**Unit 14**

**Factoring Polynomials**

**Topic A: Factoring**

* Highest / greatest common factor
* Factoring polynomials by grouping
* Factoring difference of squares

**Topic B: Factoring trinomials**

* Factoring *x*2 + *b x* + *c*
* Factoring *ax*2 + *b x* + *c*
* More on factoring *ax*2 + *b x* + *c*
* Factoring trinomials: AC method
* Factoring special products

**Topic C: Application of factoring**

* Quadratic equations
* Solving quadratic equations
* Application of quadratic equations

**Unit 14 Summary**

**Unit 14 Self-test**

**Topic A: Factoring**

**Highest / Greatest Common Factor**

**Factoring whole numbers**: write the number as a product (multiply) of its prime factors.

**Prime factor**: it is a prime number that has only two factors, 1 and itself.

**Example**: Factor 42.

**42** = 2 ∙ 3 ∙ 7 2, 3 and 7 are prime factors.

**Common factor:** a number or an expression that is a factor of each term of a group of terms.

**Greatest / highest common factor (GCF or HCF):** the product of the common factors.

**Examples**:

2 ∙ *x ∙* *y*2= 2*xy*2

2 ∙ 3 = 6

|  |  |
| --- | --- |
| **Expression** | **Factors Common factor GCF or GCF** |
| 30  42 | **2 ∙ 3** ∙ 5 2, 36  **2 ∙ 3** ∙ 7 |
| 2*xy*3  6*xy*2 | **2** ∙ ***x*** *∙**y**∙* ***y***22, *x , y*22*xy*2  **2** ∙ 3 ∙ ***x*** *∙* ***y*2** |

**Factoring a polynomial**: express a polynomial as a product of other polynomials. (Factoring is the reverse of multiplication.)

Multiplying (or expanding) Distributive property.

(*a* + *b*) *c* = *ac* + *bc*

Factoring The common factor is *c.*

**Example**:

**Multiplying Factoring GCF or HCF**

3*xy* (2*x* – 4*xy* + 3) 6*x*2*y* –12*x*2*y*2 *+* 9*xy*

= 6*x*2*y* –12*x*2*y*2 *+* 9*xy =* **3*xy***(2*x*) – **3*xy*** (4*xy*)+ **3*xy*** ∙ 3 3*xy*

*=* **3*xy***(2*x* – 4*xy* + 3)

**Examples**

|  |  |  |
| --- | --- | --- |
| **Expression** | **Factoring** | **GCF or HCF** |
| 6*a*2 – 9*a* | **3*a*** *∙* 2*a* **–3*a*** *∙* 3 *=* **3*a***(2*a* **–**3) | 3*a* |
| 4*x*4*y* + 12*x*3*y* – 16*xy* | **4*xy*** *∙ x*3 + **4*xy*** *∙* 3*x*2– **4*xy****∙* 4 = **4*xy*** (*x* 3 + 3*x*2 – 4) | 4*xy* |
| 13*z*2 (*z* + 2) *–* (3*z* + 6) | 13*z*2 **(*z* + 2)** *–* 3**(*z* + 2)** = **(*z*+ 2)** (13*z*2 *–* 3) | *z* + 2 |
| 2 – 2 + | –2 + ***w*** *∙* 1 = – 2 + 1) |  |
| - 5*x*4 – 10*x*2 + 15*x* | **-5*x*** *∙ x* 3**–** **5*x*** *∙* 2 *x*+ (-**5*x*)***∙* (-3) = **- 5*x***(*x* 3 + 2*x* – 3) | -5*x* |

**Tips:** - Factor each term and pull out the GCF.

* If the first term is negative, factor out a negative GCF to make the first term positive.

**Factoring Polynomials by Grouping**

**Steps for factoring polynomials by grouping**:

**Steps Example**

Factor **16*x*2 – 4*x* + 28*x* – 7**.

* Regroup terms with the GCF. 16*x*2 – 4*x* + 28*x* – 7 = (16*x*2 – 4*x*) + (28*x* –7)
* Factor out the GCF from each group. = 4*x* **(4*x* –1)** + 7**(4*x* –1)**
* Factor out the GCF again from last step. *=* (4*x*– 1) (4*x +* 7)

**Factoring completely:** continue factoring until nofurther factors can be found.

**Example**: Factor the following completely.

1. **35*xy*2 – 7*x*2*y* + 5*y* – *x*** *=* (35*xy*2 – 7*x*2*y*) + (5*y* – *x*) Regroup terms with the GCF.

*=* **7*xy***(5*y* – *x*) + (5*y* – *x*) 1 Factor out 7*xy*.

*=* (5*y* – *x*) (7*xy* + 1) Factor out (5*y* – *x*).

1. **3*xy*+ *yz* – 5*yz* + 6*xy*** *=*(3***xy***+ 6***xy***) + (***yz***– 5***yz***) Regroup.

*=* **3*xy*** (1 + 2) + ***yz*** (1 – 5) Factor out the GCF.

