**Unit 13**

**More about Polynomials**

**Topic A: Adding and subtracting polynomials**

* Polynomials review
* Adding and subtracting polynomials

**Topic B: Multiplication of polynomials**

* Multiplying polynomials
* Special binomial products

**Topic C: Polynomial division**

* Dividing polynomials
* Long division of polynomials
* Missing terms in long division

**Unit 13 Summary**

**Unit 13 Self-test**

**Topic A: Adding and Subtracting Polynomials**

**Polynomials Review**

**Review of basic algebraic terms:**

|  |  |  |
| --- | --- | --- |
| **Algebraic term Definition** | | **Example** |
| **Algebraic expression** | A mathematical phrase that contains numbers, variables, and arithmetic operations (+, *–* , etc.). | 5*x* + 2  3*a* – (4*b* + 6)  – *z*2 + 11 |
| **Constant** | A number. | *x* + **2** constant: 2 |
| **Variable** | A letter that can be assigned different values. | 3 – ***x*** variable: *x* |
| **Coefficient** | The number that is in front of a variable. | **-6** *x* coefficient:-6  *xz*3 coefficient: 1 |
| **Term** | A term can be a constant, a variable, or the product of a number and variable(s).  (Terms are separated by a plus or minus sign.) | 3*x* + 13*y*2+ 73*xy*  Terms: 3*x,*  , 13*y*2 , 73*xy* |
| **Like terms** | The terms that have the same variables and exponents. | 2*x y*2 + 5*x* – 7 + 13*y*2  Like terms: 2*x* and 5*x*  - *y*2 and 13*y*2 , and -7 |
| **Factor** | A number or variable that multiplies with another.  A number or expression can have many factors. | 24 = 2 3 4 factors: 2, 3, 4  5*xy* = 5 *x* *y* factors: 5, *x, y* |

|  |  |  |
| --- | --- | --- |
| **Poly**nomial | **Example** | **Coefficient** |
| **Mo**nomial(one term) | 7*a* | 7 |
| **Bi**nomial(two terms) | 3*x* – 5 | 3 |
| **Tri**nomial(three terms) | -4*x*2 + *xy +* 7 | -4, 1 |
| **Poly**nomial (one or more terms) | 2*pq* + 4*p*3 + 11 | 2, 4, 1 |

**The degree of a term with more variables:** the sum of the exponents of its variables.

**Example:** -3*x***3** *y***5** *z***2** degree: 3 + 5 + 2 = 10

**The degree of a polynomial with more variables:** the highest degree of any individual term.

**Example:** 4*ab*3 + 3*a***2***b***2***c***3** – 5*a* + 1 degree: 72 + 2 + 3 = 7

4 7 1

**Additive (or negative) inverse or opposite:** the opposite of a term (two terms whose sum is 0).

**Example**:  **1)**  The additive inverse of **5** is -5 5 + (-5) = 0

**2)** The additive inverse of  ***y*** is *y*  - + = 0

**3)** The additive inverse of **4*ab*3 – 3*a*2 +*b*3** is -4*ab*3 + 3*a*2 – *b*3

**Adding and Subtracting Polynomials**

**Add or subtract polynomials:**

**Example**: Add 4*x*3 – 5*x*2 – *x* + 3 and 3*x*3 + 3*x*2 – 5*x* + 2.

**Steps Solution**

(**4*x*3 – 5*x*2 – *x* + 3) + (3*x*3 + 3*x*2 – 5*x* + 2)**

* Regroup like terms: = (4*x*3 + 3*x*3) + (-5*x*2 + 3*x*2) + (-*x* – 5*x*) + (3 + 2)
* Combine like terms: = 7*x*3 – 2*x*2 – 6*x* + 5

**Example**: Subtract 6*x*2 + 7*x* – 5 and 3*x*2 – 4*x* + 16.

**Steps Solution**

**(6*x*2 + 7*x* – 5) – (3*x*2 – 4*x* + 16)**

* Remove parenthesis: = 6*x*2 + 7*x* – 5 **–** 3*x*2 + 4*x* **–** 16

(Reverse each sign in second parenthesis.)

