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Preface v
About the author vii
Unit R  Review of basic mathematics 1
   Topic A:  Basic math skills 2
      Numbers and place value 2
      Prime / composite numbers 3
      Prime factorization 4
      Basic mathematical symbols and terms 5
   Topic B:  Percent, decimal and fraction 6
      Fractions 6
      More about fractions 7
      Decimals 8
      Operations with decimals 9
      Percent and conversion 10
   Topic C:  Operations with fractions 11
      Least common denominator (LCD) 11
      Operations with fractions 12
      Ratio and proportion 13
Unit R  Summary 14
Unit R  Self-test 18

Unit 1  Basic statistics and calculator use 20
   Topic A:  Average 21
      Mean and range 21
      Median and mode 22
   Topic B:  Graphs 23
      Bar or column graph 23
      Line graph 24
      Circle or pie graph 25
      Create a circle graph 26
   Topic C:  Using a calculator and estimating 27
      Scientific calculator 27
      Basic functions of a scientific calculator 28
      Rounding and estimating 29
Unit 1  Summary 30
Unit 1  Self-test 32

Unit 2  Introduction to algebra 34
   Topic A:  Algebraic expressions 35
      Basic algebraic terms 35
      Evaluating algebraic expressions 36
   Topic B:  Translating words into algebraic expressions 37
      Key words in word problems 37
      Translating phrases into algebraic expressions 38
      Writing algebraic expressions 39
      Steps for solving word problems 40
   Topic C:  Exponents & order of operations 42
      Introduction to exponents 42
      Read and write exponential expressions 43
      Order of operations 44
Unit 2  Summary 45
Unit 2  Self-test 47

Unit 3  Introduction to geometry 49
   Topic A:  Perimeter, area, and volume 50
      Perimeter of plane figures 50
      Circle 51
      Perimeter 52
      Perimeters of irregular / composite shapes 54
   Topic B:  Area 55
      Areas of quadrilaterals and circles 55
      Areas of irregular / composite shapes 56
   Topic C:  Volume 57
<table>
<thead>
<tr>
<th>Topic D: Volume of solids</th>
<th>57</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topic D: Surface and lateral area</td>
<td>59</td>
</tr>
<tr>
<td>Surface and lateral area – rectangular solids</td>
<td>59</td>
</tr>
<tr>
<td>Surface and lateral area – cylinders, cones and spheres</td>
<td>60</td>
</tr>
<tr>
<td>Unit 3 Summary</td>
<td>62</td>
</tr>
<tr>
<td>Unit 3 Self-test</td>
<td>64</td>
</tr>
<tr>
<td>Unit 4 Measurement</td>
<td>68</td>
</tr>
<tr>
<td>Topic A: Metric system of measurement</td>
<td>69</td>
</tr>
<tr>
<td>International system of units</td>
<td>69</td>
</tr>
<tr>
<td>Metric conversion</td>
<td>70</td>
</tr>
<tr>
<td>The unit factor method</td>
<td>71</td>
</tr>
<tr>
<td>Topic B: Metric units for area and volume</td>
<td>72</td>
</tr>
<tr>
<td>Convert units of area and volume</td>
<td>72</td>
</tr>
<tr>
<td>The relationship between mL, g and cm³</td>
<td>73</td>
</tr>
<tr>
<td>Topic C: Imperial system</td>
<td>74</td>
</tr>
<tr>
<td>The system of imperial units</td>
<td>74</td>
</tr>
<tr>
<td>Imperial unit conversion</td>
<td>75</td>
</tr>
<tr>
<td>Topic D: Converting between metric &amp; imperial units</td>
<td>76</td>
</tr>
<tr>
<td>Imperial and metric conversion</td>
<td>76</td>
</tr>
<tr>
<td>Unit 4 Summary</td>
<td>78</td>
</tr>
<tr>
<td>Unit 4 Self-test</td>
<td>80</td>
</tr>
<tr>
<td>Unit 5 The real number system</td>
<td>82</td>
</tr>
<tr>
<td>Topic A: Rational and irrational numbers</td>
<td>83</td>
</tr>
<tr>
<td>Real numbers</td>
<td>83</td>
</tr>
<tr>
<td>Topic B: Properties of addition and multiplication</td>
<td>84</td>
</tr>
<tr>
<td>Properties of addition</td>
<td>84</td>
</tr>
<tr>
<td>Properties of multiplication</td>
<td>85</td>
</tr>
<tr>
<td>Properties of addition and multiplication</td>
<td>87</td>
</tr>
<tr>
<td>Topic C: Signed numbers and absolute value</td>
<td>88</td>
</tr>
<tr>
<td>Signed numbers</td>
<td>88</td>
</tr>
<tr>
<td>Absolute value</td>
<td>89</td>
</tr>
<tr>
<td>Topic D: Operations with signed numbers</td>
<td>90</td>
</tr>
<tr>
<td>Adding and subtracting signed numbers</td>
<td>90</td>
</tr>
<tr>
<td>Multiplying signed numbers</td>
<td>91</td>
</tr>
<tr>
<td>Dividing signed numbers</td>
<td>92</td>
</tr>
<tr>
<td>Unit 5 Summary</td>
<td>93</td>
</tr>
<tr>
<td>Unit 5 Self-test</td>
<td>95</td>
</tr>
<tr>
<td>Unit 6 Polynomials</td>
<td>98</td>
</tr>
<tr>
<td>Topic A: Introduction to polynomials</td>
<td>99</td>
</tr>
<tr>
<td>Polynomials</td>
<td>99</td>
</tr>
<tr>
<td>Degree of a polynomial</td>
<td>100</td>
</tr>
<tr>
<td>Combine like terms</td>
<td>101</td>
</tr>
<tr>
<td>Removing parentheses</td>
<td>102</td>
</tr>
<tr>
<td>Topic C: Multiplying and dividing polynomials</td>
<td>103</td>
</tr>
<tr>
<td>Multiply and dividing monomials</td>
<td>103</td>
</tr>
<tr>
<td>Multiplying / dividing polynomials by monomials</td>
<td>104</td>
</tr>
<tr>
<td>FOIL method to multiply binomials</td>
<td>105</td>
</tr>
<tr>
<td>Unit 6 Summary</td>
<td>106</td>
</tr>
<tr>
<td>Unit 6 Self-test</td>
<td>108</td>
</tr>
<tr>
<td>Unit 7 Equations</td>
<td>110</td>
</tr>
<tr>
<td>Topic A: Properties of equations</td>
<td>111</td>
</tr>
<tr>
<td>Introduction to equations</td>
<td>111</td>
</tr>
<tr>
<td>Solving one-step equations</td>
<td>112</td>
</tr>
<tr>
<td>Properties of equality</td>
<td>114</td>
</tr>
<tr>
<td>Topic B: Solving equations</td>
<td>115</td>
</tr>
<tr>
<td>Solving multi-step equations</td>
<td>115</td>
</tr>
<tr>
<td>Equation solving strategy</td>
<td>116</td>
</tr>
<tr>
<td>Equations involving decimals / fractions</td>
<td>117</td>
</tr>
<tr>
<td>Topic C: One solution, no solutions, infinite solutions</td>
<td>118</td>
</tr>
<tr>
<td>Types of equations</td>
<td>118</td>
</tr>
<tr>
<td>Topic D: Writing and solving equations</td>
<td>120</td>
</tr>
<tr>
<td>Number problems</td>
<td>120</td>
</tr>
<tr>
<td>Consecutive integers</td>
<td>122</td>
</tr>
<tr>
<td>Mixed problems</td>
<td>123</td>
</tr>
<tr>
<td>Unit 7 Summary</td>
<td>125</td>
</tr>
<tr>
<td>Unit 7</td>
<td>Self-test</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td><strong>Unit 8  Formulas</strong></td>
<td>130</td>
</tr>
<tr>
<td>Topic A: Substitution into formulas</td>
<td>131</td>
</tr>
<tr>
<td>Geometry formulas</td>
<td>131</td>
</tr>
<tr>
<td>Substituting into formulas</td>
<td>132</td>
</tr>
<tr>
<td>Topic B: Solving formulas</td>
<td>134</td>
</tr>
<tr>
<td>Solving for a specific variable</td>
<td>134</td>
</tr>
<tr>
<td>More examples for solving formulas</td>
<td>135</td>
</tr>
<tr>
<td>Topic C: Pythagorean theorem</td>
<td>136</td>
</tr>
<tr>
<td>Pythagorean theorem</td>
<td>136</td>
</tr>
<tr>
<td>Applications of the Pythagorean theorem</td>
<td>137</td>
</tr>
<tr>
<td><strong>Unit 8 Summary</strong></td>
<td>138</td>
</tr>
<tr>
<td><strong>Unit 8 Self-test</strong></td>
<td>140</td>
</tr>
<tr>
<td><strong>Unit 9  Ratio, proportion, and percent</strong></td>
<td>142</td>
</tr>
<tr>
<td>Topic A: Ratio and rate</td>
<td>143</td>
</tr>
<tr>
<td>Ratio</td>
<td>143</td>
</tr>
<tr>
<td>Rate</td>
<td>144</td>
</tr>
<tr>
<td>Topic B: Proportion</td>
<td>145</td>
</tr>
<tr>
<td>Solving proportion</td>
<td>145</td>
</tr>
<tr>
<td>Topic C: Percent</td>
<td>147</td>
</tr>
<tr>
<td>Percent review</td>
<td>147</td>
</tr>
<tr>
<td>Solving percent problems</td>
<td>148</td>
</tr>
<tr>
<td>Topic D: Similar triangles</td>
<td>149</td>
</tr>
<tr>
<td>Similar triangles</td>
<td>149</td>
</tr>
<tr>
<td>Solving similar triangles</td>
<td>150</td>
</tr>
<tr>
<td><strong>Unit 9 Summary</strong></td>
<td>151</td>
</tr>
<tr>
<td><strong>Unit 9 Self-test</strong></td>
<td>154</td>
</tr>
<tr>
<td><strong>Unit 10  Trigonometry</strong></td>
<td>156</td>
</tr>
<tr>
<td>Topic A: Angles and triangles</td>
<td>157</td>
</tr>
<tr>
<td>Angles</td>
<td>157</td>
</tr>
<tr>
<td>Triangles</td>
<td>159</td>
</tr>
<tr>
<td>Find the missing measurement</td>
<td>160</td>
</tr>
<tr>
<td>Topic B: Trigonometric functions</td>
<td>161</td>
</tr>
<tr>
<td>Sides and angles</td>
<td>161</td>
</tr>
<tr>
<td>Trigonometric functions</td>
<td>162</td>
</tr>
<tr>
<td>Sine, cosine, and tangent</td>
<td>163</td>
</tr>
<tr>
<td>Topic C: Solving right triangles</td>
<td>164</td>
</tr>
<tr>
<td>Trigonometry using a calculator</td>
<td>164</td>
</tr>
<tr>
<td>Solving triangles</td>
<td>165</td>
</tr>
<tr>
<td>Angles of depression and elevation</td>
<td>167</td>
</tr>
<tr>
<td>Applications of trigonometry</td>
<td>168</td>
</tr>
<tr>
<td><strong>Unit 10 Summary</strong></td>
<td>169</td>
</tr>
<tr>
<td><strong>Unit 10 Self-test</strong></td>
<td>171</td>
</tr>
<tr>
<td><strong>Unit 11  Exponents, roots and scientific notation</strong></td>
<td>174</td>
</tr>
<tr>
<td>Topic A: Exponents</td>
<td>175</td>
</tr>
<tr>
<td>Basic exponent properties review</td>
<td>175</td>
</tr>
<tr>
<td>Degree of a polynomial</td>
<td>176</td>
</tr>
<tr>
<td>Topic B: Properties of exponents</td>
<td>177</td>
</tr>
<tr>
<td>Properties of exponents</td>
<td>177</td>
</tr>
<tr>
<td>Properties of exponents – examples</td>
<td>179</td>
</tr>
<tr>
<td>Simplifying exponential expressions</td>
<td>180</td>
</tr>
<tr>
<td>Topic C: Scientific notation and square roots</td>
<td>181</td>
</tr>
<tr>
<td>Scientific notation</td>
<td>181</td>
</tr>
<tr>
<td>Square roots</td>
<td>182</td>
</tr>
<tr>
<td>Simplifying square roots</td>
<td>183</td>
</tr>
<tr>
<td><strong>Unit 11 Summary</strong></td>
<td>184</td>
</tr>
<tr>
<td><strong>Unit 11 Self-test</strong></td>
<td>186</td>
</tr>
<tr>
<td><strong>Unit 12  Solving word problems</strong></td>
<td>188</td>
</tr>
<tr>
<td>Topic A: Value mixture problems</td>
<td>189</td>
</tr>
<tr>
<td>Solving value mixture problems</td>
<td>189</td>
</tr>
<tr>
<td>Topic B: Concentration mixture problems</td>
<td>191</td>
</tr>
<tr>
<td>Solving mixture problems</td>
<td>191</td>
</tr>
<tr>
<td>Topic C: Motion &amp; business problems</td>
<td>193</td>
</tr>
<tr>
<td>Distance, speed and time problems</td>
<td>193</td>
</tr>
<tr>
<td>Business problems</td>
<td>194</td>
</tr>
<tr>
<td>Topic D: Mixed problems</td>
<td>196</td>
</tr>
</tbody>
</table>
If you are looking for a quick exam, homework guide, and review book in intermediate mathematics, “Key Concepts of Intermediate Level Math” is an excellent source. Skip the lengthy and distracting books and instead use this concise book as a guideline for your studies, quick reviewing and tutoring.

This unique and well-structured book is an excellent supplement and convenient reference book for intermediate mathematics. It provides concise, understandable and effective guide on intermediate level mathematics.

**Key Features**

As an aid to readers, the book provides some noteworthy features:

- A concise study guide, quickly getting to the heart of each particular topic, helping students with a quick review before doing mathematics homework as well as preparation for tests.
- Each topic, concept, term and phrase has a clear definition followed by examples on each page.
- Key terms, definitions, properties, phrases, concepts, formulae, rules, equations, etc. are easily located. Clear step-by-step procedures for applying theorems.
- Clear and easy-to-understand written format and style. Materials presented in visual and gray scale format with less text and more outlines, tables, boxes, charts, etc.
- Tables that organize and summarize procedures, methods, and equations; clearly presenting information and making studying more effective.
- Procedures and strategies for solving word problems, using realistic real-world application examples.
- Summary at the end of each unit to emphasize the key points and formulas in the unit, which is convenient for students reviewing before exams.
- Self-test at the end of each unit tests student’s understanding of the material. Students can take the self-test before beginning the unit to determine how much they know about the topic. Those who do well may decide to move on to the next unit.
Suitable Readers

This book can be used for:

- Adult Basic Education programs at colleges.
- Students in community colleges, high schools, tutoring, or resource rooms.
- Self-study readers, including new teachers to brush up on their mathematics.
- Professionals as a quick review of some basic mathematic formulas and concepts, or parents to help their children with homework.

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Bibliography

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Meizhong is also the author of several books, including:

- *Legends of Four Chinese Sages* – coauthor (Lily S.S.C Literary Ltd. – Canada, 2007).
Unit R

Review of Basic Mathematics

Topic A: Basic math skills

- Numbers and place value
- Prime / composite numbers
- Prime factorization
- Basic mathematical symbols and terms

Topic B: Percent, decimal and fraction

- Fractions
- More about fractions
- Decimals
- Operations with decimals
- Percent and conversion

Topic C: Operations with fractions

- Least common denominator (LCD)
- Operations with fractions
- Ratio and proportion

Unit R Summary

Unit R Self-test

Unit R is a review of basic math fundamentals. There is a self-test at the end of the unit that can test students’ understanding of the material. Students can take the self-test before beginning the unit to determine how much they know about the topic. Those who do well may decide to move on to the next unit without reading the lesson.
Topic A: Basic Math Skills

Numbers and Place Value

Numbers:

<table>
<thead>
<tr>
<th>Classify numbers</th>
<th>Definition</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>The ten digits</td>
<td>a symbol for numeral below 10</td>
<td>0, 1, 2, 3, 4, 5, 6, 7, 8 and 9</td>
</tr>
<tr>
<td>Whole numbers</td>
<td>the numbers used for counting</td>
<td>0, 1, 2, 3, 4, 5, 6, 7 …</td>
</tr>
<tr>
<td>Integers</td>
<td>all the whole numbers and their negatives</td>
<td>… -3, -2, -1, 0, 1, 2, 3 …</td>
</tr>
<tr>
<td>Odd numbers</td>
<td>any integer that cannot be evenly divided by 2</td>
<td>1, 3, 5, 7, 9 …</td>
</tr>
<tr>
<td>Even numbers</td>
<td>any integer that can be evenly divided by 2</td>
<td>0, 2, 4, 6, 8, 10 …</td>
</tr>
</tbody>
</table>

Number line is a straight line on which every point corresponds to an integer.

Place value: the value of the position of a digit in a number.

- Each digit in a number has a place value.
- The location in a number determines the value of a digit.