*=*3*xy* (3) + *yz* (-4) Simplify.

*=*9*xy* – 4*yz*

1. ***t*3 – *t*2*w* – *tw*2 + *w*3** *=*(***t***3 – ***t***2*w*) – (*t****w***2 –***w***3) Regroup.

*=* ***t*2**(*t* – *w*) – ***w*2**(*t* – *w*) Factor out (*t* – *w*).

*=* **(*t* – *w*)** (*t*2– *w*2) Apply *a*2 – *b*2 = (*a* + *b*) (*a* **–** *b*)

*=* **(*t* – *w*)** (*t* + *w*) **(*t* – *w*)**

*=*(*t* – *w*)2 (*t* + *w*)

**Tip**: Identify patterns of common factors such as 5*y* **–** *x*, *t – w* …

**Factoring Difference of Squares**

**Factoring difference of squares:**

|  |  |
| --- | --- |
| **Formula** | **Example** |
| ***a2* – *b*2 = (*a + b*) (*a*** – ***b*)**  or***a2* – *b*2 = (*a*** – ***b*) (*a*** + ***b*)** | *x*2 – 49 = *x*2 – 72 = (*x +* 7) (*x –* 7)  *y*2 – 81 = *y*2 – 92 = (*y –* 9) (*y +* 9) |

**Note:** - *a*2 + *b*2 cannot be factored.

* Always factor out the greatest common factor (GCF) first.
* Determine the perfect square or the square root of each term.

Recall that **factoring is the reverse of multiplication**.

Factoring

a2 – *b*2 = (*a + b*) (*a* – *b*)

Multiplying

**Example**: Factor the following completely.

1. **2*x*2 – 18** = 2 (*x*2 – 9)Factor out 2.

= 2 (*x*2 – 32)9 = 32 or

= 2 (*x +* 3) (*x –* 3) *a2 – b*2 *=* (*a + b*) (*a – b*):  *a* = *x, b* = 3

1. **1 – 64*u*2** = 12 – 82 *u*2 1 = 12 , 64 = 82 or

= 12 – (8*u*)2 *a*n *b* n =(*a b*) n

= (1*+* 8*u*) (1 *–* 8*u*) *a2 – b*2 *=* (*a + b*) (*a – b*):  *a* = 1*, b* = 8*u*

1. **100*t*2 – 256** = 102*t*2 – 162 256 = 162 or

= (10*t*)2 – 162 *a*n *b* n =(*a b*) n

= (10*t +* 16) (10*t –* 16) *a2 – b*2 *=* (*a + b*) (*a – b*):  *a* = 10*t, b* = 16

*a b*

1. **9*x*2 – 16*y*2** = 32*x*2 – 42*y*2 = (**3*x***)2 – (**4*y***)2  *a*n *b*n =(*a b*)n

= (3*x +* 4*y*) (3*x –* 4*y*) *a2 – b*2 *=* (*a + b*) (*a – b*):  *a* = 3*x, b* = 4*y*

1. ***x*8 – 4**=62 *(x*4)2 – 0.22 0.04 = 0.22 or , *x*8 =(*x*4) 2

= (6*x*4)2 – 0.22 *a*n *b*n =(*a b*) n

= (6*x*4 *+* 0.2) (6*x*4 *–* 0.2) *a2 – b*2 *=* (*a + b*) (*a – b*):  *a* = 6*x*4*, b* = 0.2

**Topic B: Factoring Trinomials**

**Factoring *x*2 + *b x* + *c***

**Factoring *x*2 + *b x* + *c*: cross-multiplication method**

StepsStandard form **Example**

***x*2 + *b x* + *c* *x*2 + 7*x* + 12**

* Setting up two sets of parenthesis. = ( ) ( ) = ( ) ( )

*x*2 + *b x* + *c* *x*2 + 7*x* + 12

* Factor the first term *x*2: *x*2 = *x ∙ x x* *c*1 *x* 3
* Factor the last term *c* (by trial and error): *c = c*1*∙* *c*2  *x* *c*2  *x* 4

*x ∙ x* = *x*2   *c*1*∙ c*2 = *c*  *x ∙ x =* *x*2  3 ∙ 4 = 12

* Cross multiply and then add up to the middle term. (*c*1)(*x*) + (*c*2)(*x*) = *b x* 3 ∙ *x* + 4 ∙ *x* = 7*x*
* Complete the parenthesis with *x + c*1 and *x* + *c*2. *x*2 + *b x* + *c* *x*2 + 3*x* + 2

**= (*x + c*1) (*x + c*2)** =(*x +* 3) (*x +* 4)

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* Check using FOIL. (*x +* 3) (*x +* 4) = *x*2 + 4*x* + 3*x* + 12

(*x +* 3) (*x +* 4) = *x*2 + 7*x* + 12 √

Check: *-*5 + (-3) = -8√

|  |  |
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| **Factoring *x*2 + *b x* + *c* using the cross-multiplication method** | |
| **In general**  ***x*2 + *b x* + *c* =** ( ) ( )  *x**c*1  *x**c*2  *x ∙ x* = *x*2  *c*1 *∙ c*2 *= c*  ?  (*c*1)(*x*) + (*c*2)(*x*) = *b x*  *x*2 + *b x* + *c =* **(*x + c*1)(*x + c2*)** | **Example**  ***x*2** – **8 *x* + 15 =** ( ) ( )  *x* -5  *x* -3  *x ∙ x* = *x*2  (-3)(-5) = 15  ?  (*-*5) *x* + (-3) *x* = -8*x*  yes!  *x*2 – 8*x* +15 = (*x* –5) (*x* –3) |

**Tips:** - Cross multiply and then add up to the middle term.

- Write the factors with their appropriate signs (+ or –) to get the right middle term.

