make his trip.

**Topic C: Exponents and Order of Operations**

**Introduction to Exponents**

**Power:** the ***product*** of a number repeatedly multiplied by itself.

**Example**: 32 = 3 ∙ 3 = 9 , the “32” is the ***product*** of 3 repeatedly multiplied by itself.

**Exponent:** the ***number of times*** a number is multiplied by itself.

**Example**: In 3**2**,the “2” means 3 is multiplied by itself ***two times***.

**Base, exponent and power:**

**Exponential notation (exponential expression)**: *an* or BaseExponent

|  |
| --- |
| **Exponential notation Example** |
| Power Exponent    ***a*n** = *a ∙ a ∙ a ∙ a … a* 24 = 2 ∙ 2 ∙ 2 ∙ 2 = 16  Base  Read “*a* to the *n*th” Read “2 to the 4th.”  or “the *n*th power of *a*.” |

2 is repeatedly multiplied

by itself 4 times.

Exponents make it easier to write very long numbers (for multiplications).

**Any non-zero number to the zero power equals 1 (*a*0 = 1).** 00 is undefined.

**Example:** 20 = 1 , 130000 = 1

**Any number raised to the power of 1 equals the number itself (*a*1 = *a*).**

**Example**: 41 = 4 , 10001 = 1000

Anything raised to the first power is itself. (4 is multiplied by itself one time)

**1 raised to any power is still 1 (1n = 1).** **Example**: 13 = 1 , 110000 = 1

13 = 1 ∙ 1 ∙ 1 = 1

**Exponents: basic properties:**

|  |  |
| --- | --- |
| **Name Property Example** | |
| Zero exponent *a*0 | *a*0 = 1 (00 is undefined) = 1 , (2*xy*)0 = 1 |
| One exponent *a*1 | *a*1 = *a*  4.51 = 4.5 , = |
| 1*n* = 1 17 = 1 ,1389 = 1 |

**Read and Write Exponential Expressions**

**How to read exponent expressions:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Base Exponent** | **Repeated multiplication** | **Product** | **Read** |
| **32** | 3 *∙* 3 | 9 | 32  3 squared |
| **103** | 10 ∙ 10 ∙ 10 | 1000 | 103  10 cubed |
| **(0.2)2** | 0.2 *∙*  0.2 | 0.04 | (0.2)2 0.2 squared |
| **110** | 1 *∙* 1 *∙* 1 *∙* 1*∙* 1 *∙* 1 *∙* 1 *∙* 1 *∙* 1 *∙* 1 | 1 | 110 1 to the tenth |
|  |  |  | two thirds cubed |
| **100000** |  | 1 | 100000 10000 to the zero |
| ***y*5** | ***y ∙ y ∙ y ∙ y ∙ y*** | *y*5 | *y*5 *y* to the fifth |

**Example**: Write the following exponential expressions in expanded form.

**Exponential expressions Expanded form**

1. 64  6 ∙ 6 ∙ 6 ∙ 6*an = a ∙ a ∙ a* …
2. (-*x*)3 (-*x*) (-*x*) (-*x*)
3. (3*x*2*y*)2 (3*x*2*y*) (3*x*2*y*)

**Example**: Write each of the following in the exponential form.

**Expanded form Exponential notation**

* + - 1. (0.2) (0.2) (0.2)  (0.2)3
      2. (5*a*) (5*a*) (5*a*) (5*a*) (5*a*)4

**Example:** Evaluate (42)(33) (60 ) (91).

42 *∙* 33 *∙* 60 *∙* 91 = (4 *∙* 4) (3 *∙* 3 *∙* 3) (1) (9) *a*0 = 1 , *a*1 = *a*

= 16 *∙* 27 *∙* 1 *∙* 9 = 3888

**Example:** Write each of the following as a base with an exponent.

1. Six to the power of eight.68
2. *x* to the seventh power. *x*7
3. Eight cubed. 83

**Example:** Evaluate , given *x* = 2 and *y* = 9.

Substitute *x* for 2 and *y* for 9. = = 14 Calculate.