Example: 2,063,946,753

2, 0 6 3, 9 4 6, 7 5 3
Prime / Composite Numbers

**Factor:** a number you multiply with others to get another number.

**Example:** \(3 \times 4 = 12\) 3 and 4 are factors.

- Some numbers can be factored in many ways:
  - **Example:** \(2 \times 4 = 8\) or \(4 \times 2 = 8\) or \(1 \times 8 = 8\) or \(8 \times 1 = 8\)

- Factors for some numbers:

<table>
<thead>
<tr>
<th>Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factors</td>
<td>1</td>
<td>1, 2</td>
<td>1, 3</td>
<td>1, 2, 4</td>
<td>1, 5</td>
<td>1, 2, 3, 6</td>
<td>1, 7</td>
<td>1, 2, 4, 8</td>
<td>1, 3, 9</td>
<td>1, 2, 5, 10</td>
</tr>
</tbody>
</table>

**Prime number:** a whole number that only has two factors, 1 and itself.

**Example:** 2, 3, 5, and 7 are prime numbers.

(7 has two factors: 1 and 7, \(1 \times 7 = 7\))

**Composite number:** a whole number that has more than two factors, and can be evenly divided.

**Example:** 4, 6, 8, 9 and 10 are composite numbers.

(6 has four factors: 1, 2, 3 and 6, \(1 \times 6 = 6\), \(2 \times 3 = 6\))

**Rules for testing a prime / composite number:**

- A prime number is always an odd number, except for 2 (but an odd number is not necessarily a prime number).
  - **Example:** The prime numbers 1, 3, 5, and 7 are odd numbers.
    The odd number 9 is a composite number.

- An even number (ends in a 0, 2, 4, 5, 6, and 8) is always a composite number (except number 2).
  - **Example:** 14, 28, 376, and 5372 are composite numbers.

- All numbers that end with five and are greater than five are composite numbers.
  - **Example:** 15, 65, and 345 are composite numbers.

**Tip:** The Prime Tester in the following website can determine if a number is a prime or a composite number.

http://www.murderousmaths.co.uk/games/primcal.htm
Prime Factorization

**Prime factorization** is finding which prime numbers can be used to multiply to get the original number.

**Prime factorization:** the product of all the prime numbers for a given number.

**Example:** $30 = 2 \times 3 \times 5$  
“Product” – the keyword for multiplication  
2, 3, and 5 are prime numbers (or prime factors).

**Find the prime factorization:**

- **Method 1:** do repeated division (or upside down division) by prime numbers, and multiply all the prime numbers around the outside to get the prime factorization.

**Example:** Find the prime factorization of 24.

\[
\begin{array}{c|c}
2 & 24 \\
2 & 12 \\
2 & 6 \\
3 & 3 \\
\end{array}
\]

Note: Stop dividing until you reach a prime number. The outside numbers are 2, 2, 2, 3.

The prime factorization for 24 is: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

- **Method 2:** *factor tree* method - split the number into two factors, then split non-prime factors until all the factors are prime, and multiply all the prime numbers.

**Example:** Find the prime factorization of 24.

```
24
 \ \ 
 2 12  \ (24 ÷ 2 = 12)
   \ \ 
   2 6  \ (12 ÷ 2 = 6)
     \ 
     2 3  \ (6 ÷ 2 = 3)
```

The prime factorization for 24 is: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$
Basic Mathematical Symbols and Terms

Basic mathematical symbols summary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>=</td>
<td>equal</td>
<td>3 = 3</td>
</tr>
<tr>
<td>≠</td>
<td>not equal</td>
<td>2 ≠ 3</td>
</tr>
<tr>
<td>≈</td>
<td>approximately</td>
<td>4 ≈ 3.89</td>
</tr>
<tr>
<td>&gt;</td>
<td>is greater than</td>
<td>4 &gt; 2</td>
</tr>
<tr>
<td>&lt;</td>
<td>is less than</td>
<td>1 &lt; 3</td>
</tr>
<tr>
<td>≥</td>
<td>is greater than or equal to</td>
<td>x ≥ 4</td>
</tr>
<tr>
<td>≤</td>
<td>is less than or equal to</td>
<td>x ≤ 8</td>
</tr>
<tr>
<td>±</td>
<td>plus or minus</td>
<td>3 ± 2 means: 3 + 2 or 3 − 2</td>
</tr>
<tr>
<td>+</td>
<td>addition</td>
<td>3 + 2</td>
</tr>
<tr>
<td>−</td>
<td>subtraction</td>
<td>7 − 3</td>
</tr>
<tr>
<td>× or • or ( )</td>
<td>multiplication</td>
<td>6 × 3 = 18 or 6 • 3 = 18 or (6)(3) = 18</td>
</tr>
<tr>
<td>÷ or / or − or — or —</td>
<td>division</td>
<td>4 ÷ 2 = 2, 4 / 2, 4 ÷ 2, (factor) / (factor) = 18</td>
</tr>
</tbody>
</table>

Arithmetic terms:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>Addend + addend = sum</td>
</tr>
<tr>
<td>Subtraction</td>
<td>Subtrahend − minuend = difference</td>
</tr>
<tr>
<td>Multiplication</td>
<td>Multiplicand × multiplier = product</td>
</tr>
<tr>
<td>Division</td>
<td>Dividend ÷ divisor = quotient &amp; remainder</td>
</tr>
</tbody>
</table>

Properties of zero

- Any number multiplied by 0 will always equal to 0.
  \[3 \times 0 = 0\]
- The number 0 divided by any nonzero number is zero.
  \[\frac{0}{6} = 0\]
  (0 apples divided by 6 kids, each kid gets 0 apples.)
- A number divided by 0 is not defined (not allowed).
  \[\frac{6}{0}\] is undefined.
  (6 apples shared by zero kids has no meaning.)

Writing whole numbers in words:

- Do not use ‘and’ when writing or reading whole numbers.
- Do not use ‘s’ at the end of trillion, million, thousand, hundred, etc.

Example: Write the following number in words: 12,023,476

Twelve million, twenty-three thousand, four hundred seventy-six.
Fraction: a fraction is a part of a whole. It is expressed in the form of \( \frac{a}{b} \). (Example: \( \frac{2}{5} \))

- Numerator: the number that represents how many parts are being dealt with.
- Denominator: the number of parts the whole is being divided into.

Three types of fractions

- Proper fraction: has a numerator smaller than (<) the denominator.
  
  Example: \( \frac{1}{2}, \frac{3}{8}, \frac{16}{237} \)

- Improper fraction: has a numerator larger than or equal to (\( \geq \)) the denominator.
  
  Example: \( \frac{7}{6}, \frac{56}{31}, \frac{9}{9} \)

- Mixed fraction (or mixed number): contains an integer and a proper fraction.
  
  Example: \( 2\frac{1}{4}, 3\frac{2}{5}, 5\frac{4}{7} \)

Conversion between a mixed number and an improper fraction

- To convert a mixed number to an improper fraction:

  \[
  \text{Improper fraction} = \frac{\text{whole number} \times \text{denominator} + \text{numerator}}{\text{Denominator}}
  \]

  Example:
  
  \[2\frac{1}{4} = \frac{2 \times 4 + 1}{4} = \frac{9}{4}\]

- To convert an improper fraction to a mixed number:

  \[
  \text{Mixed number} = \frac{\text{Numerator}}{\text{Denominator}}, \quad \text{Quotient} = \frac{\text{Remainder}}{\text{Denominator}}
  \]

  Example:
  
  \[\frac{9}{2} = 9 \div 2 = 4 \, R \, 1 = 4 \frac{1}{2}, \quad 2) \frac{9}{1} - \frac{8}{1} \]
More about Fractions

Equivalent fractions: different fractions that have the same value.

To find the equivalent fraction: divide or multiply the numerator and denominator by the same number.

- Divide by the same number (for a larger fraction):
  
  To simplify (or reduce) fractions: divide the numerator and denominator by the same number until their only common factor is 1.

  \[
  \begin{array}{c|c}
  \text{Numerator} & n \\
  \hline
  \text{Denominator} & n \\
  \end{array}
  \]

  \[
  \frac{18}{36} \div 2 = \frac{9}{18} \div 3 = \frac{3}{6} = \frac{1}{2}
  \]

  The simplest fraction of \( \frac{18}{36} \) is \( \frac{1}{2} \).

- Multiply by the same number (for a smaller fraction):

  \[
  \begin{array}{c|c}
  \text{Numerator} & n \\
  \hline
  \text{Denominator} & n \\
  \end{array}
  \]

  \[
  \frac{1}{3} \times 3 = \frac{3}{9} \times 2 = \frac{6}{18}
  \]

Like and unlike fractions:

- Like fractions: fractions that have the same denominators. Examples: \( \frac{2}{7} \), \( \frac{5}{7} \), \( \frac{4}{7} \)

- Unlike fractions: fractions that have different denominators. Examples: \( \frac{2}{3} \), \( \frac{3}{5} \), \( \frac{7}{10} \)

Classifying fractions:

<table>
<thead>
<tr>
<th>Classifying fractions</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proper fraction</td>
<td>( \frac{1}{2} ), ( \frac{3}{8} ), ( \frac{16}{237} )</td>
</tr>
<tr>
<td>Improper fraction</td>
<td>( \frac{7}{6} ), ( \frac{56}{31} ), ( \frac{9}{9} )</td>
</tr>
<tr>
<td>Mixed fraction (or mixed number)</td>
<td>( \frac{2\frac{1}{4}}{7} ), ( \frac{3\frac{2}{5}}{7} ), ( \frac{4\frac{4}{7}}{7} )</td>
</tr>
<tr>
<td>Like fractions</td>
<td>( \frac{2}{7} ), ( \frac{5}{7} ), ( \frac{4}{7} )</td>
</tr>
<tr>
<td>Unlike fractions</td>
<td>( \frac{2}{3} ), ( \frac{3}{5} ), ( \frac{7}{10} )</td>
</tr>
</tbody>
</table>
Decimals

**Decimal number:** a number contains a decimal point.

- The number to the left of the decimal is the integer part.
- The number to the right of the decimal is the fractional part.

**Example:** 34.8

(Integer part + decimal point + fractional part) (8 tenth)

**Decimal place:** a place of a digit to the right of a decimal point.

- Each digit in a decimal number has a decimal place.
- The location in a number determines the value of a digit.

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>.</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousands</th>
<th>Ten thousandths</th>
<th>Hundred thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example:** 5.40378

(Smaller)

**Write decimals in words:** Integer part + and + fractional part

**Example:**

1) 35.348

Thirty-five and three hundred forty-eight thousandths

2) 6.038

Six and thirty-eight hundredths
**Operations with Decimals**

**Operations with decimals:**

<table>
<thead>
<tr>
<th></th>
<th>+ or - decimals</th>
<th>× decimals</th>
<th>÷ decimals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Line up the decimal points.</td>
<td>× as whole numbers.</td>
<td>Move the decimal point of the divisor to the right end.</td>
</tr>
<tr>
<td></td>
<td>+ or − as whole numbers.</td>
<td>Count the numbers of the decimal places in both factors.</td>
<td>Move the decimal point of the dividend the same number of places to the right (insert zeros if necessary).</td>
</tr>
<tr>
<td></td>
<td>Insert a decimal point in the answer (in the same line as above).</td>
<td>Insert a decimal point in the product so that it matches the number of decimal places of factors (start at the far right).</td>
<td>Insert a decimal point in the quotient (directly above the decimal point in the dividend).</td>
</tr>
</tbody>
</table>

### Example 1

\[ 5.25 = \frac{525}{100} = \frac{1}{4} \]

- The fractional part = 25
- The number of decimal places = 2

### Example 2

\[ 0.045 = \frac{45}{1000} = \frac{9}{200} \]

- The number of decimal places = 3

### Example 3

\[ 384.3645 = \frac{3843645}{10000} \]

- The number of decimal places = 4

**Convert decimals to mixed numbers or fractions:**

- **Whole number does not change.**
- **Write the original term as a fraction.**
  - Numerator = the fractional part
  - Denominator = a multiple of 10
- **Simplify (reduce) if possible.**

**Example: 1)**

\[ 5.25 = \frac{25}{100} \]

The fractional part = 25

The number of decimal places = 2

**Example: 2)**

\[ 0.045 = \frac{45}{1000} \]

The number of decimal places = 3
Percent and Conversion

Percent (%): one part per hundred, or per one hundred.

The standard form of percent proportion:

\[
\text{Part} = \frac{\text{Percent}}{100} \quad \text{or} \quad \frac{\text{"is" number}}{\%} = \frac{\text{"of" number}}{100}
\]

(With the word “is”)

(With the word “of”)

Use percent proportion method to solve % problems:

- Identify the part, whole, and percent.
- Set up the proportion equation.
- Solve for the unknown.

Converting between percent, decimal and fraction:

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Step</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent to Decimal</td>
<td>Move the decimal point two places to the left, then remove %.</td>
<td>31% = 31.%, 31% = 0.31</td>
</tr>
<tr>
<td>Decimal to Percent</td>
<td>Move the decimal point two places to the right, then insert %.</td>
<td>0.317 = 0.317 = 31.7 %</td>
</tr>
<tr>
<td>Percent to Fraction</td>
<td>Remove %, divide by 100, then simplify.</td>
<td>15% = \frac{15}{100} = \frac{3}{20} (% = \text{per one hundred.})</td>
</tr>
<tr>
<td>Fraction to Percent</td>
<td>Divide, move the decimal point two places to the right, then insert %.</td>
<td>\frac{1}{4} = 1 \div 4 = 0.25 = 25 %</td>
</tr>
<tr>
<td>Decimal to Fraction</td>
<td>Convert the decimal to a percent, then convert the percent to a fraction.</td>
<td>0.35 = 35% = \frac{35}{100} = \frac{7}{20}</td>
</tr>
</tbody>
</table>

Converting repeating decimals to fractions:

- Let \( x \) equals the repeating decimal:
- Multiply both sides by 100:
- Subtract the first equation from the second:
- Solve for \( x \):

Example: \( 0.\overline{6} \to \text{Fraction} \)

\[
\begin{align*}
x &= 0.66 & (1) \\
100x &= 66 & (2) \\
100x - x &= 66 - 0.66 \\
99x &= 65.34 \\
x &= \frac{65.34}{99} \\
&\approx \frac{2}{3}
\end{align*}
\]
Topic C: Operations with Fractions

**Least Common Denominator (LCD):** the least common multiple (LCM) of the denominators of two or more given fractions.

**Find the LCD:** Use repeated division to find the LCM for all denominators of given fractions.

**Example:** Find the LCD for $\frac{4}{8}$, $\frac{5}{16}$, and $\frac{2}{42}$.

\[
\begin{array}{ccc}
2 & \mid & 8 \hspace{1em} 16 \hspace{1em} 42 \\
2 & \mid & 4 \hspace{1em} 8 \hspace{1em} 21 \\
2 & \mid & 2 \hspace{1em} 4 \hspace{1em} 21 \\
1 & \mid & 2 \hspace{1em} 21 \\
\end{array}
\]

LCM = $2 \times 2 \times 2 \times 1 \times 2 \times 21 = 336
# Operations with Fractions