* Regroup like terms: = (6*x*2 – 3*x*2) + (7*x* + 4*x*) + (-5 – 16)
* Combine like terms: = 3*x*2 + 11*x* – 21

**Add or subtract polynomials using the column method:**

**Example**: Add 4*x*3 – 3*x*2 + 6*x* – 5 and 3*x*3 + 2*x* + 3.

**Steps Solution**

* Line up like terms in columns: **4*x*3 – 3*x*2 + 6*x* – 5**
* Add: **+) 3*x*3  + 2*x* + 3**

Leave spaces for missing terms. 7*x*3 – 3*x*2 + 8*x* – 2

**Example**: Subtract (7*x*2 – 3*x* + 4) and (3*x*2 – 1).

**Steps Solution**

* Line up like terms in columns: **7*x*2 – 3*x* + 4** Subtrahend
* Change signs in the minuend **+) -3*x*2  + 1** Minuend

and **add**: 4*x*2 – 3*x* + 5 Difference

Or (7*x*2 – 3*x* + 4) – (3*x*2 – 1) = 7*x*2 – 3*x* + 4 – 3*x*2 + 1

= 4*x*2 – 3*x* + 5

**Topic B: Multiplication of Polynomials**

**Multiplying Polynomials**

**Multiplying monomials**

**Example**: **(-4*a*2 *b*3) (5*a*3 *b*5)** = (-4 ∙ 5) (*a*2 *∙ a*3) (*b*3 *∙ b*5) Multiply the coefficients and add the exponents.

= -20 *a*5 *b*8 *am* *an* = *am+n*

**Multiplying monomial and polynomial**

**Example**: **5*x*2 (4*x*3 – 3*x*)** = (5*x*2) (4*x*3) **–** (5*x*2) (3*x*) Distributive property: *a* (*b* + *c*) = *ab* + *ac*

= (5∙4) (*x*2+3) **–** (5∙3) (*x*2+1) Multiply the coefficients and add the exponents.

= 20*x*5 – 15*x*3 *am* *an* = *am+n*

**Example**: **3*xy*3 (4*xy*2 *+* *x*3 *y* – *y*)** Distribute

= (3*xy*3) (4*xy*2) *+* (3*xy*3) (*x*3*y*) + (3*xy*3) (-*y*) Multiply the coefficients and add the exponents.

= 12*x*2*y*5 *+* 3*x*4*y*4 – 3*xy*4 *am* *an* = *am+n*

**Multiplying binomials (2 terms × 2 terms)**

**Example**: Find the following product.

F O I L

**(4*a* – 5) (2*a* – 3)** = 4*a* ∙ 2*a* + 4*a* (-3)– 5 ∙ 2*a* – 5 (-3) FOIL

= 8*a*2 **–** 12*a* **–** 10*a* + 15 *am* *an* = *am+n*

= 8*a*2 – 22*a* + 15 Combine like terms.

**Multiplying binomial and polynomial**

**Example**: Multiply: 2*x* – 3*x*2 and *x*2 + *x* – 4

**Steps Solution**

**(2*x* – 3*x*2) (*x*2 + *x* – 4)**

* Use the distributive property: = 2*x* ∙ *x*2 + 2*x* ∙ *x* + 2*x* (-4) – 3*x*2 ∙ *x*2 – 3*x*2 ∙ *x* – 3*x*2 (-4)
* Multiply coefficients and add exponents: = 2*x*3 + 2*x*2 – 8*x*– 3*x*4– 3*x*3 + 12*x*2
* Combine like terms and write in descending order: = - 3*x*4 – *x*3 + 14*x*2 – 8*x*

**Multiplying polynomials mentally** (no need to write out each step).

**Example**: Multiply.

1. **2*x*3 (3*x*2 – 2)** = 6*x*5 – 4*x*3  *a* (*b* + *c*) = *ab* + *ac* , *an am = a n+ m*
2. **(*a* – 3) (2*a* – 1)** = 2*a*2 – 7*a* + 3 FOIL

**Special Binomial Products**

**Special binomial products – squaring binominals**

(*a = x , b =* 3 )

|  |  |  |  |
| --- | --- | --- | --- |
| **Special products** | **Formula** | **Initial expansion** | **Example** |
| Difference of squares | **(*a* + *b*) (*a* – *b*) = *a*2 – *b*2**  It does not matter if (*a* – *b*) comes first | (*a* + *b*) (*a* – *b*) = *a*2 – *ab* + *ba* – *b*2 = *a*2 – *b*2 | (*x* + 3) (*x* – 3) = *x*2 –32 = *x*2 – 9  or (*x –* 3) (*x +* 3) *= x2 –*32 *= x2 –* 9 |
| Square of sum | **(*a* + *b*)2 = *a*2 +2*ab + b*2**  A perfect square trinomial | (*a* +*b*)2 = (*a* + *b*) (*a* + *b*)  = *a*2 + *ab* + *ba* + *b*2  = *a*2 +2*ab + b*2 | (*y* + 2)2 = *y*2 +2 ∙ *y ∙* 2 *+* 22  = y2 +4*y +* 4 |
| Square of difference | **(*a* – *b*)2 = *a*2 –2*ab + b*2**  A perfect square trinomial | (*a* –*b*)2 = (*a* – *b*) (*a* – *b*)  = *a*2 –*ab* – *ba* + *b*2  = *a*2 –2*ab + b*2 | (*z* **–** 5)2 = *z*2 **–**2 ∙ *z ∙* 5 *+* 52  = *z*2 **–** 10*z +* 25 |