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| **Summary: Factoring *x*2 + *b x* + *c*** | **Example: *x*2** – **8*x* + 15** | |
| *x*2 + (*c*1 + *c*1) *x* + *c*1*c*2 = (*x + c*1) (*x + c*2)  *x* *c*1  ?  *x* *c*2  Check: *c*1*x* + *c*2 *x* = *b x* | | *x*2 + [-5 + (-3)] *x* + 15 = (*x* –5) (*x* –3)  *x* -5  ?  *x* -3  Check: *-*5*x* + (-3*x*) = -8*x* yes! |

**Example**: Factor the following:  **Trial and error process**

1. ***a*2** – **11*a* + 30** = ( ) ( ) *a*2 – 11*a* + 30  *a*2 – 11*a* + 30

***a*** -5 *a* 3 *a* 6

*a* -6 *a* 10 *a* 5

*a ∙ a =* *a*2 (-5)(-6) = 30

?

?

Check: *-*5 + (-6) = -11√

?

(*-*5)*a* + (-6) *a* = -11*a* yes! 3*a* + 10*a* = -11*a*  no 6*a* + 5*a* = -11*a* no

Answer: *a*2 – 11*a* + 30 = (*a* – 5) (*a* – 6)

**2)**3***x*2 + 24*x*** – **27 = 3(*x*2 + 8*x*** – **9)**  *x*2  + 8*x* – 9 *x*2 + 8*x* – 9

*x* - 1 *x* 3 *x* -3

*x*9 *x*  -3 *x* 3

?

?

?

*x ∙ x =* *x*2 (-1)(9) = -9

(-1) *x* + 9*x* = 8*x* yes!Check: *-*1 + 9 = 8√3*x*+ (-3) *x* = 8*x* no (-3)*x* + 3*x* = 8*x* no

Answer:3(*x*2 + 8*x* – 9) = 3(*x* – 1) (*x* + 9)

**Note:** Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form (*ax*2 + *b x* + *c*)first.

**Factoring *ax*2 + *b x* + *c***

**Procedure for factoring *ax*2 + *b x* + c using the cross-multiplication method:**

StepsIn general **Example**

***ax*2 + *b x* + *c* 3*x*2** – 2*x***– 8**

* Setting up two sets of parenthesis. = ( ) ( ) = ( )( )

*ax*2 + *b x* + *c* 3*x*2 – 2*x* –8

* Factor the first term *ax*2: *ax*2 = *a*1*x ∙ a*2*x a*1*x* *c*1 *x* -2
* Factor the last term *c* (by trial and error): *a*2*x* *c*2 3*x* 4

*c = c*1 *∙ c*2  *a*1*x ∙ a*2*x* = *ax*2   *c*1 *∙* *c*2 *= c*  3*x*2 = *x ∙* 3*x*  -8 = -2 ∙ 4

* Cross multiply and then add up to the middle term. *c*1 (*a*2*x*) + *c*2 (*a*1*x*) = *b x* (-2)(3*x*) + 4 (*x*) = -2*x*
* Complete the parenthesis with (*a*1*x + c*1) and (*a*2*x* + *c*2). *ax*2 + *b x* + *c* 3*x*2 – 2*x* –8

= **(*a*1*x + c*1) (*a*2*x + c*2)** =(*x* –2) (3*x +* 4)

F O I L

* Check using FOIL. (*x* – 2) (3*x +* 4) = 3*x*2 + 4*x* – 6*x* – 8

√

(*x* – 2) (3*x +* 4) = 3*x*2 – 2*x* – 8

(Original expression)

**Tip:** Write the factors with their appropriate signs (+ or –) to get the right middle term.

|  |  |
| --- | --- |
| **Factoring *ax*2 + *b x* + *c* using the cross-multiplication method** | |
| **In general**  ***ax*2 + *b x* + *c* =** ( ) ( )  *a*1*x**c*1  *a*2*x**c*2  *a*1*x ∙ a*2*x* = *ax*2  *c = c*1*∙ c*2  ?  (*c*1)(*a*2*x*) + (*c*2) (*a*1 *x*) = ***b x***  *ax*2 + *b x* + *c =* **(*a*1*x + c*1) (*a*2*x + c*2)** | **Example**  **4*x*2** + **7*x* + 3 =** ( ) ( )  4*x* 3  *x* 1  4*x ∙ x* = 4*x*2  3 ∙ 1= 3  ?  3 ∙ *x* + 4*x* ∙ 1 = **7 *x*** yes!  4*x*2 + 7*x* + 3 = (4*x +* 3) (*x +* 1) |