<table>
<thead>
<tr>
<th>Operation</th>
<th>Steps</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adding and subtracting like fractions</td>
<td>- Add / subtract the numerators.</td>
<td>[ \frac{3}{13} + \frac{5}{13} = \frac{3+5}{13} = \frac{8}{13} ]</td>
</tr>
<tr>
<td></td>
<td>- Denominators do not change.</td>
<td>[ \frac{7}{12} - \frac{3}{12} = \frac{7-3}{12} = \frac{4}{12} = \frac{1}{3} ]</td>
</tr>
<tr>
<td></td>
<td>- Simplify if necessary.</td>
<td></td>
</tr>
<tr>
<td>Adding and subtracting unlike fractions</td>
<td>- Determine the LCD.</td>
<td>[ \frac{5}{12} + \frac{3}{8} = \frac{10}{24} + \frac{9}{24} = \frac{10+9}{24} = \frac{19}{24} ]</td>
</tr>
<tr>
<td></td>
<td>- Rewrite fractions with the LCD, and add or subtract the numerators.</td>
<td>[ \frac{5}{12} \times \frac{3}{8} = \frac{9}{24} ] [ (\text{LCD} = 24) ]</td>
</tr>
<tr>
<td></td>
<td>- Simplify if necessary.</td>
<td>[ \frac{4}{9} - \frac{2}{6} = \frac{8}{18} - \frac{6}{18} = \frac{8-6}{18} = \frac{2}{18} = \frac{1}{9} ]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ \frac{9}{2} \times \frac{3}{2} = \frac{9}{18} = \frac{1}{2} ] [ (\text{LCD} = 18) ]</td>
</tr>
<tr>
<td>Adding and subtracting mixed numbers with like denominators</td>
<td>- Add / subtract integers.</td>
<td>[ 2\frac{3}{5} + 5\frac{1}{5} = (2 + 5)\frac{3+1}{5} = \frac{7}{5} ]</td>
</tr>
<tr>
<td></td>
<td>- Add / subtract as fractions.</td>
<td>[ 5\frac{9}{14} - 3\frac{5}{14} = (5 - 3)\frac{9-5}{14} = \frac{2}{14} = \frac{1}{7} ]</td>
</tr>
<tr>
<td></td>
<td>- Simplify if necessary.</td>
<td></td>
</tr>
<tr>
<td>Adding and subtracting mixed numbers with unlike denominators</td>
<td>- Rewrite fractions with the LCD.</td>
<td>[ \frac{3\frac{5}{12}}{\frac{8}{3}} = \frac{3}{8} \frac{10}{24} - \frac{9}{24} ]</td>
</tr>
<tr>
<td></td>
<td>- Add / subtract as fractions.</td>
<td>[ = (3-2)\frac{10-9}{24} = \frac{1}{24} ]</td>
</tr>
<tr>
<td></td>
<td>- If the sum/difference created an improper fraction → a mixed number.</td>
<td></td>
</tr>
<tr>
<td>Multiplying fractions</td>
<td>- Cross simplify if the fraction is not in lowest terms.</td>
<td>[ \frac{2\frac{1}{9}}{\frac{3}{5}} = \frac{2\times1}{3\times5} = \frac{2}{15} ]</td>
</tr>
<tr>
<td></td>
<td>- Multiply the numerators.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Multiply the denominators.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Simplify the result if necessary.</td>
<td></td>
</tr>
<tr>
<td>Multiplying mixed numbers</td>
<td>- Convert mixed numbers to improper fractions.</td>
<td>[ \frac{1\frac{6}{5}}{2\frac{3}{2}} = \frac{3}{5} \frac{1}{\frac{1}{2}} = \frac{3\times1}{\frac{1\times2}{2}} = \frac{3}{1} = \frac{3}{1} ]</td>
</tr>
<tr>
<td></td>
<td>- Cross simplify if the fractions is not in lowest terms.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Multiply the numerators.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Multiply the denominators.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Simplify the result if necessary.</td>
<td></td>
</tr>
<tr>
<td>Dividing fractions</td>
<td>- Change the divisor to its reciprocal (switch the numerator and denominator).</td>
<td>[ \frac{2}{7} \div \frac{3}{5} = \frac{2}{7} \times \frac{5}{3} = \frac{2\times5}{7\times3} = \frac{10}{21} ]</td>
</tr>
<tr>
<td>Dividing mixed numbers</td>
<td>- Convert mixed numbers to improper fractions.</td>
<td>[ \frac{8}{5} \div \frac{1}{5} = \frac{8\times5}{5\times1} = \frac{5}{1} \times \frac{1\times5}{\frac{2}{2}} = \frac{5}{2} = \frac{21}{2} ]</td>
</tr>
<tr>
<td></td>
<td>- Divide fractions.</td>
<td></td>
</tr>
</tbody>
</table>
Ratio and Proportion

Ratio, rate and proportion:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Unit</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratio</strong></td>
<td>$a \div b \text{ or } \frac{a}{b}$</td>
<td>with the same unit.</td>
</tr>
<tr>
<td><strong>Rate</strong></td>
<td>$a \div b \text{ or } \frac{a}{b}$</td>
<td>with different units.</td>
</tr>
<tr>
<td><strong>Proportion</strong></td>
<td>$\frac{a}{b} = \frac{c}{d}$</td>
<td>an equation with a ratio on each side.</td>
</tr>
</tbody>
</table>

**Note:** the units for both numerators must match and the units for both denominators must match.

Example: $\frac{\text{in}}{\text{ft}} = \frac{\text{in}}{\text{ft}}, \frac{\text{minutes}}{\text{hours}} = \frac{\text{minutes}}{\text{hours}}$

Solving a proportion:

- Cross multiply: multiply along two diagonals. $\frac{a}{b} \times \frac{c}{d}$
- Solve for the unknown.

**Example:**
John’s height is 1.75 meters, and his shadow is 1.09 meters long. A building’s shadow is 10 meters long at the same time. How tall is the building?

- Facts and unknown:

<table>
<thead>
<tr>
<th>John’s height = 1.75 m</th>
<th>Let $x$ = Building’s height (unknown)</th>
</tr>
</thead>
<tbody>
<tr>
<td>John’s shadow = 1.09 m</td>
<td>Building’s shadow = 10 m</td>
</tr>
</tbody>
</table>

- Equation: $\frac{1.75m}{1.09m} = \frac{x m}{10m}$

- Cross multiply: $\frac{1.75m}{1.09m} \times \frac{x m}{10m}$

- Solve for $x$: $x = \frac{(1.75)(10)}{10.9} = 16.055$ m

The building’s height is 16.055 m.
Unit R: Summary

Review of Basic Mathematics

Numbers:

<table>
<thead>
<tr>
<th>Classify Numbers</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole numbers</td>
<td>0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, …</td>
</tr>
<tr>
<td>Odd numbers</td>
<td>1, 3, 5, 7, …</td>
</tr>
<tr>
<td>Even numbers</td>
<td>0, 2, 4, 6, 8, …</td>
</tr>
<tr>
<td>Digits</td>
<td>0, 1, 2, 3, 4, 5, 6, 7, 8, 9.</td>
</tr>
<tr>
<td>Expanded form</td>
<td>345 = 300 + 40 + 5</td>
</tr>
<tr>
<td>Prime number</td>
<td>A whole number that only has two factors, 1 and itself.</td>
</tr>
<tr>
<td>Composite number</td>
<td>A whole number that has more than two factors.</td>
</tr>
</tbody>
</table>

Place value: the value of the position of a digit in a number.

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>Trillions</th>
<th>Billions</th>
<th>Millions</th>
<th>Thousands</th>
<th>Ones</th>
</tr>
</thead>
</table>

Factor: a number you multiply with others to get another number.

Prime factorization: the product of all the prime factors for a given number.

Find the prime factorization: do repeated division (or upside-down division) by prime numbers, and multiply all the prime numbers around the outside to get the prime factorization.

Properties of zero:

- Any number multiplied by 0 will always equal to 0.
- The number 0 divided by any nonzero number is zero.
- A number divided by 0 is not defined (not allowed).

Basic mathematical symbol summary:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>=</td>
<td>equal</td>
</tr>
<tr>
<td>≠</td>
<td>not equal</td>
</tr>
<tr>
<td>≈</td>
<td>approximately</td>
</tr>
<tr>
<td>&gt;</td>
<td>is greater than</td>
</tr>
<tr>
<td>&lt;</td>
<td>is less than</td>
</tr>
<tr>
<td>≥</td>
<td>is greater than or equal to</td>
</tr>
<tr>
<td>≤</td>
<td>is less than or equal to</td>
</tr>
<tr>
<td>±</td>
<td>plus or minus</td>
</tr>
<tr>
<td>+</td>
<td>addition</td>
</tr>
<tr>
<td>−</td>
<td>subtraction</td>
</tr>
<tr>
<td>× or • or ( )</td>
<td>multiplication</td>
</tr>
<tr>
<td>÷ or / or — or ——</td>
<td>division</td>
</tr>
</tbody>
</table>
Writing whole numbers in words:

- Do not use ‘and’ when writing or reading whole numbers.
- Do not use ‘s’ at the end of trillion, million, thousand, hundred, etc.

**Fraction:** a fraction is a part of a whole. It is expressed in the form of \( \frac{a}{b} \).

\[
\text{Fraction: } \frac{a}{b} = \frac{\text{Numerator}}{\text{Denominator}}
\]

**Decimal number:** a number contains a decimal point.

Integer part + decimal point + fractional part

**Decimal place:** a place of a digit to the right of a decimal point.

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<tr>
<th>Hundreds</th>
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<th></th>
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<th>Hundredths</th>
<th>Thousandths</th>
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<th>Hundred thousandths</th>
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</table>

**Write decimals in words:** Integer part + and + fractional part

**Convert decimals to mixed numbers or fractions:**

- Whole number does not change.
- Write the original term as a fraction.
  - Numerator = the fractional part
  - Denominator = a multiple of 10
    (The number of zeros = The number of decimal places)
- Simplify if possible.

**Classifying fractions:**

<table>
<thead>
<tr>
<th>Classifying fractions</th>
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<tbody>
<tr>
<td>Proper fraction</td>
</tr>
<tr>
<td>numerator &lt; denominator</td>
</tr>
<tr>
<td>Improper fraction</td>
</tr>
<tr>
<td>numerator ≥ denominator</td>
</tr>
<tr>
<td>Mixed fraction (or mixed number)</td>
</tr>
<tr>
<td>A number made up of an integer and a fraction.</td>
</tr>
<tr>
<td>Like fractions</td>
</tr>
<tr>
<td>Fractions that have the same denominators.</td>
</tr>
<tr>
<td>Unlike fractions</td>
</tr>
<tr>
<td>Fractions that have different denominators.</td>
</tr>
</tbody>
</table>

**Arithmetic terms:**

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<td>Subtrahend – minuend = difference</td>
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<tr>
<td>Multiplication</td>
<td>Multiplicand (\times) multiplier = product (factor) (factor)</td>
</tr>
<tr>
<td>Division</td>
<td>Dividend ÷ divisor = quotient &amp; remainder (factor)</td>
</tr>
</tbody>
</table>
To convert a mixed number to an improper fraction:

\[
\text{Improper fraction} = \frac{\text{whole number} \times \text{denominator} + \text{numerator}}{\text{Denominator}}
\]

To convert an improper fraction to a mixed number:

\[
\text{Mixed number} = \frac{\text{Numerator}}{\text{Denominator}} \Rightarrow \text{Quotient} \text{ Remainder} \text{ Quotient} \text{ Remainder}
\]

The standard form of percent proportion:

\[
\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100} \quad \text{or} \quad \frac{\text{"is" number}}{\text{"of" number}} = \frac{\%}{100}
\]

Converting between percent, decimal and fraction:

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent to Decimal</td>
<td>Move the decimal point two places to the left, then remove % .</td>
</tr>
<tr>
<td>Decimal to Percent</td>
<td>Move the decimal point two places to the right, then insert % .</td>
</tr>
<tr>
<td>Percent to Fraction</td>
<td>Remove % , divide by 100, then simplify.</td>
</tr>
<tr>
<td>Fraction to Percent</td>
<td>Divide, move the decimal point two places to the right, then insert % .</td>
</tr>
<tr>
<td>Decimal to Fraction</td>
<td>Convert the decimal to a percent, then convert the percent to a fraction.</td>
</tr>
</tbody>
</table>

Least common multiple (LCM): the lowest number that is divisible by each given number without a remainder.

Least common denominator (LCD): the least common multiple (LCM) of the denominators of two or more given fractions.

Find the LCD: Use repeated division to find the LCM for all denominators of given fractions.

Ratio, rate and proportion:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio</td>
<td>With the same unit.</td>
</tr>
<tr>
<td>Rate</td>
<td>With different units.</td>
</tr>
<tr>
<td>Proportion</td>
<td>The units for both numerators must match and the units for both denominators must match.</td>
</tr>
</tbody>
</table>

Solving a proportion:

- Cross multiply: multiply along two diagonals.
- Solve for the unknown.

To find the equivalent fraction: divide or multiply the numerator and denominator by the same number.
To simplify (or reduce) fractions: divide the numerator and denominator by the same number until their only common factor is 1.

\[
\text{Numerator } \div n \quad \text{Denominator } r \div n
\]

“n” is any whole number that does not equal to 0.

Operations with fractions:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adding and subtracting <em>like</em> fractions</td>
<td>- Add / subtract the numerators.</td>
</tr>
<tr>
<td></td>
<td>- Denominators do not change.</td>
</tr>
<tr>
<td></td>
<td>- Simplify if necessary.</td>
</tr>
<tr>
<td>Adding and subtracting <em>unlike</em> fractions</td>
<td>- Determine the LCD.</td>
</tr>
<tr>
<td></td>
<td>- Rewrite fractions with the LCD, and add or subtract the numerators.</td>
</tr>
<tr>
<td></td>
<td>- Simplify if necessary.</td>
</tr>
<tr>
<td>Adding and subtracting mixed numbers with <em>like</em> denominators</td>
<td>- Add / subtract whole numbers.</td>
</tr>
<tr>
<td></td>
<td>- Add / subtract as fractions.</td>
</tr>
<tr>
<td></td>
<td>- Simplify if necessary.</td>
</tr>
<tr>
<td>Adding and subtracting mixed numbers with <em>unlike</em> denominators</td>
<td>- Rewrite fractions with the LCD.</td>
</tr>
<tr>
<td></td>
<td>- Add / subtract as fractions.</td>
</tr>
<tr>
<td></td>
<td>- If the sum/difference created an improper fraction → a mixed number.</td>
</tr>
<tr>
<td>Multiplying fractions</td>
<td>- Cross - simplify if the fraction is not in lowest terms.</td>
</tr>
<tr>
<td></td>
<td>- Multiply the numerators.</td>
</tr>
<tr>
<td></td>
<td>- Multiply the denominators.</td>
</tr>
<tr>
<td></td>
<td>- Simplify the result if necessary.</td>
</tr>
<tr>
<td>Multiplying mixed numbers</td>
<td>- Convert mixed numbers to improper fractions.</td>
</tr>
<tr>
<td></td>
<td>- Cross - simplify if the fractions is not in lowest terms.</td>
</tr>
<tr>
<td></td>
<td>- Multiply the numerators.</td>
</tr>
<tr>
<td></td>
<td>- Multiply the denominators.</td>
</tr>
<tr>
<td></td>
<td>- Simplify the result if necessary.</td>
</tr>
<tr>
<td>Dividing fractions</td>
<td>- Change the divisor to its reciprocal (switch the numerator and denominator).</td>
</tr>
<tr>
<td></td>
<td>- Multiply the resulting fractions.</td>
</tr>
<tr>
<td>Dividing mixed numbers</td>
<td>- Convert mixed numbers to improper fractions.</td>
</tr>
<tr>
<td></td>
<td>- Divide fractions.</td>
</tr>
</tbody>
</table>
Unit R: Self-Test

Review of Basic Mathematics

Topic A

1. Find the prime factorization of 36.

2. a) Write the number in words: 10, 024, 526  
   b) Write the decimal in words: 47.268

3. Calculate the following without using a calculator:  
   a) 0.463 + 2.456 + 3.52  
   b) 3.21 × 2.5  
   c) 6.48 ÷ 2.4

Topic B

4. a) Convert a mixed number to an improper fraction: $4 \frac{2}{7}$  
    b) Convert an improper fraction to a mixed number: $\frac{9}{5}$

5. Reduce to lowest terms: $\frac{12}{48}$

6. 12 percent of what number is 48?

7. Convert between percent, decimal and fraction:  
   a) 45% to decimal  
   b) 0.436 to %  
   c) 25% to fraction

d) \( \frac{5}{25} \) to \%  

e) 0.4 to fraction  

f) \( 0.\overline{3} \) to Fraction  

**Topic C**  

8. a) Find the LCM of 24 and 64.  

b) Find the LCD for \( \frac{2}{5}, \frac{3}{15}, \) and \( \frac{24}{35} \)  

9. Calculate:  

a) \( \frac{1}{6} + \frac{4}{6} \)  

b) \( \frac{11}{14} - \frac{4}{14} \)  

c) \( \frac{3}{8} + \frac{5}{4} \)  

d) \( \frac{6}{7} - \frac{4}{21} \)  

e) \( 2\frac{3}{7} + 4\frac{2}{7} \)  

f) \( 7\frac{8}{12} - 5\frac{7}{12} \)  

g) \( 4\frac{9}{12} - 3\frac{2}{4} \)  

h) \( \frac{8}{10} \times \frac{5}{2} \)  

i) \( 2\frac{1}{4} \times 4\frac{4}{3} \)  

j) \( \frac{4}{9} \div \frac{8}{3} \)  

k) \( 3 \div 2\frac{5}{2} \)
Unit 1

Basic Statistics and Calculator Use

Topic A: Average
- Mean and range
- Median and mode

Topic B: Graphs
- Bar or column graph
- Line graph
- Circle or pie graph
- Create a circle graph

Topic C: Using a calculator and estimating
- Scientific calculator
- Basic functions of a scientific calculator
- Estimating and rounding

Unit 1 Summary

Unit 1 Self-test
**Topic A: Average**

**Statistics:** the mathematical branch that deals with data collection, organization, description, and analysis to draw conclusions.

**Average:** it refers to the statistical mean, median, mode, or range of a group of numbers or a set of data.

- **Mean** = average.
- **Median** = middle number.
- **Mode** = the number that occurs most often.
- **Range** = the difference between the largest and smallest values.

**Mean** (or arithmetic mean): the standard average value of a group of numbers or a set of data.

It is the most common expression for the average.

- Determine the mean: add up all the numbers in the group and divide by the number of values.

\[
\text{Mean} = \frac{\text{Sum of numbers}}{\text{Number of values}}
\]

- **Example:** Find the mean of 2, 3, 4, 0, 1.

\[
\text{Mean} = \frac{2 + 3 + 4 + 0 + 1}{5} = \frac{10}{5} = 2
\]

There are 5 numbers.