**Special binomial products:**  special forms of binomial products that are worth memorizing.

**Memory aid:** (*a* ± *b*)2 = (*a*2 ± 2*ab + b*2) Notice the reversed plus or minus sign in the second term.

**Example**: Find the following products.

*a b*

*b b*

1. **(5*x* + 3) (5*x* – 3)** = (5*x*)2 –32(*a* **+** *b*) (*a* – *b*) = *a*2 – *b2*

= 25*x*2 –9 *a* = 5*x , b* = 3

1. **(2*t* –2** *=* (2*t*)2 – 2(2t) + 12 (*a* – *b*)2 = *a*2 –2*ab* + *b*2

= 4*t*2 –4*t* + 1 *a* = 2*t , b* = 1

1. **(3*w* + )2**= (3*w*)2 + 2(3*w*) (*+* (2 (*a* **+** *b*)2 = *a*2 **+** 2*ab* + *b*2

= 9*w*2 + 2*w* + *a* = 3*w , b* =

1. **(5*u* – )2**= (5*u*)2 – 2(5*u*) (*+* (2 (*a* – *b*)2 = *a*2 –2*ab* + *b*2

= 25*u*2 *–* 5*uv* +2 *a* = 5*u , b* =

**5)** **(*t* – ) (*t* + )** = (*t* )2 –)2(*a* **+** *b*) (*a* – *b*) = *a*2 – *b2*

*=*  *t*2 – *a* =  *, b* =

**Topic C: Polynomial Division**

**Dividing Polynomials**

**Dividing a monomial by a monomial**

**Example**: Apply  *= am-n*

**Example**:

**Steps Solution**

* Divide the coefficients:
* Divide like variables (apply  *= am-n*):  *= v* 2 - 5= *v* -3

*a* -m *=*

**Dividing a polynomial by a monomial**

**Example**:

**Steps Solution**

* Split the polynomial into three parts:
* Divide a monomial by a monomial: = 3*a* + 1

**Example**:

**Steps Solution**

* Group:
* Factor out the greatest common factor (GCF):
* Split the polynomial into two parts: =
* Divide a monomial by a monomial: = 4*x* + 2 = 2 (2*x* + 1)

**Long Division of Polynomials**

**Long division for numbers:**

**Example**:

Quotient 7

Divisor) Dividend 3 )22

21

  Remainder 1

**Polynomial long division:** a method used for dividing a polynomial by another polynomial of the same or lower degree (it is very similar to long division for numbers).

**Example**:

**Steps Solution Long division for numbers**

* Write in ***divisor*** *)* ***Dividend*** form: **2*x* )**  2) 481

2*x* 2

* Divide the first term: 2*x* ) 2) 481

–4*x*2 (2*x*)(2*x*) = 4*x*2 –4 2 ∙ 2 = 4

2*x +* 4240

* Divide the second term: 2*x* ) 2)481

4*x*2 4

Bring 8*x* down 8*x* 8 Bring 8 down

(2*x*)(4) = 8*x* – 8*x* – 8 2 ∙ 4 = 8

1 1

Remainder

* Quotient = 2*x* + 4, remainder = 1
* Tip: continue until the degree of the remainder is less than the degree of the divisor.

(i.e. 1 = 1 ∙ *x***0**  and 2*x* = 2*x***1**,0 < 1) Quotient Divisor) Dividend

* Check: Dividend = Quotient ∙ Divisor + Remainder   Remainder

**?**

= (2*x* + 4) (2*x*) + 1 Distribute

**√**

= 4*x*2 + 8*x* + 1 Correct!

**Missing Terms in Long Division**

If there is a missing consecutive power term in a polynomial (i.e. if there are *x*3 and *x* but

not *x*2), add in the missing term with a coefficient of 0.

**Example**:

**Steps Solution**

* Rewrite both polynomials in descending order:

Descending order: ,

* Write in **divisor ) Dividend** form and insert *x* + 1 )

a 0 coefficient for the missing power term. Missing power

* Divide as usual: ***x* + 1 )**  (*x*2)(*x*) = *x*3

) (*x*2)(1) = *x*2

(-5*x*)(*x*) = -5*x*2

(-5*x*)(1) = -5*x*

5*x*  + 7 (5)(*x*) = 5*x*

) 5*x* + 5 (5)(1) = 5

2

* Quotient = , remainder = 2

* Check: Dividend = Quotient **∙** Divisor + Remainder Quotient Divisor) Dividend

  Remainder

Distribute

**?**

Combine like terms.

**√**

Correct!