**Tip:** Cross multiply and then add up to the middle term.

|  |
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| **Summary: Factoring *ax*2 + *b x* + *c*** |
| *a*1 *a*2*x*2 + (*c*1*a*2 + *c*2*a*1) *x* + *c*1 *c*2 = (*a*1*x + c*1) (*a*2*x + c*2)  *a*1*x* *c*1  *a*2*x* *c*2 |

**Note:** Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form (*ax*2 + *b x* + *c*)first.

**More on Factoring *ax*2 + *b x* + *c***

**Example**: Factor **6*y*2 – 17*y* – 14**.

**Trial and error process**

**6*y*2 – 17*y* – 14** = ( ) ( ) 1) 6*y*2 – 17*y* – 14

3*y* 2 *y* -7

2*y* -7 6*y* 2

?

3*y* ∙ 2*y* = 6*y*2 2(-7) = -14 (-7) (6*y*) + 2*y* = -17*y* no

? 2) 6*y*2 – 17*y* – 14

(2) (2*y*) + (-7) (3*y*) = -**17*y*** yes! 3*y* 7

2*y* -2

?

6*y*2 – 17*y* – 14 = (3*y* + 2) (2*y* – 7) 7(2*y*) + (-2)(3*y*) = -17*y* no

3) 6*y*2 – 17*y* – 14

6*y* 2

*y* -7

?

2*y* + (-7) (6y) = -17*y* no

Check: (3*y* + 2) (2*y* – 7) = 6*y*2 – 21*y +* 4*y* –14

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**√**

(2*y* + 2) (2*y* – 7) = 6*y*2 – 17*y* –14 Correct!

**Example**: Factor the following completely.

1. **28*x* – 24** + **20*x*2** = 20*x*2 + 28*x* – 24 Rewrite in descending order or standard form (*ax*2 + *bx* + *c*)*.*

= 4(5*x*2 + 7*x* – 6) = 4 ( ) ( ) Factor out 4.

*x* 2

5*x* -3 10*x* + (– 3*x*) = 7*x*  **√**  4 (5*x*2 + 7*x* – 6) = 4(*x* + 2) (5*x* – 3)

**Note:** Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form (*ax*2 + *b x* + *c*)first.

1. **8*a*2 – 6*ab* – 5*b*2 =** ( ) ( )

2*a* *b* -10*ab* + 4*ab* = -6*ab* **√**

4*a* -5*b*

8*a*2 – 6*ab* – 5*b*2= (2*a* + *b*) (4*a* – 5*b*)

1. **2*t*4 + 14*t*2+ 20** = 2(*t*4 + 7*t*2+ 10) **=** 2( ) ( ) Factor out 2.

*t*2 2

*t*2 5

2*t*4 + 14*t*2+ 20 = 2(*t*2 + 2) (*t*2 + 5)5*t*2+ 2*t*2 = 7*t*2 **√**

**Factoring Trinomials: AC Method**

**AC method for factoring trinomials: *ax*2 + *bx* + *c***

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| **Factoring *ax*2 + *bx +* *c* = 0 by Grouping Example** |
| **Steps**  **Solve 14*x* + 6 = -8*x*2**   * Convert to standard form (descending order) if necessary. 8*x*2 + 14*x* + 6 = 0 * Factor out the greatest common factor (GCF). 2 (**4***x*2 + 7*x* **+ 3**) = 0 * Multiply ***a*** and ***c*** in ***a****x*2 + *bx* + ***c*** . *ac* = **4** **3** = 12 * Factor the product ***ac***that sum to the middle coefficient ***b***. 4 3 = **12**, 4 + 3 = **7** * Rewrite the middle term as the sum using the 2 (4*x*2 + **7***x* + 3) = 0   factors found in last step. 2 (4*x*2 + **4***x* **+ 3***x* + 3) = 0   * Factor by grouping. 2 [4*x* **(*x*****+ 1**) + **3 (*x*** **+ 1)**] = 0   2 (*x* + 1) (4*x* + 3) = 0 Factor out (*x* + 1). |

**Example**: Factor **6*x*2** – **16 = 4*x*** using *ac* method.

**Steps Solution**

**6*x*2** – **16 = 4*x***

* Write in standard form: 6*x*2 – 4*x* – 16 = 0
* Factor out the greatest common factor: 2(**3***x*2 – 2*x* **– 8**) = 0
* Multiply ***a*** and ***c***in ***a****x*2 + *bx* + ***c*** : *ac* = **3** ∙ (**-8**) = -24
* Factor the product ***ac*** that sum to the middle coefficient ***b***.