**Order of Operations**

**Basic operations:** addition, subtraction, multiplication, division, exponent, etc.

**The order of operations are the rules** of which calculation comes first in an expression (when doing expressions with more than one operation).

**Order of operations:**

|  |  |
| --- | --- |
| **Order of operations** | |
| 1. the brackets or parentheses (innermost first) | ( ) , [ ] , { } |
| 2. exponent (power) | *a****n*** |
| 3. multiplication and division (from left-to-right) | **×**  and  **÷** |
| 4. addition and subtraction (from left-to-right) | +and |

**Example:**  432 + 5 + **(*2 + 1*)** – 2= 4***3*2** + 5 + 3 – 2 ( ), *a*n

= ***4 9*** + 5 + 3 – 2 ×

= ***36 + 5*** + 3 – 2 +

= ***41 + 3*** – 2 +

= 44 – 2 –

= 42

**Memory aid - BEDMAS**

|  |  |  |  |
| --- | --- | --- | --- |
| **B** | **E** | **DM** | **AS** |
| **B**rackets | **E**xponents | **D**ivide or **M**ultiply | **A**dd or **S**ubtract |

**Grouping symbols:** if parentheses are inside one another, calculate the inside set first.

* + Parentheses ( ) are used in the inner most grouping.
  + Square brackets [ ] are used in the second higher level grouping.

**Example:**  43 + [5 + **(*2 + 1*)**] – 32 = 43 + **[*5 + 3*]** – 32 ( ), [ ]

= 43 + 8 – ***32*** *a*n

= ***4 3*** + 8 – 9 ×

= ***12 + 8*** – 9 +

= 20 – 9 –

= 11

**Unit 2: Summary**

**Introduction to Algebra**

**Basic algebraic terms**

|  |  |  |
| --- | --- | --- |
| **Algebraic term Description** | | **Example** |
| **Algebraic expression** | A mathematical phrase that contains numbers, letters, grouping symbols (parentheses) and arithmetic operations. | 5*x* + 2 , 3*a* + (4*b* 6), |
| **Constant** | A number. | *x* + ***2*** constant: 2 |
| **Variable** | A letter that can be assigned different values. | 3 – ***x*** variable: *x* |
| **Coefficient** | The number in front of a variable. | ***-6*** *x* coefficient:-6  *x* coefficient: 1 |
| **Term** | A term can be a constant, variable, or the product of a number and variable(s).  (Terms are separated by addition or subtraction signs.) | 3*x* + 13*y*2+ 7*xy*  Terms: 3*x,* , 13*y*2 , 7*xy* |
| **Like terms** | The terms that have the same variables and exponents. | 2*x* – *y*2 + 5*x* – 7 + 13*y*2  Like terms: 2*x* and 5*x*  -*y*2 and 13*y*2 , and -7 |

**Evaluating an algebraic expression:** substitute a specific value for a variable and perform the mathematical operations (+,, , ÷, etc.).

**To evaluate an expression:**

* + Replace the variable(s) with number(s).
  + Calculate.

**Key or clue words in word problems:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Addition**  **(+)** | **Subtraction**  **()** | **Multiplication**  **(×)** | **Division**  **(÷)** | **Equals to**  **(=)** |
| add | subtract | times | divided by | equals |
| sum (of) | difference | product | quotient | is |
| plus | take away | multiplied by | over | was |
| total (of) | minus | double | split up | are |
| altogether | less (than) | twice | fit into | were |
| increased by | decreased by | triple | per | amounts to |
| gain (of) | loss (of) | of | each | totals |
| combined | balance | how much (total) | goes into | results in |
| entire | (amount) left | how many | as much as | the same as |
| in all | savings |  | out of | gives |
| greater than | withdraw |  | ratio (of) | yields |
| complete | reduced by |  | percent |  |
| together | fewer (than) |  | share |  |
| more (than) | how much more |  | distribute |  |
| and | how many extra |  | average |  |
| additional | how long |  |  |  |

**Steps for solving word problems:**

|  |
| --- |
| **Steps for solving word problems** |
| * ***Organize*** the ***facts*** given from the problem (create a ***table*** or ***diagram*** if it will make the problem clearer). * Identify and label the unknown quantity (***let x* = *unknown***). * Convert words into mathematical symbols, and ***determine the******operation*** – write an ***equation*** (looking for ‘key’ or ‘clue’ words). * ***Estimate*** and**solve** the equation and find the solution(s). * ***Check*** and state the ***answer***.   (Check the solution with the equation and check it back into the problem – is it logical?) |

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**Exponent:** the ***number of times*** a number is multiplied by itself.