**Unit 13: Summary**

**More about Polynomials**

**Basic algebraic terms:**

|  |  |
| --- | --- |
| **Algebraic term Definition** | |
| **Algebraic expression** | A mathematical phrase that contains numbers, variables, and arithmetic operations. |
| **Constant** | A number. |
| **Variable** | A letter that can be assigned different values. |
| **Coefficient** | The number that is in front of a variable. |
| **Term** | A term can be a constant, a variable, or the product of a number and variable(s).  (Terms are separated by a plus or minus sign.) |
| **Like terms** | The terms that have the same variables and exponents. |
| **Factor** | A number or variable that multiplies with another. |

|  |  |
| --- | --- |
| **Poly**nomial | **Description** |
| **Mo**nomial | One term. |
| **Bi**nomial | Two terms. |
| **Tri**nomial | Three terms. |
| **Poly**nomial | One or more terms. |

**The degree of a term with more variables:** the sum of the exponents of its variables.

**The degree of a polynomial with more variables:** the highest degree of any individual term.

**Additive (or negative) inverse or opposite:** the opposite of a term.

**Add or subtract polynomials:**

* Regroup like terms.
* Combine like terms.

**Add polynomials using the column method:**

* Line up like terms in columns.
* Add.

**Subtract polynomials using the column method:**

* Line up like terms in columns.
* Change signs in minuend and add.

**Multiplying binomial and polynomial:**

* Use the distributive property. *a* (*b* + *c*) = *ab* + *ac*
* Multiply coefficients and add exponents. Apply  *= am-n*
* Combine like terms and write in descending order.

**Special binomial products – squaring binominals**

|  |  |
| --- | --- |
| **Special products** | **Formula** |
| Difference of squares | (*a* + *b*) (*a* – *b*) = *a*2 – *b*2 |
| Square of sum | (*a* + *b*)2 = *a*2 +2*ab + b*2 |
| Square of difference | (*a* – *b*)2 = *a*2 –2*ab + b*2 |

Memory aid: (*a* ± *b*)2 = (*a*2 ± 2*ab + b*2)

**Dividing a monomial by a monomial**

* Divide coefficients.
* Divide like variables (apply  *= am-n*).

**Dividing a polynomial by a monomial**

* Split the polynomial into parts.
* Divide a monomial by a monomial.

**Polynomial long division:** a method used for dividing a polynomial by another polynomial of the same or lower degree (it is very similar to long division for numbers).

**Example**:

**Steps Solution**

* Rewrite both polynomials in descending order:

Descending order: ,

* Write in **divisor ) Dividend** form and insert *x* + 2 )

a 0 coefficient for the missing power term. Missing power

* Divide as usual: *x* + 2 ) (*x*2)(*x*) = *x*3

) (*x*2)(2) = 2*x*2

(-2*x*)(*x*) = -2*x*2

(-2*x*)(2) = -4*x*

*x* + 8 (1)(*x*) = *x*

) *x* + 2 (1)(2) = 2

6

* Quotient = , remainder = 6
* Tip: continue until the degree of the remainder is less than the degree of the divisor.
* Check: Dividend = Quotient ∙ Divisor + Remainder Quotient Divisor) Dividend

  Remainder

**Unit 13: Self-Test**

**More about Polynomials**

**Topic A**

1. Determine the degree of the following.
2. -8*x*4 *y*3 *z*5
3. 21*x*5*y* + 32*x*2*y*3*z* + 6*x*3*y*4*z*2
4. 3.5*a*4*b* + 6.1*a*4*b*3*c* – 7.3*a* + 5.4
5. Determine the additive inverse.
6. 8y
7. *x*
8. 9*xy*2 – 4*x*2 +*y*3
9. Add 5*x*4 – 3*x*3 – *x* + 7 and 4*x*4 + 2*x*3 – 7*x* + 3.
10. Subtract 8*x*2 + 5*x* – 4 and 4*x*2 – 2*x* + 14.
11. Add or subtract polynomials using the column method:

**a)** Add 7*a*3 – 4*a*2 + 3*a* – 6 and 4*a*3 + 6*a* + 8.

**b)** Subtract (9*x*2 – 4*x* + 8) and (4*x*2 – 3).

**Topic B**

1. Multiply.
2. (-6*x*3 *y*2) (4*x*4 *y*3)
3. 4*a*2 (3*a*4– 6*a*)
4. 7*xy*2 (2*xy*4 *+* *x*3 *y* – 3*y*)
5. (3*x* – 4) (4*x* – 5)
6. (3*a* – 2*a*2) (*a*2 + *a* – 5)
7. Find the following product.
8. 4*t*4 (2*t*3 – 5)
9. (*x* – 5) (3*x* – 2)
10. (6*a* + 5) (6*a* – 5)
11. (3*w* –2
12. (5*u* + )2
13. (6*x* – )2
14. **(*z*** – **) (*z* + )**

**Topic C**

1. Divide the following.


5. Use long division to divide the following.