(There are different pairs to get the product of ***ac***of**-24**. Try to find two

numbers that multiply to ***ac*** and add to obtain ***b =* -2**.)

|  |  |
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| Some factors of *ac*  (-24) | Sum of factors ( *b* = -2) |
| -3 & 8 | -3 + 8 = 5 |
| -4 & 6 | -4 + 6 = 2 |
| 8 & -3 | 8 + (-3) = 5 |
| 4 & -6 | 4 + (- 6) = **-2** Correct! |

The right choices are 4 an -6, since they both add up to *b* = -2. 4 (-6) = -24, 4 + (-6) = -2

* Rewrite the middle term as **4***x* **– 6***x*. 2(3*x*2 **– 2***x* – 8) = 0

2 (3*x*2 **+ 4***x* **– 6***x* – 8) = 0

* Factor by grouping. 2 [*x* (3***x* + 4**) – 2(3***x* + 4**) = 0 Factor out (3*x* + 4)

2(3*x* + 4) (*x* – 2) = 0

**Factoring Special Products**

Factoring

***a*2 + 2*ab* + *b*2 = (*a + b*)2**

Multiplying

Recall that **factoring is the reverse of multiplication**.

Recognize some polynomials as special products can factor more quickly.

**Special products:**

|  |  |  |
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| **Name** | **Formula** | **Example** |
| Square of sum  (perfect square trinomial) | ***a*2 + 2*ab* + *b*2 = (*a + b*)2** | *x*2 + 10*x* + 25 = (*x +* 5)2 *a* = *x*, *b* = 5  *x* 5  *x* 5  Check: (*x +* 5)2= *x*2 + 2 ∙ *x* ∙ 5 + 52 = *x*2 + 10*x* + 25 **√** |
| Square of difference  (perfect square trinomial) | ***a*2** – **2*ab* + *b*2 = (*a*** – ***b*)2** | 9*y*2 – 24*y* + 16= (3*y*– 4)2 *a* = 3*y*, *b* = 4  3*y* -4  3*y*-4  Check: (3*y* – 4)2= (3*y*)2 – 2(3*y*) (4) + 42 = 9*y*2 – 24*y* + 16 **√** |

**Note:** The quickest way to factor an expression is to recognize it as a special product.

**Memory aid:** (*a*2 ± 2*ab + b*2) = (*a* ± *b*)2  *v* Notice the reversed plus or minus sign in the second term.

**To use perfect square trinomial formulas:** use cross-multiplication method to factor a perfect square. Then use the square formula to check.

**Example**: Factor the following completely.

1. **28*z* + 49 + 4*z*2**= 4*z*2 + 28*z* + 49 Rewrite in standard form:  *ax*2 + *bx* + *c*

2*z*7

2*z* 7

= (2*z +* 7) (2*z +* 7) 7(2*z*)+ 7(2*z*) = 28*x*

=(2*z +* 7)2

Check:(2*z +* 7)2=(2*z*)2 + 2 ∙ 2*z* ∙ 7 + 72 = 4*z*2 + 28*z* + 49 **√** *a*2 + 2*ab* + *b*2 = (*a + b*)2 : *a* = 2*z , b* = 7

1. **50*p*2 – 40*p* + 8**= 2(25*p*2 – 20*p* + 4)Factor out 2.

2**(**25*p*2 – 20*p* + 4)= 2(5*p*– 2)2

5*p* -2 -2 (5*p*)+ -2 (5*p*)= -20 *p*

5*p* -2

Check:2(5*p –* 2)2=2[(5*p*)2 – 2(5*p*) (2) + (2)2]= 2(25*p*2 – 20*p* + 4)**√** *a*2 *–* 2*ab* + *b*2 = (*a – b*)2: *a* = 5*p , b* = 2

1. **16*n*10 – 48*n*5+36** = 4(4*n*10 **–** 12*n*5 *+* 9) Factor out 4.

2*n*5  - 3

2*n*5  - 3(2*n*5) (-3)+ (2*n*5) (-3)= -12*n*5

= 4 (2*n*5 – 3)2 *a*m *a*n =*am+*n

Check: (2*n5 –* 3)2= (2*n*5)2 *–* 2(2*n*5) (3) + (3)2 = 4*n*10 – 12*n*5 *+* 9 **√** *a*2 – 2*ab* + *b*2 = (*a* – *b*)2 : *a* = 2*n*5*, b* = 3

**Topic C: Application of Factoring**

**Quadratic Equations**

**Quadratic equation:** an equation that has a squared term, such as 7*x***2** + 3*x* **–** 5 = 0.

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| **Quadratic equations in standard form** |
| *ax*2 + *b x* + *c* = 0 *a* ≠ 0 |

**Incomplete quadratic equation**

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| **Incomplete quadratic equation** | **Example a b c** |
| *ax*2 + *b x* = 0 (*c =* 0) | 4*x*2 **–** 3*x* = 0 4 -3 0 |
| *ax*2 + *c* = 0 (*b =* 0) | 8*x*2 + 5 = 0 8 0 5 |