**Base, exponent and power:**

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|  |
| --- |
| **Exponential notation Example** |
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**Exponents: basic properties:**

|  |  |
| --- | --- |
| **Name Property** | |
| Zero Exponent *a*0 | *a*0 = 1 (00 is undefined) |
| One Exponent *a*1 | *a*1 = *a* |
| 1*n* = 1 |

**Order of operations:**

|  |  |
| --- | --- |
| **Order of operations** | |
| 1. the brackets or parentheses (innermost first) | ( ) , [ ] , { } |
| 2. exponent (power) | *a****n*** |
| 3. multiplication or division (from left-to-right) | **×**  and  **÷** |
| 4. addition or subtraction (from left-to-right) | +and |

**Memory aid - BEDMAS**

|  |  |  |  |
| --- | --- | --- | --- |
| **B** | **E** | **DM** | **AS** |
| **B**rackets | **E**xponents | **D**ivide or **M**ultiply | **A**dd or **S**ubtract |

**Grouping symbols:** if parentheses are inside one another, calculate the inside set first.

* + Parentheses ( ) are used in the inner most grouping.
  + Square brackets [ ] are used in the second higher level grouping.

**Unit 2: Self-Test**

**Introduction to Algebra**

**Topic A**

1. Identify the constant, coefficient and the variable:
2. 2*x* – 3
3. -4*t* + 13 +
4. Identify the terms for each of the following:

**a)** 5*x* + 3 – *y*

**b)** 2*r* + 16r2 – *r* + 1

1. Identify the like terms in the following expressions:

**a)** 7+2*y*2 + 5*x* 1 + 13*y*2

**b)** 0.6*t* + 9*uv* −7*t* + 1.67*uv*

1. Evaluate the following algebraic expressions.
2. 7*x* – 4 + 13*x*, given *x* = 4.
3. , given *a* = 10 and *b* = 5.

**Topic B**

1. Write an expression/equation for each of the following:
2. The product of ten and *y.*
3. The quotient of *t* and six.
4. The difference between fifteen and a number more than

the quotient of three by seven is six.

1. Seven less than six times a number is fifteen.
2. Write an expression for each of the following:
3. Susan has $375 in her checking account. If she makes a deposit

of *y* dollars, how much in total will she have in her account?

1. Mark weighs 175 pounds. If he loses *y* pounds, how much

will he weigh?

1. A piece of wire 45 meters long was cut in two pieces and

one piece is *w* meters long. How long is the other piece?

1. Emily made 4 dozen muffins. If it cost her *x* dollars, what was

her cost per dozen muffins? What was her cost per muffin?

**Topic C**

1. **a**) In *x*3, the base is ( ).

**b**) In y4, the exponent is ( ).

1. Write the following exponential expressions in expanded form.
2. 93
3. (-*y*)4
4. (0.5*a*3b)2
6. Write each of the following in the exponential form.
7. (0.06) (0.06) (0.06) (0.06)
8. (12*y*) (12*y*) (12*y*)
10. Evaluate (32) (24)(230 )(101).
11. Write each of the following as a base with an exponent.
12. *y* to the eighth power.
13. Five cubed.
14. Evaluate the following:
15. , if *a* = 1 and *b* = 3.
16. 8*xy* + 7*y*4 , if *x* = and *y* = 1.
17. Calculate the following:
18. 243 + 7 – (4 + 3) + 5
19. 57 + [11 + (4 – 3)] + 42
20. 