**Range:** the difference between the highest and lowest values in a group of numbers.

- Determine the range:

\[
\text{Range} = \text{highest value} - \text{lowest value}
\]

- **Example:** Find the range: 3, 5, 2, 9, 4, 8, 1

\[
\text{Range} = 9 - 1 = 8
\]
Mode: the value(s) that occurs most frequently in a group of numbers.

Example: Find the mode:
2, 4, 5, 3, 7, 8, 4, 1  Mode = 4 The value that occurs most frequently is 4.

- If no value is repeated, the mode does not exist.

Example: 13, 27, 30, 49, 47  No mode. No value is repeated.

- A bimodal has 2 modes in a group of numbers.

Example: 1, 3, 8, 17, 9, 8, 4, 6, 11, 3  Modes = 3 and 8  It has two modes.

- If more than one value occurs the same number of times, each value is a mode.

Median: the middle number of an ordered group of numbers.

Example: 1, 3, 5, 7, 9

- Determine the median: arrange the values in order (ascending or descending).
  - Ascending order: numbers are arranged from the smallest to the largest number.
  - Descending order: numbers are arranged from the largest to the smallest number.

- If the total number of terms in the group is odd, the median is the middle number.

Example: Find the median of  2, 8, 7, 1, 6, 5, 3, 4, 8, 1, 9  11 numbers (odd)
  - Ascending order: 1, 1, 2, 3, 4, 5, 6, 7, 8, 8, 9
  - Median = 5 5 is the middle number.

- If the total number of terms in the group is even, the median is the average of the two values in the middle (add two middle numbers and divide by 2):

  \[
  \text{Median} = \frac{\text{Add two middle values}}{2}
  \]

Example: Find the median of 5, 4, 9, 0, 2, 6  6 numbers (even)
  - Ascending order: 0, 2, 4, 5, 6, 9
  - Median = \(\frac{4+5}{2} = 4.5\) 4 and 5 are the middle numbers.
  - Or descending order: 9, 6, 5, 4, 2, 0
  - Median = \(\frac{5+4}{2} = 4.5\)
Topic B: Graphs

Bar or Column Graph

Bar or column graph: a chart with rectangular bars whose heights or lengths display the values. (It used to compare information between different groups.)

A bar graph can be vertical (column graph) or horizontal (bar graph).

Create a bar (or column) graph:

- Put data into tabular form (make a table).
- Label each axis and make up a title for the graph.
- Create a scale (number) for each axis starting from zero.
- Draw bars or columns (use the data from the table).

Table: a group of numbers arranged in a condensed form of columns and rows. It is a more effective way to present information.

Interpolate and extrapolate from the information provided:

Example: Make a graph from the table and answer questions.

<table>
<thead>
<tr>
<th>Student</th>
<th>Test score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adam</td>
<td>75%</td>
</tr>
<tr>
<td>John</td>
<td>65%</td>
</tr>
<tr>
<td>Karen</td>
<td>90%</td>
</tr>
<tr>
<td>Mike</td>
<td>85%</td>
</tr>
<tr>
<td>Steve</td>
<td>60%</td>
</tr>
<tr>
<td>Alice</td>
<td>80%</td>
</tr>
</tbody>
</table>

Example: Bar’s height displays the student score.

a) How many students earned 80% or greater?  
3 students (80, 85, 90)

b) How many students earned 60%?  
1 student (60)

c) How many more students earned between 59% and 81%?  
4 students (60, 65, 75, 80)
**Line Graph**

**Line graph:** a chart that displays information by connecting lines between data points. It is used to track changes over periods of time.

A line graph consists of a horizontal x-axis and a vertical y-axis.
- Horizontal x-axis: represents the independent variable (such as time).
- Vertical y-axis: represents the dependent variable (such as temperature, population, sales, rainfall, etc.).

Create a line graph:
- Put data into tabular form (make a table).
- Label each axis and make up a title for the graph. Example: horizontal axis — months of the year, vertical axis — temperature
- Create a scale for each axis. Example: horizontal axis — Jan., Feb., Mar., April ..., vertical axis — 0°C, 5°C, 10°C, 15°C ...
- Plot the data points (use the data from the table).
- Draw a curve (or a line) that best fits the data points (connect the points).

Example of a line graph:

**Average temperatures in Prince George**

<table>
<thead>
<tr>
<th>Month</th>
<th>Temperature °C (Low)</th>
<th>Temperature °C (High)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>-16</td>
<td>-5</td>
</tr>
<tr>
<td>Feb</td>
<td>-14</td>
<td>-1</td>
</tr>
<tr>
<td>March</td>
<td>-8</td>
<td>6</td>
</tr>
<tr>
<td>April</td>
<td>-3</td>
<td>12</td>
</tr>
<tr>
<td>May</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>June</td>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>July</td>
<td>7</td>
<td>24</td>
</tr>
<tr>
<td>Aug</td>
<td>6</td>
<td>23</td>
</tr>
<tr>
<td>Sept</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>Oct</td>
<td>-1</td>
<td>11</td>
</tr>
<tr>
<td>Nov</td>
<td>-6</td>
<td>3</td>
</tr>
<tr>
<td>Dec</td>
<td>-13</td>
<td>-4</td>
</tr>
</tbody>
</table>

**Average Temperatures in Prince George (°C)**
Circle or Pie Graph

**Circle (or pie) graph:** a chart made by dividing a circle into sections (parts) that each represent a percentage of the total. It is used to compare parts of a whole.

- Entire pie: represents the total amount ($360^\circ$).
- Sectors: represent percentages of the total.

Create a circle graph:

- Put data into tabular form (make a table).
- Calculate the total amount.
- Determine the percentage of each sector or part.

\[
\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100} \quad \text{or} \quad \text{Percent} = \frac{\text{Part}}{\text{Whole}} \cdot 100
\]

- Determine the angle of each sector (convert the percent to a decimal first).

\[
\text{Angle for each part} = (\text{Decimal}) \cdot (360^\circ)
\]

- Draw a circle (use a compass) and a radius ($r$).

- Draw in the sectors of the circle (use a protractor), and add colors to the sectors (this will help to make them easier to distinguish).
- Label the sectors and make up a title for the graph.

**How to use a protractor:**

- Place the protractor on the circle so that the center mark of the protractor at the center of the circle.
- Ensure that the radius of the circle is lined up on the zero line at the end of the protractor.
- Draw the sector by using the calculated angle.

Each time you add a sector the radius changes to the line you just drew.
Create a Circle Graph

Example: Create a circle graph using the following table — final grades in a math class.

<table>
<thead>
<tr>
<th>Final grades in a math class</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
</tr>
<tr>
<td><strong>Total number of students:</strong></td>
<td><strong>10</strong></td>
</tr>
</tbody>
</table>

- The total number of students: \(1 + 2 + 4 + 3 = 10\)
  There are 10 students in the class.

- Determine the percentage of each sector (convert the percent to a decimal):
  - First sector in the circle chart:
    \[
    \frac{1}{10} = \frac{\text{Percent}}{100}, \quad \% = \frac{1 \times 100}{\frac{10}{10}} = 10\% = 0.1
    \]
  - Second sector in the circle chart:
    \[
    \frac{2}{10} = \frac{\text{Percent}}{100}, \quad \% = \frac{2 \times 100}{\frac{10}{10}} = 20\% = 0.2
    \]
  - Third sector in the circle chart:
    \[
    \frac{4}{10} = \frac{\text{Percent}}{100}, \quad \% = \frac{4 \times 100}{\frac{10}{10}} = 40\% = 0.4
    \]
  - Fourth sector in the circle chart:
    \[
    \frac{3}{10} = \frac{\text{Percent}}{100}, \quad \% = \frac{3 \times 100}{\frac{10}{10}} = 30\% = 0.3
    \]

- Determine the angle of each sector:
  - First sector in the circle chart: \((\text{Decimal}) (360^\circ) = (0.1) (360^\circ) = 36^\circ\)
  - Second sector in the circle chart: \((\text{Decimal}) (360^\circ) = (0.2) (360^\circ) = 72^\circ\)
  - Third sector in the circle chart: \((\text{Decimal}) (360^\circ) = (0.4) (360^\circ) = 144^\circ\)
  - Fourth sector in the circle chart: \((\text{Decimal}) (360^\circ) = (0.3) (360^\circ) = 108^\circ\)

<table>
<thead>
<tr>
<th>Percent</th>
<th>Decimal</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>(0.1)</td>
<td>36^\circ</td>
</tr>
<tr>
<td>20%</td>
<td>(0.2)</td>
<td>72^\circ</td>
</tr>
<tr>
<td>40%</td>
<td>(0.4)</td>
<td>144^\circ</td>
</tr>
<tr>
<td>30%</td>
<td>(0.3)</td>
<td>108^\circ</td>
</tr>
<tr>
<td><strong>Total:</strong> 100%</td>
<td>(1)</td>
<td>Total: 360^\circ</td>
</tr>
</tbody>
</table>

Check: The sum of the percentages = 100%. The sum of all the degrees should be = 360^\circ.

- Draw the circle graph:
Scientific calculator: a calculator with advanced functions that can solve mathematics, science, and engineering problems.

**Basic functions of a scientific calculator**

- Basic functions (+, −, ×, ÷)
- Parentheses
- Absolute values (abs)
- Order of operations
- Exponents or powers
- Pi problems ($\pi = 3.141592654...$)
- Fractions
- Scientific notation
- Trigonometry functions (sine, cosine, tangent)
- Etc.

**Identify main keys:**
Basic Functions of a Scientific Calculator

**Basic features:**

<table>
<thead>
<tr>
<th>Operation</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>Addition</td>
</tr>
<tr>
<td>-</td>
<td>Subtraction</td>
</tr>
<tr>
<td>×</td>
<td>Multiplication</td>
</tr>
<tr>
<td>÷</td>
<td>Division</td>
</tr>
<tr>
<td>(−) or neg</td>
<td>Negative number</td>
</tr>
<tr>
<td>$x^2$</td>
<td>Squaring</td>
</tr>
<tr>
<td>$x^n$ or $y^x$ or $^*$</td>
<td>Exponent or power</td>
</tr>
<tr>
<td>√ or Sqrt</td>
<td>Square root</td>
</tr>
<tr>
<td>$\sqrt[3]{i}$</td>
<td>Cube root</td>
</tr>
<tr>
<td>$\sqrt[n]{i}$</td>
<td>nth root</td>
</tr>
<tr>
<td>( )</td>
<td>Parentheses</td>
</tr>
<tr>
<td>π</td>
<td>Pi</td>
</tr>
<tr>
<td>Shift or 2nd F or INV</td>
<td>Converting between degrees and radians</td>
</tr>
<tr>
<td>$\frac{a}{b}$ or d/c</td>
<td>Fraction</td>
</tr>
<tr>
<td>$\frac{a}{c}$ or a b/c</td>
<td>Mixed number</td>
</tr>
<tr>
<td>Exp or $\times 10^i$</td>
<td>Scientific notation</td>
</tr>
<tr>
<td>sin, cos, tan</td>
<td>Trigonometry functions</td>
</tr>
<tr>
<td>sin$^{-1}$, cos$^{-1}$, tan$^{-1}$</td>
<td>Inverse trigonometry functions</td>
</tr>
</tbody>
</table>

**Determine what order you need to press the keys** (it may vary with different calculators).

**Examples:**

1) $21 + 34 \times 5 = ?$

\[
21 + 34 \times 5 \quad \quad \quad \text{Display: 191}
\]

2) $\frac{432}{6} + \pi = ?$

\[
\frac{432}{6} + \pi \quad \quad \quad \text{Display: 75.14159…}
\]

3) $27^2 + 38 \times 17 = ?$

\[
27^2 + 38 \times 17 \quad \quad \quad \text{Display: 1375}
\]

4) $3\frac{1}{4} + 2\frac{3}{5} = ?$

\[
\text{Shift} \quad \frac{3}{4} + 2 \frac{3}{5} \quad \quad \quad \text{Display: 5} \frac{17}{20} \quad \text{or} \quad 5.85
\]

5) $\sqrt[3]{27} + 2^3 = ?$

\[
\text{Shift} \quad \sqrt[3]{27} + 2^3 \quad \quad \quad \text{Display: 11}
\]

or 2nd F $\sqrt[3]{27} + 2^3 \quad \quad \quad \text{Display: 11}$
**Rounding whole numbers:** choose an approximation for a whole number (making a number simpler).

**The method of rounding:**

- If the rounding digit (next digit) is ≥ 5 (greater than or equals to), round-up (add 1 to the left digit of the rounding digit and replace all the digits to the right of the rounding digit with 0).
- If the rounding digit is < 5 (less than), round down (do not change the left digit of the rounding digit, replace the rounding digit and all the digits to the right of it with 0).

**Example:**

1) Round to the nearest **largest place.** 3,459,567 ≈ \(3,000,000\)  
   The rounding digit is 4, 4 < 5 round down

2) Round to the nearest **ten.** 345 ≈ \(350\)  
   The rounding digit is 5, 5 ≥ 5 round-up

3) Round to the nearest **hundred.** 3,429 ≈ \(3,400\)  
   The rounding digit is 2, 2 < 5 round down

4) Round to the nearest **thousand.** 27,656 ≈ \(28,000\)  
   The rounding digit is 6, 6 > 5 round-up

**Estimate:** find a value that can be used to check if an answer is reasonable (approximating).

**Method of estimating:** round to the largest place value.

- If the next digit is ≥ 5, round-up.
- If the next digit is < 5, round down.

**Example:** Estimate the following.

1) \[7656 \approx 8000\]
   \[+ 4358 \approx + 4000\]
   \[\approx 12000\]
   The next digit of 7 is 6 (6 > 5, round-up).
   The next digit of 4 is 3 (3 < 5, round down).

2) \[8756 \approx 9000\]
   \[− 5432 \approx − 5000\]
   \[\approx 4000\]
   The next digit of 8 is 7 (7 > 5, round-up).
   The next digit of 5 is 4 (4 < 5, round down).

3) \[5378 \times 367 \approx 5000 \times 400 = 2,000,000\]

4) \[7576 \div 237 \approx 8000 \div 200 = 40\]
Unit 1: Summary

Basic Statistics and Calculator Use

Graphs

- **Bar or column graph**: a chart with rectangular bars whose heights or lengths display the values. (It used to compare values between different groups.)
  
  Construct a bar or column graph: page 23.

- **Line graph**: a chart that displays information by connecting lines between data points. (It is used to track changes over periods of time).
  
  Construct a line graph: page 24.

- **Circle graph**: a chart made by dividing a circle into sections (parts) that each represent a percentage of the total. (It is used to compare parts of a whole.)
  

- **Average**:

<table>
<thead>
<tr>
<th>Average/Range</th>
<th>Description / Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>The &quot;standard&quot; average value of a group of numbers or a set of data.</td>
</tr>
</tbody>
</table>
  |               | Mean = \[
  |               | \frac{\text{Sum of numbers}}{\text{Number of values}} \] |
  | **Median**   | The middle number of an ordered group of numbers. |
  |               | - Arrange the values in order. |
  |               | - If the total number of terms in the group is odd, the median is the middle number. |
  |               | - If the total number of terms in the sample is even: |
  |               | Median = \[
  |               | \frac{\text{Add two middle values}}{2} \] |
  | **Mode**     | The value(s) that occurs most frequently in a group of numbers. |
  |               | - If no value is repeated, the mode does not exist. |
  |               | - If more than one value occurs with the same frequency, each value is a mode. |
  |               | - A bimodal has 2 modes in a group of numbers. |
  | **Range**    | The difference between the highest and lowest values in a group of numbers. |
  |               | Range = \[
  |               | \text{highest value} - \text{lowest value} \] |

Scientific calculator

- **Scientific calculator**: a calculator with advanced functions that can solve mathematics, science, and engineering problems.
• **Basic functions of a scientific calculator:**

<table>
<thead>
<tr>
<th>Operation</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>Addition</td>
</tr>
<tr>
<td>−</td>
<td>Subtraction</td>
</tr>
<tr>
<td>×</td>
<td>Multiplication</td>
</tr>
<tr>
<td>÷</td>
<td>Division</td>
</tr>
<tr>
<td>(−) or neg</td>
<td>Negative number</td>
</tr>
<tr>
<td>(x^2)</td>
<td>Squaring</td>
</tr>
<tr>
<td>(x^y) or (y^x)</td>
<td>Exponent or power</td>
</tr>
<tr>
<td>√ or Sqrt</td>
<td>Square root</td>
</tr>
<tr>
<td>(\sqrt[3]{\phantom{0}})</td>
<td>Cube root</td>
</tr>
<tr>
<td>(\sqrt[n]{\phantom{0}})</td>
<td>nth root</td>
</tr>
<tr>
<td>( )</td>
<td>Parentheses</td>
</tr>
<tr>
<td>π</td>
<td>Pi</td>
</tr>
<tr>
<td>Shift or 2nd F or INV</td>
<td>Converting between degrees and radians</td>
</tr>
<tr>
<td>– or d/c</td>
<td>Fraction</td>
</tr>
<tr>
<td>or a b/c</td>
<td>Mixed number</td>
</tr>
<tr>
<td>Exp or (\times 10^x)</td>
<td>Scientific notation</td>
</tr>
<tr>
<td>sin, cos, tan</td>
<td>Trigonometry functions</td>
</tr>
<tr>
<td>sin(^{-1}), cos(^{-1}), tan(^{-1})</td>
<td>Inverse trigonometry functions</td>
</tr>
</tbody>
</table>

**Rounding**

• Rounding whole numbers: choose an approximation for a number.