**Zero-product property:**

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| **Zero-product property** |
| If *A ∙ B =* 0, then either *A* = 0 or *B =* 0 (or both)  (*A* and *B* are algebraic expressions.) |

**Note:** “or” means possibility of both.

**Solving incomplete quadratic equations**

Add 6*x.*

|  |  |  |
| --- | --- | --- |
| **Incomplete quadratic equation** | **Steps** | **Example** |
| **Use the zero-product property to solve**  ***ax*2 *+ bx* = 0** | * Express in *ax*2 + ***b*** *x =* 0 * Factor: *x* (*ax + b*) = 0 * Apply the zero-product property:   *x* = 0 *ax + b* = 0   * Solve for *x*: *x* = 0 | **Solve 11*x*2 = -6*x***  **11***x*2 + **6***x* = 0  *x* (11*x +* 6)= 0  *x* = 0 11*x +* 6= 0  *x* = 0 |
| **Use the square root method to solve**  ***ax*2– *c* = 0**  **(or *ax*2 *= c*)** | * Express in ***a****x*2 *=* ***c*** * Divide both sides by *a*: * Take the square root of both sides: | **Solve 64*x*2 – 9 = 0**  **64***x*2 = **9**    = |

**Solving Quadratic Equations**

**Solve a quadratic equation**: a quadratic equation *ax*2 + *b x* + *c* = 0can be written as:

(*x + a*) (*x + b*) = 0 Factor.

Set each term equal to zero:  *x + a* = 0 *x + b* = 0 Zero-product property.

Solutions：  *x = -a* *x* = -*b* Solve for *x.*

**Example**: Solve for *x.*  **(*x* + 6) (*x*** –**11) = 0**

*x* + 6 = 0 *x* – 11 = 0 Zero-product property.

***x*** = -6 ***x*** =11 Solve for *x.*

**Example**: Solve the quadratic equation ***x*2 – *x* – 20 = 0.**

1. ***x*2 – *x* – 20 = 0**

*x*4  Factor. *x* - 5 4*x* + (-5) *x* = -*x*

(*x* + 4) (*x –* 5) = 0

*x +* 4 = 0 *x* **–** 5 = 0 Zero-product property.

***x*** =-4 ***x*** =5

1. **6*x*2** *–* **13*x* = 15** Rewrite in standard form:  *ax*2 + *bx* + *c* = 0

6*x*2 *–* 13*x* *–* 15 = 0 Set the equation equal to 0.

6*x*5  Factor.

*x* -3 5*x* + (-3) (6*x*) = -13*x*

(6*x* + 5) (*x –* 3) = 0

6*x +* 5 = 0 *x* **–**3 = 0 Zero-product property.

***x*** =- ***x* =** 3

1. ***x*2 *–*  = *x*** Rewrite in standard form:  *ax*2 + *bx* + *c* = 0

*x*2 *– x* *–*  = 0 Set the equation equal to 0.

*x*  Factor.

*x* - ,

(*x* + ) (*x –* ) = 0

*x +*  = 0 *y* **–** = 0 Zero-product property.

***x* =** - ***x* =**

**Application of Quadratic Equations**

**Review number problems - examples**

|  |  |
| --- | --- |
| **English phrase Algebraic expression/equation** | |
| 6 more than the difference of the square of a number and 11 is 32. | (*x*2 – 11) + 6 = 32 |
| The quotient of 5 and the product of 9 and a number is 7 less than the number. | = *x* – 7 |
| The product of 9 and the square of a number decreased by 13 is 21. | 9*x*2 – 13 = 21 |
| 15 more than the quotient of 4*x* by 7 is 5 times the square of a number. | 15 + = 5*x*2 |

Let *x* = a number; *y* = a number

**Review consecutive integers**

|  |  |
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| **English phrase Algebraic expression Example** | |
| **Three** consecutive **odd** integers | *x , x +* 2*, x* + 4If *x =* **1***, x* + 2 = **3***, x* + 4 = **5** |
| **Three** consecutive **even** integers | *x* , *x* + 2 , *x* + 4 If *x* = **2**, *x* + 2 = **4**,  *x* + 4 = **6** |
| The product of two consecutive odd integers is 35. | *x* (*x* + 2) = 35 |
| Three consecutive even integers whose sum is 12. | *x* + (*x* + 2) + (*x* + 4) = 12 |

**Examples**:

1. The product of a number and 4 more than the square of the number is 21. Find the number(s).

* Let *x* = the number
* Equation ***x*2 + 4*x* = 21**
* Solve for *x*:  *x***2** + 4*x* – 21 = 0 Rewrite in standard form.

*x*7  Factor. *x* - 3 7*x* + (-3) *x* = 4*x*

(*x* + 7) (*x –* 3) = 0

*x +* 7 = 0 *x* **–** 3 = 0 Zero-product property.

***x*** =-7 ***x*** =3

1. The product of two consecutive even integers is 48. Find the integers.

* Let *x* = the first even integer
* Equation ***x* (*x* + 2) = 48** The 2nd integer is *x* + 2.
* Solve for *x*:  *x*2 + 2*x* – 48 = 0 Rewrite in standard form.