• The method of rounding:
  - If the rounding digit (next digit) is \(\geq 5\), round-up.
  - If the rounding digit is \(< 5\) (less than), round down.

**Estimating**

• Estimate: find a value that can be used to check if an answer is reasonable.

• Method of estimating: round to the largest place value.
  - If the next digit is \(\geq 5\), round-up.
  - If the next digit is \(< 5\), round down.
Unit 1: Self-Test

Basic Statistics and Calculator Use

Topic A

1. Find the mean: 4, 0, 5, 10, 9, 2
2. Find the range: 11, 7, 2, 6, 9, 13, 3
3. Find the mode:
   a) 12, 4, 7, 3, 9, 51, 6, 7
   b) 21, 13, 4, 16, 54, 100
4. Find the median:
   a) 4, 6, 7, 10, 9, 11, 3, 8, 5, 1, 14, 2, 23
   b) 6, 14, 10, 11, 0, 19, 5, 4

Topic B

5. Create a column graph from the table and answer the following questions:

<table>
<thead>
<tr>
<th>Student</th>
<th>Test score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evan</td>
<td>85%</td>
</tr>
<tr>
<td>Jon</td>
<td>75%</td>
</tr>
<tr>
<td>Alice</td>
<td>90%</td>
</tr>
<tr>
<td>Tom</td>
<td>65%</td>
</tr>
<tr>
<td>Damon</td>
<td>95%</td>
</tr>
<tr>
<td>Steve</td>
<td>70%</td>
</tr>
</tbody>
</table>

   a) How many students earned 85% or greater?
   b) How many students earned 75%?
   c) How many more students earned between 64% and 91%?

6. Create a line graph from the table (average temperatures in Vancouver):

<table>
<thead>
<tr>
<th>Month</th>
<th>Temperature °C (High)</th>
<th>Temperature °C (Low)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Feb</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>March</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>April</td>
<td>13</td>
<td>6</td>
</tr>
<tr>
<td>May</td>
<td>17</td>
<td>9</td>
</tr>
<tr>
<td>June</td>
<td>20</td>
<td>12</td>
</tr>
<tr>
<td>July</td>
<td>22</td>
<td>14</td>
</tr>
<tr>
<td>Aug</td>
<td>22</td>
<td>14</td>
</tr>
<tr>
<td>Sept</td>
<td>19</td>
<td>11</td>
</tr>
<tr>
<td>Oct</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>Nov</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Dec</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>
7. Create a circle graph from the table (Tom’s monthly expenses):

<table>
<thead>
<tr>
<th>Tom</th>
<th>Monthly Expenses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent</td>
<td>$600</td>
</tr>
<tr>
<td>Food</td>
<td>$300</td>
</tr>
<tr>
<td>Transportation</td>
<td>$60</td>
</tr>
<tr>
<td>Utilities</td>
<td>$80</td>
</tr>
<tr>
<td>Clothing</td>
<td>$85</td>
</tr>
<tr>
<td>Entertainment</td>
<td>$165</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>$35</td>
</tr>
</tbody>
</table>

**Topic C**

8. Complete the following with your calculator:
   a) \(78 + 43 \times 11\)
   b) \(\frac{2468}{8} + \pi\)
   c) \(42^2 + 43 \times 25\)
   d) \(4 \frac{1}{6} + 3 \frac{4}{7}\)
   e) \(3\sqrt{125} + 3^5\)

9. Rounding:
   a) Round to the nearest largest place. 6,345,789
   b) Round to the nearest ten. 567
   c) Round to the nearest hundred. 8,649
   d) Round to the nearest thousand. 47,567

10. Estimate the following:
    a) \(79,215 + 784\)
    b) \(11,345 - 372\)
    c) \(4,738 \times 624\)
    d) \(8,345 \div 382\)
Unit 2
Introduction to Algebra

Topic A: Algebraic expressions
- Basic algebraic terms
- Evaluating algebraic expressions

Topic B: Translating words into algebraic expressions
- Key words in word problems
- Translating phrases into algebraic expressions
- Writing algebraic expressions
- Steps for solving word problems

Topic C: Exponents and order of operations
- Introduction to exponents
- Read and write exponential expressions
- Order of operations

Unit 2 Summary

Unit 2 Self-test


**Topic A: Algebraic Expressions**

### Basic Algebraic Terms

**Algebra:** a branch of mathematics containing numbers, letters and arithmetic operators (+, −, ×, ÷, etc.) with the letters used to represent unknown quantities (variables).

**Example:** 3 + 2 = 5 in algebra may look like $x + 2 = 5$  \(x\) represents 3.

**Constant:** a *number* stands for a fixed value that does not change.

**Example:** 2 in $x + 2$ is a constant.

**Variable:** a *letter* that can be assigned different values (it represents an unknown quantity).

**Example:** $x + 2$ when \(x = 0\), \(x + 2 = 0 + 2 = 2\)

when \(x = 3\), \(x + 2 = 3 + 2 = 5\)

**Coefficient:** the *number* that in front of a letter (variable).

**Example:**
- \(9x\) coefficient: \(9\)
- \(-\frac{2}{7}x\) coefficient: \(-\frac{2}{7}\)
- \(x\) coefficient: \(1\)  \(x = 1 \cdot x\)

**Algebraic expression:** a mathematical phrase that contains numbers, letters, grouping symbols (parentheses) and arithmetic operations (+, −, ×, ÷, etc.)

**Example:** \(5x + 2, \ \frac{2y}{3} + 4, \ (3x - 4y^2) + 6\)

**Term:** a term can be a number, letter, or the product (multiplication) of a number and letter. (Terms are separated by addition or subtraction signs.)

**Example:**

- a) \(3x - 4 + \frac{2}{5} + y\) has four terms: \(3x, -4, \frac{2}{5},\) and \(y\).

- b) \(7xyz + 12 - \frac{4}{19}z^2\) has three terms: \(7xyz, 12,\) and \(-\frac{4}{19}z^2\).

**Like terms:** the terms that have the same variables and exponents.

**Example:** \(2x - 3y^2 - \frac{6}{7} + 5x + 9 + 4y^2\)

Like terms: \(2x\) and \(5x\)  \(\text{The same variable: } x\)

- \(-3y^2\) and \(4y^2\)  \(\text{The same variable raised to the same power: } y^2\)

- \(-\frac{6}{7}\) and \(9\)  \(\text{All constants are like terms.}\)
Evaluating an algebraic expression: substitute a specific value for a variable and perform the mathematical operations (+, −, ×, ÷, etc.).

Note:
- In algebra, a multiplication sign “×” is usually omitted to avoid confusing it with the letter x.
- If there is no symbol or sign between a number and letter, it means multiplication, such as $3x = 3 \cdot x$.

Steps to evaluate an algebraic expression:
- Replace the variable(s) with number(s).
- Calculate.

Example: Evaluate the following algebraic expressions.

1) $3x - 4$, given $x = 5$.

\[
3x - 4 = 3 \cdot 5 - 4 \\
= 15 - 4 \\
= 11
\]

2) $\frac{x}{y} + 8$, given $x = -9$ and $y = 3$.

\[
\frac{x}{y} + 8 = \frac{-9}{3} + 8 \\
= 5
\]

3) $3a - 4 + 2$, given $a = 5$.

\[
3a - 4 + 2 = 3 \cdot 5 - 4 + 2 \\
= 15 - 4 + 2 \\
= 13
\]

4) $\frac{6x}{y} + 7x - 2$, given $x = 1$ and $y = 9$.

\[
\frac{6x}{y} + 7x - 2 = \frac{6 \cdot 1}{9 - 3} + 7 \cdot 1 - 2 \\
= \frac{6}{6} + 7 - 2 \\
= 6
\]
Topic B: Translating Words into Algebraic Expressions

### Key Words in Word Problems

#### Identifying keywords:
- When trying to figure out the correct operation (+, −, ×, ÷, etc.) in the word problem it is important to pay attention to keywords (clues to what the problem is asking).
- Identifying keywords and pulling out relevant information that appear in the word problem are effective ways for solving mathematical word problems.

#### Key or clue words in word problems

<table>
<thead>
<tr>
<th>Addition (+)</th>
<th>Subtraction (−)</th>
<th>Multiplication (×)</th>
<th>Division (÷)</th>
<th>Equals to (=)</th>
</tr>
</thead>
<tbody>
<tr>
<td>add</td>
<td>subtract</td>
<td>times</td>
<td>divided by</td>
<td>equals</td>
</tr>
<tr>
<td>sum (of)</td>
<td>difference</td>
<td>product</td>
<td>quotient</td>
<td>is</td>
</tr>
<tr>
<td>plus</td>
<td>take away</td>
<td>multiplied by</td>
<td>over</td>
<td>was</td>
</tr>
<tr>
<td>total (of)</td>
<td>minus</td>
<td>double</td>
<td>split up</td>
<td>are</td>
</tr>
<tr>
<td>altogether</td>
<td>less (than)</td>
<td>twice</td>
<td>fit into</td>
<td>were</td>
</tr>
<tr>
<td>increased by</td>
<td>decreased by</td>
<td>triple</td>
<td>per</td>
<td>amounts to</td>
</tr>
<tr>
<td>gain (of)</td>
<td>loss (of)</td>
<td>of</td>
<td>each</td>
<td>totals</td>
</tr>
<tr>
<td>combined</td>
<td>(amount) left</td>
<td>how much (total)</td>
<td>goes into</td>
<td>results in</td>
</tr>
<tr>
<td>in all</td>
<td>savings</td>
<td>how many</td>
<td>as much as</td>
<td>the same as</td>
</tr>
<tr>
<td>greater than</td>
<td>withdraw</td>
<td>out of</td>
<td>gives</td>
<td></td>
</tr>
<tr>
<td>complete</td>
<td>reduced by</td>
<td>ratio /rate</td>
<td>yields</td>
<td></td>
</tr>
<tr>
<td>together</td>
<td>fewer (than)</td>
<td>percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>more (than)</td>
<td>how much more</td>
<td>share</td>
<td></td>
<td></td>
</tr>
<tr>
<td>additional</td>
<td>how long</td>
<td>average</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Examples:

1) Edward drove from Prince George to Williams Lake (235 km), then to Cache Creek (203 km) and finally to Vancouver (390 km). How many kilometers in total did Edward drive? 

\[ 235 \text{ km} + 203 \text{ km} + 390 \text{ km} = 828 \text{ km} \]

The key word: total (+)

2) Emma had $150 in her purse on Friday. She bought a pizza for $15, and a pair of shoes for $35. How much money does she have left?

\[ $150 - 15 - 35 = $100 \]

The key word: left (−)

3) Lucy received $950 per month of rent from Mark for the months September to November. How much rent in total did she receive?

\[ $950 \cdot 3 = $2850 \]

The key word: how much total (×)

4) Julia is going to buy a $7500 used car from her uncle. She promises to pay $500 per month, in how many months can she pay it off?

\[ $7500 \div 500 = 15 \text{ month} \]

The key word: per (+)
Translating Phrases into Algebraic Expressions

Method to translate words into algebraic expression:

- Look for basic key words for translating word problems from English into algebraic expressions.
- Translate English words into mathematical symbols (the language of mathematics).

Translate words into algebraic expression:

<table>
<thead>
<tr>
<th>Algebraic expression</th>
<th>Word phrases</th>
<th>Algebraic expression</th>
<th>Word phrases</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 + y</td>
<td>the sum of 7 and y</td>
<td>t – 8</td>
<td>8 less than t</td>
</tr>
<tr>
<td></td>
<td>7 more than y</td>
<td></td>
<td>t decreased (or reduced) by 8</td>
</tr>
<tr>
<td></td>
<td>y increased by 7</td>
<td></td>
<td>subtract 8 from t</td>
</tr>
<tr>
<td></td>
<td>7 plus y</td>
<td></td>
<td>the difference between t and 8</td>
</tr>
<tr>
<td>2x or 2 · x</td>
<td>the product of 2 and x</td>
<td>z ÷ 3 or z/3</td>
<td>The quotient of z and 3</td>
</tr>
<tr>
<td></td>
<td>2 multiplied by x</td>
<td></td>
<td>z divided by 3</td>
</tr>
<tr>
<td></td>
<td>double (or twice) of x</td>
<td></td>
<td>One third of z</td>
</tr>
<tr>
<td>y³</td>
<td>The third power of y</td>
<td>4y – 9</td>
<td>9 less than 4 times y</td>
</tr>
<tr>
<td></td>
<td>y cubed</td>
<td>2(t – 5)</td>
<td>Twice the difference of t and 5</td>
</tr>
<tr>
<td></td>
<td>y raised to the third power</td>
<td>6 + 2x/3</td>
<td>6 more than the quotient of 2x by 3</td>
</tr>
</tbody>
</table>

Note:

- The order of the subtraction and division is important when translate words into algebraic expression.
- Place the numbers in the correct order for subtraction and division.

Example:

1) The difference between t and 7 means t – 7 not 7 – t. t appears first.
2) 8 less than t means t – 8 not 8 – t. 8 less than t not t less than 8.
3) The quotient of z and 3 means z/3 not 3/z. z appears first.
Writing Algebraic Expressions

**Example:** Write a mathematical equation for each of the following:

1) Five greater than four divided by a number is seventeen.

\[ 5 + \frac{4}{x} = 17 \]

(Let \( x = a \) number)

2) A number is 7 times the number \( y \) added to 23.

\[ x = 7y + 23 \]

(Let \( x = a \) number)

---

**Example:** Write an algebraic expression for each of the following:

1) The difference of \( y \) and 3.45.

\[ y - 3.45 \]

2) The difference of \( \frac{4}{23} \) and \( w \).

\[ \frac{4}{23} - w \]

3) \( z \) less than the number 67.

\[ 67 - z \]

4) 27 minus the product of 18 and a number.

\[ 27 - 18x \]

(Let \( x = a \) number)

5) The sum of a number and 7 divided by 2.

\[ \frac{x + 7}{2} \]

6) Steve has $200 in his saving account. If he makes a deposit of \( x \) dollars, how much in total will he have in his account?

\[ 200 + x \]

7) Ann weighs 150 pounds. If she loses \( y \) pounds, how much will she weigh?

\[ 150 - y \]

8) A piece of wire 30 centimeters long was cut in two pieces and one piece is \( z \) centimeters long. How long is the other piece?

\[ 30 - z \]

9) Alice made 3 dozen cupcakes. If it costs her \( y \) dollars, what was her cost per dozen cupcakes? What was her cost per cupcake?

\[ \frac{y}{3} , \frac{y}{36} \]

(1 dozen = 12, \( 3 \cdot 12 = 36 \))
Steps for solving word problems:

<table>
<thead>
<tr>
<th>Steps for solving word problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>- <strong>Organize</strong> the facts given from the problem (create a <strong>table</strong> or <strong>diagram</strong> if it will make the problem clearer).</td>
</tr>
<tr>
<td>- Identify and label the unknown quantity (<strong>let</strong> ( x ) = unknown).</td>
</tr>
<tr>
<td>- Convert words into mathematical symbols, and <strong>determine the operation</strong> – write an <strong>equation</strong> (looking for ‘key’ or ‘clue’ words).</td>
</tr>
<tr>
<td>- <strong>Estimate</strong> and <strong>solve</strong> the equation and find the solution(s).</td>
</tr>
<tr>
<td>- <strong>Check</strong> and state the <strong>answer</strong>.</td>
</tr>
</tbody>
</table>

(Check the solution to the equation and check it back into the problem – is it logical?)

**Example to illustrate the steps involved**

**Example:** William bought 5 pairs of socks for $4.35 each. The cashier charged him an additional $2.15 in sales tax. He left the store with a measly $5.15. How much money did William start with?

- **Organize the facts** (make a table):

<table>
<thead>
<tr>
<th>5 socks</th>
<th>$4.35 each</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales tax</td>
<td>$2.15</td>
</tr>
<tr>
<td>Money left</td>
<td>$5.15</td>
</tr>
</tbody>
</table>

- **Determine the unknown**: How much did William start with? \( (x = ?) \)

- **Convert words into math symbols, and determine the operation** (find **key words**):
  
  - The **total** cost without the sales tax: \( $4.35 \times 5 \)
  
  - With an **additional** $2.15 sales tax: \( ($4.35 \times 5) + $2.15 \)
  
  - William started with: \( x = [($4.35 \times 5) + $2.15] + $5.15 \)

- **Estimate and solve the unknown**:
  
  - Estimate: \( x = [($4 \times 5) + $2] + $5 = $27 \)
  
  - Actual solution: \( x = [($4.35 \times 5) + $2.15] + $5.15 = $29.05 \)
• Check: If William started with $29.05, and subtract 5 socks for $4.35 each and sales tax in $2.15 to see if it equals $5.15.

\[ ? \]
\[ $29.05 - [\left( $4.35 \times 5 \right) + $2.15] = $5.15 \]
\[ $29.05 - $23.9 = $5.15 \]
Correct!