*x*-6  Factor. *x* 8 -6*x* + 8*x* = 2*x*

(*x* **–** 6) (*x +* 8) = 0

*x* **–**6 = 0 *x* **+** 8 = 0 Zero-product property.

***x*** = 6 ***x*** =-8

If *x* = 6, *x* + 2 = 8 If *x* = -8, *x* + 2 = -6

**Dimension (length and width) problems:**

**Example**: Robert is going to replace the old carpet in his bedroom, which is a rectangle and has a ***length 3 meters greater than its width***. If the ***area*** of his bedroom is ***54*** square meters (***m2***), what will be the dimensions of the carpet?

StepsSolution

* Organize the facts.

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| **Facts** | Area *A* = 54m2  Length = Width + 3m |
| **Unknowns** | Width = *x ,* Length = *x* + 3m |

* Draw a diagram.

Width = *x*

Length = *x* + 3

* Equation: ***x* (*x* + 3) = 54**  Area: *A* = *w l*
* Solve the equation. *x*2 + 3*x* = 54 Distribute.

- Standard form: *x*2 + 3*x* – 54 = 0 *ax*2 + *bx* + *c* = 0

*x*9

- Factor:  *x* -6 9*x* + (-6) *x* = 3*x*

(*x* + 9) (*x* – 6) = 0

- Zero-product property: *x* + 9 = 0 *x* – 6 = 0

- Solutions: *x* = -9 *x* = 6

Since the width of a rectangle cannot be negative, eliminate *x* = -9.

* Answer (the size of the carpet): **Width** = ***x*** =6 m

**Length** **= *x* + 3** = 6 + 3 =9 m

**Triangle problem:**

**Example**: A triangle is 1 meter wider than it is tall. The area is 36 m2. Find the base and the height.

* Organize the facts.

|  |  |
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| **Facts** | Area *A* = 36m2  Base = Height + 1m |
| **Unknowns** | Height = *x ,* Base = *x* + 1m |

* Equation. (*x* + 1) *x* = 36 Area: *A* =  *bh*
* Solve the equation. *x*2 + *x* = 72 Multiply both sides by 2.

- Standard form: *x*2 + *x* – 72 = 0 *ax*2 + *bx* + *c* = 0

*x*9

- Factor:  *x* -8 9*x* + (-8) *x* = *x*

(*x* + 9) (*x* – 8) = 0

- Zero-product property: *x* + 9 = 0 *x* – 8 = 0

- Solutions: *x* = -9 *x* = 8

* Answer: **Heigh**t = ***x*** =8 m

(Since the height of a triangle cannot be negative, eliminate *x* = -9.)

**Base** = ***x* + 1** = 8 + 1 =9m

**Unit 14: Summary**

**Factoring Polynomials**

**Factoring whole numbers**: write the number as a product of its prime factors.

**Common factor:** a number or an expression that is a factor of each term of a group of terms.

**Greatest / highest common factor (GCF or HCF):** the product of the common factors.

**Factoring a polynomial**: express a polynomial as a product of other polynomials. It is the

reverse of multiplication.

**Steps for factoring polynomials by grouping:**

* Regroup terms with the GCF.
* Factor out the GCF from each group.
* Factor out the GCF again from last step.

**Special products:**

|  |  |
| --- | --- |
| **Name** | **Formula** |
| Difference of squares | ***a2* – *b*2 = (*a + b*) (*a*** – ***b*)** |
| Square of sum | ***a*2 + 2*ab* + *b*2 = (*a + b*)2** |
| Square of difference | ***a*2** – **2*ab* + *b*2 = (*a*** – ***b*)2** |

Memory aid: (*a*2 ± *ab + b*2) = (*a* ± *b*)2

**Cross-multiplication method:**

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| **Factoring *x*2 + *b x* + *c* using the cross-multiplication method** | |
| **In general**  ***x*2 + *b x* + *c* =** ( ) ( )  *x**c*1  *x**c*2  *x ∙ x* = *x*2  *c*1 *∙ c*2 *= c*  ?  (*c*1)(*x*) + (*c*2)(*x*) = *b x*  *x*2 + *b x* + *c=* **(*x + c*1)(*x + c2*)** | **Example**  ***x*2** – **8 *x* + 15 =** ( ) ( )  *x* -5  *x* -3  *x ∙ x* = *x*2  (-3)(-5) = 15  ?  (*-*5) *x* + (-3) *x* = -8*x*  yes!  *x*2 – 8*x* +15 = (*x* –5) (*x* –3) |

**Tips:** - Cross multiply and then add up to the middle term.

- Write the factors with their appropriate signs (+ or –) to get the right middle term.