• State the answer: William started with $29.05.

More examples:

Example: James had 96 toys. He sold 13 on first day, 32 on second day, 21 on third day, 14 on fourth day and 7 on the last day. What percentage of the toys were not sold?

• Organize the facts:

<table>
<thead>
<tr>
<th>James had</th>
<th>96 toys</th>
</tr>
</thead>
<tbody>
<tr>
<td>The total number of toys sold</td>
<td>13 + 32 + 21 + 14 + 7</td>
</tr>
<tr>
<td>The toys not sold</td>
<td>96 – the total number of toys sold</td>
</tr>
</tbody>
</table>

• Determine the unknown: Let \( x \) = percentage of the toys were not sold

• The total number of toys sold: \( 13 + 32 + 21 + 14 + 7 = 87 \)

• The toys not sold: \( 96 - 87 = 9 \)

• Percentage of the toys were not sold: \( x = \frac{\text{Toys not sold}}{\text{Total number of toys}} = \frac{9}{96} \approx 0.094 = 9.4\% \)

• State the answer: 9.4\% percentage of the toys were not sold.

Example: The 60-liter gas tank in Robert’s car is 1/2 full. Kelowna is about 390 km from Vancouver and his car averages 7 liters per 100 km. Can Robert make his trip to Vancouver?

• Let \( x \) = liters of fuel are required to get to Vancouver.

• The 60-liter gas tank in Robert’s car is 1/2 full:

\[ 60 \text{ L} \times \frac{1}{2} = 30 \text{ L} \]
Robert has 30 liters gas in his car.

• Robert’s car averages 7 liters per 100 km, and Vancouver is about 390 km from Kelowna.

\[ \frac{7 \text{ L}}{100 \text{ km}} = \frac{x}{390 \text{ km}} \]
\[ (x)(100\text{km}) = (7 \text{ L})(390 \text{ km}) \]
Proportion: \( \frac{a}{b} = \frac{c}{d} \)
Cross multiply and solve for \( x \).

\[ x = \frac{(7 \text{ L})(390 \text{ km})}{100 \text{ km}} = \frac{27.3 \text{ L}}{} \]
Robert needs 27.3 liters gas to get to Vancouver.

• State the answer: 30L > 27.3L  Yes, Robert can make his trip.
**Topic C: Exponents and Order of Operations**

### Introduction to Exponents

**Power:** the *product* of a number repeatedly multiplied by itself.

**Example:** \(3^2 = 3 \cdot 3 = 9\), the “3²” is the *product* of 3 repeatedly multiplied by itself.

**Exponent:** the *number of times* a number is multiplied by itself.

**Example:** In \(3^2\), the “2” means 3 is multiplied by itself *two times*.

**Base, exponent and power:**

\[a^n \begin{cases} 
    a & \text{is the base.} \\
    n & \text{is the exponent.} \\
    a^n & \text{is the power}
\end{cases}\]

**Exponential notation (exponential expression):** \(a^n\) or \(\text{Base}^\text{Exponent}\)

<table>
<thead>
<tr>
<th>Exponential notation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a^n = a \cdot a \cdot a \ldots a)</td>
<td>(2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16)</td>
</tr>
<tr>
<td>Read “a to the nth” or “the nth power of a.”</td>
<td>Read “2 to the 4th.”</td>
</tr>
</tbody>
</table>

Exponents make it easier to write very long numbers (for multiplications).

**Any non-zero number to the zero power equals 1 \((a^0 = 1)\).**

**Example:** \(2^0 = 1\), \(13000^0 = 1\)  
\(0^0\) is undefined.

**Any number raised to the power of 1 equals the number itself \((a^1 = a)\).**

**Example:** \(4^1 = 4\), \(1000^1 = 1000\)  
Anything raised to the first power is itself.  
\((4 \text{ is multiplied by itself one time})\)

**1 raised to any power is still 1 \((1^n = 1)\).**

**Example:** \(1^3 = 1\), \(1^{10000} = 1\)  
\(1^3 = 1 \cdot 1 \cdot 1 = 1\)

**Exponents: basic properties:**

<table>
<thead>
<tr>
<th>Name</th>
<th>Property</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero exponent</td>
<td>(a^0)</td>
<td>(a^0 = 1)</td>
</tr>
<tr>
<td></td>
<td>((0^0) is undefined)</td>
<td>(\left(\frac{3}{4}\right)^0 = 1) , ((2xy)^0 = 1)</td>
</tr>
<tr>
<td>One exponent</td>
<td>(a^1 = a)</td>
<td>(4.5^1 = 4.5) , ((3x)^1 = 3x)</td>
</tr>
<tr>
<td></td>
<td>(1^a = 1)</td>
<td>(1^7 = 1) , (1^{389} = 1)</td>
</tr>
</tbody>
</table>
Read and Write Exponential Expressions

How to read exponent expressions:

<table>
<thead>
<tr>
<th>Base</th>
<th>Exponent</th>
<th>Repeated multiplication</th>
<th>Product</th>
<th>Read</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>3 \cdot 3</td>
<td>9</td>
<td>3(^2) = 3 squared</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>10 \cdot 10 \cdot 10</td>
<td>1000</td>
<td>10(^3) = 10 cubed</td>
</tr>
<tr>
<td>(0.2)</td>
<td>2</td>
<td>0.2 \cdot 0.2</td>
<td>0.04</td>
<td>(0.2)(^2) = 0.2 squared</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1</td>
<td>1</td>
<td>1(^{10}) = 1 to the tenth</td>
</tr>
<tr>
<td>(\frac{2}{3})</td>
<td>3</td>
<td>(\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3})</td>
<td>(\frac{8}{27})</td>
<td>(\left(\frac{2}{3}\right)^3) = two thirds cubed</td>
</tr>
<tr>
<td>10000</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>10000(^0) = 10000 to the zero</td>
</tr>
<tr>
<td>(y)</td>
<td>5</td>
<td>(y \cdot y \cdot y \cdot y \cdot y)</td>
<td>(y^5)</td>
<td>(y) to the fifth</td>
</tr>
</tbody>
</table>

Example: Write the following exponential expressions in expanded form.

**Exponential expressions** | **Expanded form** |
---|---|
1) \(6^4\) | \(6 \cdot 6 \cdot 6 \cdot 6\) |
2) \((-x)^3\) | \((-x) \cdot (-x) \cdot (-x)\) |
3) \((3x^2y)^2\) | \((3x^2y) \cdot (3x^2y)\) |
4) \(\left(\frac{3}{4}u\right)^4\) | \(\left(\frac{3}{4}u\right) \cdot \left(\frac{3}{4}u\right) \cdot \left(\frac{3}{4}u\right) \cdot \left(\frac{3}{4}u\right)\) |

Example: Write each of the following in the exponential form.

**Expanded form** | **Exponential notation** |
---|---|
1) \((0.2) \cdot (0.2) \cdot (0.2)\) | \((0.2)^3\) |
2) \((5a) \cdot (5a) \cdot (5a) \cdot (5a)\) | \((5a)^4\) |
3) \(\left(\frac{5}{7}t\right) \cdot \left(\frac{5}{7}t\right)\) | \(\left(\frac{5}{7}t\right)^2\) |

Example: Evaluate \((4^2) \cdot (3^3) \cdot (6^0) \cdot (9^1)\).

\[
4^2 \cdot 3^3 \cdot 6^0 \cdot 9^1 = (4 \cdot 4) (3 \cdot 3 \cdot 3) (1) (9) = 16 \cdot 27 \cdot 1 \cdot 9 = 3888
\]

Example: Evaluate \(\frac{6x^2}{y+3} + 7x - 2\), given \(x = 2\) and \(y = 9\).

\[
\frac{6x^2}{y+3} + 7x - 2 = \frac{6 \cdot 2^2}{9+3} + 7 \cdot 2 - 2 = \frac{24}{12} + 14 - 2 = 14
\]

Substitute \(x = 2\) and \(y = 9\). Calculate.
**Order of Operations**

**Basic operations:** addition, subtraction, multiplication, division, exponent, etc.

The **order of operations are the rules** of which calculation comes first in an expression (when doing expressions with more than one operation).

**Order of operations:**

<table>
<thead>
<tr>
<th>Order of operations</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. the brackets or parentheses (innermost first)</td>
<td>$( )$, $[ ]$, ${ }$</td>
</tr>
<tr>
<td>2. exponent (power)</td>
<td>$a^n$</td>
</tr>
<tr>
<td>3. multiplication and division (from left-to-right)</td>
<td>$\times$ and $\div$</td>
</tr>
<tr>
<td>4. addition and subtraction (from left-to-right)</td>
<td>$+$ and $-$</td>
</tr>
</tbody>
</table>

**Example:** $4 \cdot 3^2 + 5 + (2 + 1) - 2 = 4 \cdot 3^2 + 5 + 3 - 2$

\[
\begin{align*}
4 \cdot 3^2 + 5 + (2 + 1) - 2 &= 4 \cdot 9 + 5 + 3 - 2 \\
&= 36 + 5 + 3 - 2 \\
&= 41 + 3 - 2 \\
&= 44 - 2 \\
&= 42
\end{align*}
\]

**Memory aid - BEDMAS**

<table>
<thead>
<tr>
<th>B</th>
<th>E</th>
<th>DM</th>
<th>AS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brackets</td>
<td>Exponents</td>
<td>Divide or Multiply</td>
<td>Add or Subtract</td>
</tr>
</tbody>
</table>

**Grouping symbols:** if parentheses are inside one another, calculate the inside set first.

- Parentheses $( )$ are used in the inner most grouping.
- Square brackets $[ ]$ are used in the second higher level grouping.

**Example:** $4 \cdot 3 + [5 + (2 + 1)] - 3^2 = 4 \cdot 3 + [5 + 3] - 3^2$

\[
\begin{align*}
4 \cdot 3 + [5 + (2 + 1)] - 3^2 &= 4 \cdot 3 + [5 + 3] - 3^2 \\
&= 4 \cdot 3 + 8 - 3^2 \\
&= 4 \cdot 3 + 8 - 9 \\
&= 12 + 8 - 9 \\
&= 20 - 9 \\
&= 11
\end{align*}
\]
## Unit 2: Introduction to Algebra

### Basic algebraic terms

<table>
<thead>
<tr>
<th>Algebraic term</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Algebraic expression</strong></td>
<td>A mathematical phrase that contains numbers, letters, grouping symbols (parentheses) and arithmetic operations.</td>
<td>$5x + 2$, $3a + (4b - 6)$, $\frac{2}{3} + 4$</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>A number.</td>
<td>$x + 2$ constant: 2</td>
</tr>
<tr>
<td><strong>Variable</strong></td>
<td>A letter that can be assigned different values.</td>
<td>$3 - x$ variable: $x$</td>
</tr>
<tr>
<td><strong>Coefficient</strong></td>
<td>The number in front of a variable.</td>
<td>$-6x$ coefficient: $x$ coefficient: $1$</td>
</tr>
<tr>
<td><strong>Term</strong></td>
<td>A term can be a constant, variable, or the product of a number and variable(s). (Terms are separated by addition or subtraction signs.)</td>
<td>$3x - \frac{2}{5}, 13y^2 + 7xy$ Terms: $3x, -\frac{2}{5}, 13y^2, 7xy$</td>
</tr>
<tr>
<td><strong>Like terms</strong></td>
<td>The terms that have the same variables and exponents.</td>
<td>$2x - y^2 - \frac{2}{5} + 5x - 7 + 13y^2$ Like terms: $2x$ and $5x$, $-y^2$ and $13y^2$, $-\frac{2}{5}$ and $-7$</td>
</tr>
</tbody>
</table>

**Evaluating an algebraic expression:** substitute a specific value for a variable and perform the mathematical operations (+, −, ×, ÷, etc.).

**To evaluate an expression:**

- Replace the variable(s) with number(s).
- Calculate.

**Key or clue words in word problems:**

<table>
<thead>
<tr>
<th>Addition (+)</th>
<th>Subtraction (−)</th>
<th>Multiplication (×)</th>
<th>Division (+)</th>
<th>Equals to (=)</th>
</tr>
</thead>
<tbody>
<tr>
<td>add</td>
<td>subtract</td>
<td>times</td>
<td>divided by</td>
<td>equals</td>
</tr>
<tr>
<td>sum (of)</td>
<td>difference</td>
<td>product</td>
<td>quotient</td>
<td>is</td>
</tr>
<tr>
<td>plus</td>
<td>take away</td>
<td>multiplied by</td>
<td>over</td>
<td>was</td>
</tr>
<tr>
<td>total (of)</td>
<td>minus</td>
<td>double</td>
<td>split up</td>
<td>are</td>
</tr>
<tr>
<td>altogether</td>
<td>less (than)</td>
<td>twice</td>
<td>fit into</td>
<td>were</td>
</tr>
<tr>
<td>increased by</td>
<td>decreased by</td>
<td>triple</td>
<td>per</td>
<td>amounts to</td>
</tr>
<tr>
<td>gain (of)</td>
<td>loss (of)</td>
<td>of</td>
<td>each</td>
<td>totals</td>
</tr>
<tr>
<td>combined</td>
<td>balance</td>
<td>how much (total)</td>
<td>goes into</td>
<td>results in</td>
</tr>
<tr>
<td>entire</td>
<td>(amount) left</td>
<td>how many</td>
<td>as much as</td>
<td>the same as</td>
</tr>
<tr>
<td>in all</td>
<td>savings</td>
<td>out of</td>
<td>gives</td>
<td></td>
</tr>
<tr>
<td>greater than</td>
<td>withdraw</td>
<td>ratio (of)</td>
<td>yields</td>
<td></td>
</tr>
<tr>
<td>complete</td>
<td>reduced by</td>
<td>percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>together</td>
<td>fewer (than)</td>
<td>share</td>
<td></td>
<td></td>
</tr>
<tr>
<td>more (than)</td>
<td>how much more</td>
<td>distribute</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and</td>
<td>how many extra</td>
<td>average</td>
<td></td>
<td></td>
</tr>
<tr>
<td>additional</td>
<td>how long</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Steps for solving word problems:

<table>
<thead>
<tr>
<th>Steps for solving word problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>- <strong>Organize</strong> the facts given from the problem (create a <strong>table</strong> or <strong>diagram</strong> if it will make the problem clearer).</td>
</tr>
<tr>
<td>- Identify and label the unknown quantity (<em>let</em> ( x = \text{unknown} )).</td>
</tr>
<tr>
<td>- Convert words into mathematical symbols, and <strong>determine the operation</strong> — write an <strong>equation</strong> (looking for ‘key’ or ‘clue’ words).</td>
</tr>
<tr>
<td>- <strong>Estimate</strong> and <strong>solve</strong> the equation and find the solution(s).</td>
</tr>
<tr>
<td>- <strong>Check</strong> and state the <strong>answer</strong>.</td>
</tr>
</tbody>
</table>

(Check the solution with the equation and check it back into the problem — is it logical?)

**Power:** the **product** of a number repeatedly multiplied by itself.

**Exponent:** the **number of times** a number is multiplied by itself.

**Base, exponent and power:**

\[
a^n \quad \begin{cases} 
    a \text{ is the base.} \\
    n \text{ is the exponent.} \\
    a^n \text{ is the power}
\end{cases}
\]

**Exponential notation (exponential expression):** \( a^n \) or \( \text{Base}^\text{Exponent} \)

<table>
<thead>
<tr>
<th>Exponential notation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^n )</td>
<td>( 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16 )</td>
</tr>
</tbody>
</table>

Read “\( a \) to the \( n \)th” or “the \( n \)th power of \( a \).”

**Exponents: basic properties:**

<table>
<thead>
<tr>
<th>Name</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero Exponent ( a^0 )</td>
<td>( a^0 = 1 ) (( 0^0 ) is undefined)</td>
</tr>
<tr>
<td>One Exponent ( a^1 )</td>
<td>( a^1 = a )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Power</th>
<th>Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>( a^n )</td>
</tr>
</tbody>
</table>

Read “2 to the 4th.”