|  |  |
| --- | --- |
| **Factoring *ax*2 + *b x* + *c* using the cross-multiplication method** | |
| **In general**  ***ax*2 + *b x* + *c* =** ( ) ( )  *a*1*x**c*1  *a*2*x**c*2  *a*1*x ∙ a*2*x* = *ax*2  *c = c*1*∙ c*2  ?  (*c*1)(*a*2*x*) + (*c*2) (*a*1 *x*) = ***b x***  *ax*2 + *b x* + *c =* **(*a*1*x + c*1) (*a*2*x + c*2)** | **Example**  **4*x*2** + **7*x* + 3 =** ( ) ( )  4*x* 3  *x* 1  4*x ∙ x* = 4*x*2  3 ∙ 1= 3  ?  3 ∙ *x* + 4*x* ∙ 1 = **7 *x*** yes!  4*x*2 + 7*x* + 3 = (4*x +* 3) (*x +* 1) |

**Tips:** - Cross multiply and then add up to the middle term.

**-** Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form (*ax*2 + *b x* + *c*)first.

**Factoring polynomials:**

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| --- | --- |
| **Polynomial** | **Method** |
| Two terms  (binomial) | * If it is a perfect square:   *a*2 – *b*2 =(*a* + *b*)(*a* – *b*)   * If not, use the distributive property:   *ac* + *bc = c* (*a* + *b*) |
| Three terms  (trinomial) | *ax*2 + *b x* + c : use the cross-multiplication or AC methods. |
| Four terms | Factor by grouping |

**Quadratic equation:** an equation that has a squared term.

|  |
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| **Quadratic equations in standard form** |
| *ax*2 + *b x* + *c* = 0 *a* ≠ 0 |

**Incomplete quadratic equation**

|  |
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| **Incomplete quadratic equation** |
| *ax*2 + *b x* = 0 (*c =* 0) |
| *ax*2 + *c* = 0 (*b =* 0) |

**Zero-product property:**

|  |
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| **Zero-product property** |
| If *A ∙ B =* 0, then either *A* = 0 or *B =* 0 (or both)  (*A* and *B* are algebraic expressions.) |

**Solving incomplete quadratic equations**

|  |  |
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| **Incomplete quadratic equation** | **Steps** |
| **Use the zero-product**  **property to solve**  ***ax*2 *+ bx* = 0** | * Express in *ax*2 + ***b*** *x =* 0 * Factor: *x* (*ax + b*) = 0 * Apply the zero-product property:   *x* = 0 *ax + b* = 0   * Solve for *x*: *x* = 0 |
| **Use the square root method to solve**  ***ax*2– *c* = 0** | * Express in ***a****x*2 *=* ***c*** * Divide both sides by *a*: * Take the square root of both sides: |

**Unit 14: Self-Test**

**Factoring Polynomials**

**Topic A**

1. Factor 60.
2. Find the greatest common factor (GCF) for the following.
3. 5*x*2 – 20*x*
4. 3*a*3*b* + 15*a*4*b* – 21*ab*
5. 17*y*2 (*y* + 4) *–* (2*y* + 8)
6. 3 – 2 +
7. – 4*y*3 – 8*y*2 + 20*y*
8. Factor the following completely.
9. 25*x*2 – 5*x* + 20*x* – 4
10. 48*ab*2 – 8*a*2*b* + 6*b* – *a*
11. 4*uv*+ *vw* – 7*vw* + 21*uv*
12. *x*3 – *x*2*y* – *xy*2 + *y*3
13. 5*y*2 – 20
14. 1 – 49*w*2
15. 81*u*2 – 121
16. 25*a*2 – 36*b*2
17. *y*6 – 9

**Topic B**

1. Factor the following:
2. *x*2 + 9*x* + 20
3. *x*2 – 10*x* + 24
4. *x*2 – 3*x* – 18
5. 2*x*2 + 10*x* – 28
6. 4*x*2 – 7*x* – 15
7. 5*y*2 + 9*y* – 18
8. 24*ab*2 – 4*a*2*b* + 6*b* – *a*
9. 6*uv*+ *vs* – 7*vs* + 11*uv*
10. Factor the following using the *ac* method.
11. 6*x*2 – 60 = 9*x*
12. 6*x*2 +4*x* – 16
13. Factor the following completely.

(Use the cross-multiplication method to factor a perfect square. Then use the square formula to check.)

1. 9*x*2 + 30*x* + 25

1. 27 + 12*y*2 **–** 36*y*

1. 18*t*8 **–** 24*t*4 *+* 8

**Topic C**

1. Solve for *x*.
2. 23*x*2 = -7*x*
3. 81*x*2 – 49 = 0
4. (*x* + 9) (*x* –17) = 0
5. Solve the following quadratic equations.
6. *x*2 – *x* – 42 = 0
7. 7*x*2 *–* 31*x* = 20
8. *x*2 *+*  = *x*
9. The product of a number and 5 more than the square of the number is 36. Find the number(s).
10. The product of two consecutive even integers is 24. Find the integers.
11. Lisa is going to replace old carpet in her living room, which is a rectangle and has a length 2 meters greater than its width. If the area of her living room is 63 square meters (m2), what will be the dimensions of the carpet?
12. A triangle is 2 meters wider than it is tall. The area is 24m2. Find the base and the height.