**Order of operations:**

<table>
<thead>
<tr>
<th>Order of operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. the brackets or parentheses (innermost first)</td>
</tr>
<tr>
<td>2. exponent (power)</td>
</tr>
<tr>
<td>3. multiplication or division (from left-to-right)</td>
</tr>
<tr>
<td>4. addition or subtraction (from left-to-right)</td>
</tr>
</tbody>
</table>

**Memory aid - BEDMAS**

<table>
<thead>
<tr>
<th>B</th>
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<th>DM</th>
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<td>Divide or Multiply</td>
<td>Add or Subtract</td>
</tr>
</tbody>
</table>

**Grouping symbols:** if parentheses are inside one another, calculate the inside set first.

- Parentheses ( ) are used in the inner most grouping.
- Square brackets [ ] are used in the second higher level grouping.
Unit 2: Self-Test

Introduction to Algebra

Topic A

1. Identify the constant, coefficient and the variable:
   a) $2x - 3$
   b) $-4t + 13 + \frac{5}{7}t$

2. Identify the terms for each of the following:
   a) $5x + 3 - y$
   b) $2r + 16r^2 - \frac{3}{14}r + 1$

3. Identify the like terms in the following expressions:
   a) $7 + 2y^2 - \frac{5}{9}x + 5x - 1 + 13y^2$
   b) $0.6t + 9uv - 7t + 1.67uv$

4. Evaluate the following algebraic expressions.
   a) $7x - 4 + 13x$, given $x = 4$.
   b) $\frac{3}{a^2} + 9b + 12$, given $a = 10$ and $b = 5$.

Topic B

5. Write an expression/equation for each of the following:
   a) The product of ten and $y$.
   b) The quotient of $t$ and six.
   c) The difference between fifteen and a number more than the quotient of three by seven is six.
   d) Seven less than six times a number is fifteen.

6. Write an expression for each of the following:
   a) Susan has $375 in her checking account. If she makes a deposit of $y$ dollars, how much in total will she have in her account?
b) Mark weighs 175 pounds. If he loses \( y \) pounds, how much will he weigh?

c) A piece of wire 45 meters long was cut in two pieces and one piece is \( w \) meters long. How long is the other piece?

d) Emily made 4 dozen muffins. If it cost her \( x \) dollars, what was her cost per dozen muffins? What was her cost per muffin?

**Topic C**

7. a) In \( x^3 \), the base is (   ).

b) In \( y^4 \), the exponent is (   ).

8. Write the following exponential expressions in expanded form.
   a) \( 9^3 \)
   b) \( (-y)^4 \)
   c) \( (0.5a^3b)^2 \)
   d) \( \left(\frac{2}{7}x\right)^1 \)

9. Write each of the following in the exponential form.
   a) \( (0.06)(0.06)(0.06)(0.06) \)
   b) \( (12y)(12y)(12y) \)
   c) \( \left(\frac{-2}{9}x\right)\left(\frac{-2}{9}x\right) \)

10. Evaluate \( (3^2)(2^4)(23^3)(10^3) \).

11. Write each of the following as a base with an exponent.
   a) \( y \) to the eighth power.
   b) Five cubed.

12. Evaluate the following:
   a) \( \frac{9a^2}{b+6} + 3a + 4\) , if \( a = 1 \) and \( b = 3 \).
   b) \( 8xy + 7y^4\) , if \( x = \frac{1}{4} \) and \( y = 1 \).

13. Calculate the following:
   a) \( 2 \cdot 4^3 + 7 - (4 + 3) + 5 \)
   b) \( 5 \cdot 7 + [11 + (4 - 3)] + 4^2 \)
   c) \( \frac{104-4^2}{6+5} \)
Unit 3

Introduction to Geometry

Topic A: Perimeter, area, and volume
- Perimeter of plane figures
- Circle
- Perimeter
- Perimeters of irregular / composite shapes

Topic B: Area
- Areas of quadrilaterals and circles
- Areas of irregular / composite shapes

Topic C: Volume
- Volume of solids

Topic D: Surface and lateral area
- Surface and lateral area – rectangular solids
- Surface and lateral area – cylinders, cones and spheres

Unit 3 Summary

Unit 3 Self-test
Topic A: Perimeter, Area, and Volume

Perimeter of Plane Figures

**Polygon:** a closed figure made up of three or more line segments.

**Regular polygon:** a polygon that has all angles equal and all sides equal.

Classify regular polygons:

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Name of polygon</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
<td><img src="image" alt="Triangle" /></td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
<td><img src="image" alt="Quadrilateral" /></td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
<td><img src="image" alt="Pentagon" /></td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
<td><img src="image" alt="Hexagon" /></td>
</tr>
<tr>
<td>8</td>
<td>Octagon</td>
<td><img src="image" alt="Octagon" /></td>
</tr>
<tr>
<td>10</td>
<td>Decagon</td>
<td><img src="image" alt="Decagon" /></td>
</tr>
</tbody>
</table>

**Quadrilateral:** a four-sided polygon.

Classify quadrilaterals:

<table>
<thead>
<tr>
<th>Name of quadrilateral</th>
<th>Definition</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>A four-sided figure that has four right angles ($90^\circ$).</td>
<td><img src="image" alt="Rectangle" /></td>
</tr>
<tr>
<td>Square</td>
<td>A four-sided figure that has four equal sides and four right angles.</td>
<td><img src="image" alt="Square" /></td>
</tr>
<tr>
<td>Parallelogram</td>
<td>A four-sided figure that has opposite sides parallel ($//$) and equal. $(a // b, \ c // d; \ a = b, \ c = d)$</td>
<td><img src="image" alt="Parallelogram" /></td>
</tr>
<tr>
<td>Rhombus (diamond)</td>
<td>A four-sided figure that has four equal sides, but no right angle.</td>
<td><img src="image" alt="Rhombus" /></td>
</tr>
<tr>
<td>Trapezoid</td>
<td>A four-sided figure that has one pair of parallel sides.</td>
<td><img src="image" alt="Trapezoid" /></td>
</tr>
</tbody>
</table>
Circle: a round shape bounded by a curved line that is always the same distance from the center.

Circumference (C): the line bounding the edge of a circle.

Diameter (d): a straight line between any two points on the circle through the center of the circle.

Radius (r): a straight line between any point on the circle to the center of the circle (half of the diameter, \( r = \frac{1}{2} d \) (or \( d = 2r \)).

Example: Identify the parts of a circle (what is a, b and c?).

a. Circumference  b. Radius  c. Diameter

Example:

1) Find the radius of a circle with a diameter of 12 meters.
   \[ d = 12 \text{ m} \quad r = \frac{1}{2} d = \frac{1}{2} \cdot 12 \text{ m} = 6 \text{ m} \]

2) If the radius of a circle is 15 meters, what is the diameter of this circle?
   \[ d = 2r = 2 \cdot 15 \text{ m} = 30 \text{ m} \]
Perimeter ($P$): the total length of the outer boundary of a figure.

Find the perimeter: add together the length of each side.

Example: To find the perimeter ($P$) of the following figure, add the lengths of all 4 sides.

\[
P = 3\text{ in} + 1\text{ in} + 4\text{ in} + 1.5\text{ in} \\
= 9.5\text{ in}
\]

The perimeter of any regular (equal sided) polygon: the number of sides ($n$) times the length of any side ($s$) of that polygon. \(P = ns\)

Example: The perimeter ($P$) of a square is \(P = 4s\)

Units of perimeter: the meter (m), centimeter (cm), foot (ft), inch (in), yard (yd), etc. (The same units as length.)

The perimeter of regular polygons: \(s\) – the length of the side

<table>
<thead>
<tr>
<th>Name of the figure</th>
<th>Perimeter ($P = ns$)</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral triangle</td>
<td>$P = 3s$</td>
<td><img src="image" alt="Equilateral Triangle" /></td>
</tr>
<tr>
<td>(A triangle with three equal sides.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square</td>
<td>$P = 4s$</td>
<td><img src="image" alt="Square" /></td>
</tr>
<tr>
<td>Pentagon</td>
<td>$P = 5s$</td>
<td><img src="image" alt="Pentagon" /></td>
</tr>
<tr>
<td>Hexagon</td>
<td>$P = 6s$</td>
<td><img src="image" alt="Hexagon" /></td>
</tr>
<tr>
<td>Octagon</td>
<td>$P = 8s$</td>
<td><img src="image" alt="Octagon" /></td>
</tr>
<tr>
<td>Decagon</td>
<td>$P = 10s$</td>
<td><img src="image" alt="Decagon" /></td>
</tr>
</tbody>
</table>

Example:

1) What is the perimeter ($P$) of the following triangle?

\[
s = 3.5\text{ m} \\
P = 3s = (3)(3.5\text{ m}) = 10.5\text{ m}
\]
2) What is the perimeter \((P)\) of the following square?

\[ s = 2.3\text{ cm} \]

\[ P = 4s = 4(2.3 \text{ cm}) = 9.2 \text{ cm} \]

3) What is the perimeter \((P)\) of the following hexagon?

\[ s = 5\text{ ft} \]

\[ P = 6s = (6)(5 \text{ ft}) = 30 \text{ ft} \]

4) What is the perimeter \((P)\) of the following octagon?

\[ s = \frac{3}{4} \text{ yd} \]

\[ P = 8s = 8\left(\frac{3}{4} \text{ yd}\right) = 6 \text{ yd} \]

The perimeter of some basic geometric shapes:

<table>
<thead>
<tr>
<th>Name of the figure</th>
<th>Perimeter formula</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>(P = 2w + 2l) ((w - \text{width}, \ l - \text{length})) (w) (l)</td>
<td></td>
</tr>
<tr>
<td>Parallelogram</td>
<td>(P = 2a + 2b) ((a \text{ and } b - \text{the length of the sides})) (a) (b)</td>
<td></td>
</tr>
<tr>
<td>Trapezoid</td>
<td>(P = a + b + c + d) (a) (b) (c) (d)</td>
<td></td>
</tr>
<tr>
<td>Circle</td>
<td>(C = \pi d) or (C = 2\pi r) ((\pi \approx 3.14)) (\pi) ((\text{pi})) is the ratio of circle’s circumference (C) to its diameter (d), that is approximately 3.14. ((\pi = \frac{\text{Circumference}}{\text{diameter}} = \frac{C}{d} \approx 3.14159265359 \ldots))</td>
<td></td>
</tr>
</tbody>
</table>

Example: What is the perimeter \((P)\) of the following polygons?

1) \(w = 5\text{ ft}\) \(l = 7\text{ ft}\)

\[ P = 2w + 2l = 2(5 \text{ ft}) + 2(7 \text{ ft}) = 24 \text{ ft} \]

2) \(a = 3.4\text{ cm}\)

\[ P = 2a + 2b = 2(3.4 \text{ cm}) + 2(5.2 \text{ cm}) = 17.2 \text{ cm} \]

3) \(2.4\text{ m}\)

\[ P = a + b + c + d = 2.4 \text{ m} + 1.8 \text{ m} + 4.3 \text{ m} + 5.8 \text{ m} = 14.3 \text{ m} \]

Example: What are the circumferences \((C)\) of the circles shown below?

1) \(d = 5\text{ cm}\)

\[ C = \pi d = 3.14(5\text{ cm}) = 15.7 \text{ cm} \]

2) \(r = 2.8\text{ cm}\)

\[ C = 2\pi r = 2(3.14)(2.8\text{ cm}) \approx 17.58 \text{ cm} \]
Perimeters of Irregular / Composite Shapes

Example: What are the perimeters \((P)\) of the following figures?

1)

\[
P = 2m + 3m + 1m + 2m + (3m + 2m) + (1m + 2m) = 16 \text{ m}
\]

2)

\[
P = (2cm + 2cm) + \frac{3}{4} (2\pi r)
\]
\[
= 4 \text{ cm} + \frac{3}{4} (2\pi \cdot 2 \text{ cm}).
\]
\[
\approx 13.42 \text{ cm}
\]

3)

\[
P = (3 \text{ ft} + 3\text{ ft}) + \frac{1}{2} (\pi d)
\]
\[
= 6 \text{ ft} + \frac{1}{2} (\pi \cdot 3 \text{ ft})
\]
\[
\approx 10.71 \text{ ft}
\]

4)

\[
P = 4 \cdot \frac{1}{2} (\pi d)
\]
\[
= 4 \cdot \frac{1}{2} (\pi \cdot 2 \text{ yd})
\]
\[
\approx 12.57 \text{ yd}
\]

Example: Damon is renovating his living room that is the shape indicated in the diagram below. He wishes to put molding around the base of the walls of the living room. How much molding does he need?

\[
P = 3(4.5\text{ m}) + 2.2\text{ m} + 4.3 \text{ m} = 20 \text{ m}
\]
**Topic B: Area**

**Areas of Quadrilaterals and Circles**

**Area (A):** the size of the outermost surface of a shape (space within its boundaries).

**Units of area:** the units of measurement of area are always expressed as square units.

Such as square meter (m²), square centimeter (cm²), square foot (ft²), square inch (in²), square yard (yd²), etc.

**Areas of some basic geometric shapes:**

<table>
<thead>
<tr>
<th>Name of the figure</th>
<th>Area formula (A)</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$A = wl$</td>
<td><img src="image" alt="Rectangle" /></td>
</tr>
<tr>
<td>Square</td>
<td>$A = s^2$</td>
<td><img src="image" alt="Square" /></td>
</tr>
<tr>
<td>Triangle</td>
<td>$A = \frac{1}{2}bh$</td>
<td><img src="image" alt="Triangle" /></td>
</tr>
<tr>
<td>Parallelogram</td>
<td>$A = bh$</td>
<td><img src="image" alt="Parallelogram" /></td>
</tr>
<tr>
<td>Trapezoid</td>
<td>$A = \frac{1}{2}h(b + B)$</td>
<td><img src="image" alt="Trapezoid" /></td>
</tr>
<tr>
<td>Circle</td>
<td>$A = \pi r^2$</td>
<td><img src="image" alt="Circle" /></td>
</tr>
</tbody>
</table>

**Example:** What are the areas (A) of the following figures?

1) \(3.8 \text{ m}\) \(A = s^2 = (3.8 \text{ m}) (3.8 \text{ m}) = 14.44 \text{ m}^2\) \(\text{m} \cdot \text{m} = \text{m}^2\)

2) \(\frac{2}{3} \text{ cm}\) \(A = w l = \left(\frac{2}{3} \text{ cm}\right) \left(\frac{3}{4} \text{ cm}\right) = \frac{1}{2} \text{ cm}^2\) \(\text{cm} \cdot \text{cm} = \text{cm}^2\)

3) \(4.2 \text{ yd}\) \(A = \frac{1}{2} bh = \frac{1}{2} \left(5.3 \text{ yd}\right)\left(4.2 \text{ yd}\right) = 11.13 \text{ yd}^2\)

4) \(\frac{2}{5} \text{ in}\) \(A = bh = \left(\frac{2}{5} \text{ in}\right) \left(\frac{1}{4} \text{ in}\right) = \frac{1}{10} \text{ in}^2\)

5) \(h = 5 \text{ ft}\) \(A = \frac{1}{2} h (b + B) = \frac{1}{2} \left(5 \text{ ft}\right) (2 \text{ ft} + 6 \text{ ft}) = 20 \text{ ft}^2\)

6) \(r = 0.25 \text{ cm}\) \(A = \pi r^2 \approx (3.14) (0.25 \text{ cm})^2 \approx 0.2 \text{ cm}^2\)
Areas of Irregular / Composite Shapes

Example: Find the areas \( (A) \) of the following figures.

1) \[ \begin{array}{c}
\text{4m} \\
\text{3m} \\
\text{2m} \\
\text{1m}
\end{array} \]

Total area = Area of parallelogram + Area of triangle
\[
A = (bh) + \left( \frac{1}{2} bh \right) = (3m)(4m) + \left( \frac{1}{2} \right)(1m)(2m) = 12m^2 + 1m^2 = 13m^2
\]

2) \[ \begin{array}{c}
\text{1ft} \\
\text{3ft} \\
\text{2.5ft}
\end{array} \]

Total area = Area of trapezoid + Area of \( \left( \frac{1}{2} \right) \) circle
\[
A = \left( \frac{1}{2} \right)(h(b + B)) + \left( \frac{1}{2} \pi r^2 \right) = \left( \frac{1}{2} \right)(3ft)(1ft + 2.5ft) + \left( \frac{1}{2} \right)(3.14)(0.5ft)^2 \approx 5.64 \text{ ft}^2
\]

Example: Damon is renovating his living room that is the shape indicated in the diagram below. He wishes to purchase new flooring. How much does he need to order to cover the entire living room floor?

Total area = Area of square + Area of triangle
\[
A = S^2 + \frac{1}{2} bh = (4.5 \text{ m})^2 + \frac{1}{2} (4.5 \text{ m})(2.2\text{ m}) = 25.2 \text{ m}^2
\]

Example: William built a wooden deck at the back of his home. It is shown in the following diagram. He decides to insert a circular hot tub that has a diameter of 2.4 m. Calculate the area of the remaining exposed wooded floor of the deck.

Shaded area = Area of rectangle - Area of circle
\[
A = (wl) - (\pi r^2) = (5\text{ m})(7\text{ m}) - (3.14)(1.2\text{ m})^2 \approx 30.48 \text{ m}^2
\]
**Topic C: Volume**

**Volume** ($V$): the amount of space a solid object (three-dimensional) occupies.

**Example:** the volume of a can of food is the amount of food inside.

**Units of volume:** the units of measurement of volume are always expressed as cubic units. Such as the cubic meter ($m^3$), cubic centimeter ($cm^3$), cubic foot ($ft^3$), cubic inch ($in^3$), cubic yard ($yd^3$), etc.

**Volumes of basic geometric shapes:**

<table>
<thead>
<tr>
<th>Name</th>
<th>Figure</th>
<th>Volume formula ($V$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td>$s$</td>
<td>$V = s^3$ ($s$ – the length of the side)</td>
</tr>
<tr>
<td>Rectangular solid</td>
<td>$h$ $l$ $w$</td>
<td>$V = w \cdot l \cdot h$ ($w$ – width, $l$ – length, $h$ - height)</td>
</tr>
<tr>
<td>Cylinder</td>
<td>$h$ $r$</td>
<td>$V = \pi r^2 h$ ($r$ - radius, $h$ – height, $\pi \approx 3.14$)</td>
</tr>
<tr>
<td>Sphere</td>
<td>$r$</td>
<td>$V = \frac{4}{3} \pi r^3$ ($r$ - radius)</td>
</tr>
<tr>
<td>Cone</td>
<td>$h$</td>
<td>$V = \frac{1}{3} \pi r^2 h$ ($r$ - radius, $h$ - height)</td>
</tr>
<tr>
<td>Pyramid</td>
<td>$h$ $l$ $w$</td>
<td>$V = \frac{1}{3} w \cdot l \cdot h$ ($w$ – width, $l$ – length, $h$ - height)</td>
</tr>
</tbody>
</table>

**Example:** Find the volumes ($V$) of the following figures.

1) $V = s^3 = (1.4 \text{ m}) (1.4 \text{ m}) (1.4 \text{ m})$  
   $= (1.4 \text{ m})^3 = 2.744 \text{ m}^3$  
   $\text{m} \cdot \text{m} \cdot \text{m} = \text{m}^3$

2) $V = w \cdot l \cdot h = (4.2\text{ in}) (1.3\text{ in}) (2.4\text{ in}) \approx 13.1 \text{ in}^3$  
   $\text{in} \cdot \text{in} \cdot \text{in} = \text{in}^3$

3) $V = \pi r^2 h = \pi (3\text{ m})^2 (8\text{ m}) \approx 226.2 \text{ m}^3$

**Unit 3 Introduction to Geometry** 57
4) \[ d = 4\text{cm} \]
\[ V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (2\text{cm})^3 \approx 33.51 \text{cm}^3 \]
\[ (d = 4\text{ cm}, \ r = \frac{1}{2}d = 2\text{ cm}) \]

5) \[ r = 3\text{ft} \]
\[ V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (3\text{ft})^2 (5\text{ft}) \approx 47.1 \text{ft}^3 \]

6) \[ h=4\text{m}, \ w=2\text{m}, \ l=3\text{m} \]
\[ V = \frac{1}{3} wh = \frac{1}{3} (2\text{m})(3\text{m})(4\text{m}) = 8 \text{m}^3 \]

7) Determine the amount of water that will fill the following bucket.
\[ V = \pi r^2 h = \pi (5\text{cm})^2 (25\text{cm}) \approx 1963.5 \text{cm}^3 \]
\[ (d = 10\text{ cm}, \ r = \frac{1}{2}d = 5\text{ cm}) \]
\[ d = 10\text{cm} \]

**Volume of composite shapes**

**Example:** Find the volume \((V)\) of the following figure.

\[ r = 2.5 \text{in} \]

Total volume = Volume of the cylinder + Volume of the cone

\[ V = (\pi r^2 h) + \left(\frac{1}{3} \pi r^2 h\right) = \left[\pi (2.5 \text{ in})^2 (4 \text{ in})\right] + \left[\frac{1}{3} \pi (2.5 \text{ in})^2 (4.5 \text{ in})\right] \]
\[ = 107.99 \text{in}^3 \]

**Example:** Find the volumes \((V)\) of the following figure (a rectangular solid with a cylinder removed from inside).

\[ (\text{Cylinder: } h = 4\text{m}, \ r = 1\text{m}) \]

Unknown volume = Volume of the rectangular solid − Volume of the cylinder

\[ V = (wlh) - (\pi r^2 h) = [(2\text{m})(5\text{m})(4\text{m})] - [\pi (1\text{m})^2 (4\text{m})] \approx 27.43 \text{m}^3 \]
**Topic D: Surface and Lateral Area**

**Surface and Lateral Area**

---

**Surface area (SA):** the total area on the surface of a solid object (a three-dimensional object).

**Lateral area (LA):** the surface area of a solid object excluding its top and bottom.

**Lateral area (LA) of a rectangular solid:** the sum of the surface areas of the four sides excluding its top and bottom.

\[
\text{LA of a rectangular solid} = \text{front side} + \text{back side} + 2\text{ sides} \\
= 2 \,(l \, h) + 2 \,(w \, h) \\
\text{The front and back sides.} \quad \text{The left and right sides.}
\]

**Example:** Determine the lateral area (LA) of the rectangular solid.

\[
\text{LA} = 2(5\text{ft} \cdot 2\text{ft}) + 2(1\text{ft} \cdot 2\text{ft}) \\
= 20 \text{ ft}^2 + 4 \text{ ft}^2 \\
= 24 \text{ ft}^2
\]

**Surface area (SA) of a rectangular solid:** the sum of the areas of the top, bottom and the four sides.

\[
\text{SA of a rectangular solid} = \text{top area} + \text{bottom area} + 4\text{ sides} \\
= (l \, w) + (l \, w) + 2(lh) + 2(wh) \\
= 2 \,(l \, w) + 2(lh) + 2(wh) \\
\text{The top & bottom.} \quad \text{The front and back sides.} \quad \text{The left and right sides.}
\]

**Example:** Determine the SA of the rectangular solid.

\[
\text{SA} = 2 \,(3\text{m} \cdot 1\text{m}) + 2(3\text{m} \cdot 2\text{m}) + 2(1\text{m} \cdot 2\text{m}) \\
= 6\text{m}^2 + 12\text{m}^2 + 4\text{m}^2 = 22\text{m}^2
\]

**Example:** How many square centimeters of glass are needed to make a fish tank which is 15 cm long by 10 cm wide by 12 cm high if the top is left open?

\[
A = 2 \,(15\text{cm} \cdot 12\text{cm}) + 2(12\text{cm} \cdot 10\text{cm}) + (15\text{cm} \cdot 10\text{cm}) = 750 \text{ cm}^2
\]

The front and back sides. The left and right sides. The bottom part.
Surface and Lateral Area
– Cylinders, Cones and Spheres

Cylinders

- Lateral area (LA) of a cylinder: the area of the rectangular side that wraps around the cylinder’s side (the rectangular side folded around).

\[ \text{LA of a cylinder} = \pi dh \text{ or } 2\pi rh \]

- Imagine a fruit can that is cut down the side and rolled flat.
- Recall: the circumference of a circle \( C = \pi d \text{ or } 2\pi r \) (\( r \) – radius, \( d \) – diameter)

- Surface area (SA) of a cylinder: the sum of the surface areas of the top, bottom and the side (the lateral area).

\[ \text{SA of a cylinder} = \text{top area} + \text{bottom area} + \text{LA of a cylinder} \]

Recall: the area of a circle: \( A = \pi r^2 \)

\((r \text{ – radius, } d \text{ – diameter, } h \text{ - height})\)

Example: Determine the lateral area and surface area of the following cylinder.

\[ \text{LA} = \pi dh = \pi (3\text{m})(3.5\text{m}) \approx 32.99 \text{ m}^2 \]

\[ \text{SA} = 2(\pi r^2) + \pi dh \]

\[ = 2[\pi (1.5 \text{ m}^2)] + 32.99 \text{ m}^2 \quad d = 3\text{m}, \quad r = \frac{1}{2}d = 1.5\text{m} \]

\[ \approx 14.137 \text{ m}^2 + 32.99 \text{ m}^2 \]

\[ \approx 47.13 \text{ m}^2 \]

Cones

- Lateral area of a cone:

\[ \text{LA of a cone} = (\pi) (\text{radius}) (\text{slant height}) = \pi rs \]

Slant height \((s)\): the height from the vertex to a point on the circle base.

\((r \text{ – radius, } s \text{ – slant height})\)
- Surface area (SA) of a cone:

\[ \text{SA of a cone} = \pi rs + \pi r^2 \]

\( s \) - slant height, \( r \) - radius

**Example:** Determine the lateral area and total area of a cone whose diameter is 2m and slant height is 4m.

\[ \begin{align*}
\text{LA} &= \pi rs = \pi (1m)(4m) \approx 12.57 \text{ m}^2 \\
\text{SA} &= \pi rs + \pi r^2 = 12.57 \text{ m}^2 + \pi(1m)^2 \approx 15.71 \text{ m}^2
\end{align*} \]

**Spheres**

Surface area (SA) of a sphere:

\[ \text{SA of a sphere} = 4\pi r^2 \]

**Example:** Determine the surface area of a sphere whose radius is 4.5cm.

\[ \text{SA} = 4\pi r^2 = 4\pi (4.5\text{cm})^2 \approx 254.47 \text{ cm}^2 \]

**Example:** Mary wishes to paint 5 balls with green paint. The diameter of each ball is 18 cm. What area should Mary tell the paint store she needs to cover?

\[ \begin{align*}
\text{SA} &= 4\pi r^2 = 4\pi (9\text{cm})^2 \approx 1017.88 \text{ cm}^2 \\
5 \text{ (SA)} &= 5(1017.88 \text{ cm}^2) = 5089.4 \text{ cm}^2
\end{align*} \]

**Surface and lateral area summary:**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Lateral area (LA)</th>
<th>Surface area (SA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular Solid</td>
<td>Front side + back side + 2 sides $2(lh) + 2(wh)$</td>
<td>Top area + bottom area + 4 sides $l w + (l w) + 2(lh) + 2(wh)$</td>
</tr>
<tr>
<td>Cylinder</td>
<td>$\pi dh$ or $2\pi rh$</td>
<td>$2(\pi r^2) + \pi dh$</td>
</tr>
<tr>
<td>Cone</td>
<td>$\pi rs$</td>
<td>$\pi rs + \pi r^2$</td>
</tr>
<tr>
<td>Sphere</td>
<td>$4\pi r^2$</td>
<td></td>
</tr>
</tbody>
</table>

There is no difference between lateral area and surface area in a sphere.
Unit 3: Summary

Introduction to Geometry

Classify quadrilaterals (four-sided shapes):

<table>
<thead>
<tr>
<th>Name of quadrilateral</th>
<th>Definition</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>A four-sided figure that has four right angles (90°).</td>
<td><img src="image" alt="Rectangle" /></td>
</tr>
<tr>
<td>Square</td>
<td>A four-sided figure that has four equal sides and four right angles.</td>
<td><img src="image" alt="Square" /></td>
</tr>
<tr>
<td>Parallelogram</td>
<td>A four-sided figure that has opposite sides parallel (∥) and equal. ((a \parallel b, \ c \parallel d; \ a = b, \ c = d))</td>
<td><img src="image" alt="Parallelogram" /></td>
</tr>
<tr>
<td>Rhombus (diamond)</td>
<td>A four-sided figure that has four equal sides, but no right angle.</td>
<td><img src="image" alt="Rhombus" /></td>
</tr>
<tr>
<td>Trapezoid</td>
<td>A four-sided figure that has one pair of parallel sides.</td>
<td><img src="image" alt="Trapezoid" /></td>
</tr>
</tbody>
</table>

Terms of geometry:

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter ((P))</td>
<td>The total length of the outer boundary of a shape.</td>
</tr>
<tr>
<td>Circumference ((C))</td>
<td>The line bounding the edge of a circle.</td>
</tr>
<tr>
<td>Diameter ((d))</td>
<td>A straight line between any two points on the circle through the center of the circle.</td>
</tr>
<tr>
<td>Radius ((r))</td>
<td>A straight line between any point on the circle to the center of the circle (half of the diameter, (r = \frac{1}{2}d) or (d = 2r)).</td>
</tr>
<tr>
<td>Area ((A))</td>
<td>The size of the outermost surface of a shape.</td>
</tr>
<tr>
<td>Volume ((V))</td>
<td>The amount of space a solid object (3D) occupied.</td>
</tr>
<tr>
<td>Surface area ((SA))</td>
<td>The total area on the surface of a solid object (a 3D object).</td>
</tr>
<tr>
<td>Lateral area ((LA))</td>
<td>The surface area of a solid object excluding its top and bottom.</td>
</tr>
</tbody>
</table>

Units of perimeter: the meter (m), centimeter (cm), foot (ft or’), inch (in or”), yard (yd), etc. The same units as length.

Units of area: the units of measurement of area are always expressed as square units.

Units of volume: the units of measurement of volume are always expressed as cubic units.

Surface and lateral area summary:

<table>
<thead>
<tr>
<th>Figure</th>
<th>Lateral area ((LA))</th>
<th>Surface area ((SA))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular Solid</td>
<td>(2l \cdot h + 2wh)</td>
<td>(l \cdot w + 2lh + 2wh)</td>
</tr>
<tr>
<td>Cylinder</td>
<td>(\pi dh) or (2\pi rh)</td>
<td>(2(\pi r^2) + \pi dh)</td>
</tr>
<tr>
<td>Cone</td>
<td>(\pi rs)</td>
<td>(\pi rs + \pi r^2)</td>
</tr>
<tr>
<td>Sphere</td>
<td></td>
<td>(4\pi r^2)</td>
</tr>
</tbody>
</table>
### Geometry Formulas

- $s$ – side
- $P$ – perimeter
- $C$ – Circumference
- $A$ – area
- $V$ – volume

<table>
<thead>
<tr>
<th>Name of the figure</th>
<th>Formula</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral triangle</td>
<td>$P = 3s$</td>
<td><img src="image" alt="Equilateral Triangle" /></td>
</tr>
<tr>
<td>Pentagon</td>
<td>$P = 5s$</td>
<td><img src="image" alt="Pentagon" /></td>
</tr>
<tr>
<td>Hexagon</td>
<td>$P = 6s$</td>
<td><img src="image" alt="Hexagon" /></td>
</tr>
<tr>
<td>Octagon</td>
<td>$P = 8s$</td>
<td><img src="image" alt="Octagon" /></td>
</tr>
<tr>
<td>Decagon</td>
<td>$P = 10s$</td>
<td><img src="image" alt="Decagon" /></td>
</tr>
</tbody>
</table>
| Square             | $P = 4s$  
                     $A = s^2$ | ![Square](image) |
| Rectangle          | $P = 2w + 2l$  
                     $A = wl$ | ![Rectangle](image) |
| Parallelogram      | $P = 2a + 2b$  
                     $A = bh$ | ![Parallelogram](image) |
| Circle             | $C = \pi d = 2\pi r$  
                     $A = \pi r^2$ | ![Circle](image) |
| Triangle           | $< X + < Y + < Z = 180^\circ$  
                     $A = \frac{1}{2}bh$ | ![Triangle](image) |
| Trapezoid          | $A = \frac{1}{2}h(b + B)$ | ![Trapezoid](image) |
| Cube               | $V = s^3$ | ![Cube](image) |
| Rectangular solid  | $V = whl$ | ![Rectangular Solid](image) |
| Cylinder           | $V = \pi r^2h$ | ![Cylinder](image) |
| Sphere             | $V = \frac{4}{3}\pi r^3$ | ![Sphere](image) |
| Cone               | $V = \frac{1}{3}\pi r^2h$ | ![Cone](image) |
| Pyramid            | $V = \frac{1}{3} whl$ | ![Pyramid](image) |
**Unit 3: Self - Test**

**Introduction to Geometry**

**Topic A**

1. Find the radius of a circle with a diameter of 42 centimeters.
2. What is the perimeter ($P$) of the following triangle?
   
   ![Triangle]
   
   $s = 4.7$ cm

3. What is the perimeter ($P$) of the following polygons?
   
   a) $s = 1.4$ in
   
   b) $w = 2.3$ ft
      
      ![Rectangle]
      
      $l = 3.2$ ft

   c) $a = 7.2$ cm
      
      ![Parallelogram]
      
      $b = 10.4$ cm

   d) $s = \frac{3}{19}$ yd
      
      ![Hexagon]

4. What is the circumferences ($C$) of the circle shown below?

   ![Circle]
   
   $d = 2.5$ in

5. What are the perimeters ($P$) of the following figures?

   a) ![Complex Figure]

   b) ![Quarter Circle]

   c) ![Quarter Circle]

   ![Quadrilateral]
   
   5 in

   5 in
6. A flower bed in the shape of a parallelogram has sides of 5.5 inches and 3.4 inches. What is its perimeter?

7. The floor of a rectangular room measures 5.2 m by 4.3 m. The doorway is 1 m wide. Baseboard is to be installed around the perimeter of the room, except in the doorway. What length of baseboard needs to be purchased?

8. Tom’s rectangular yard is 10 meters wide and 15 meters long.
   a. If Tom wants to fence the whole lot, how many meters of fencing would Tom have to buy?
   b. If the fencing cost $15 per meter, estimate the cost of fencing the yard.

9. A rectangular swimming pool is 8 m long and 4 m wide. It is surrounded by concrete deck 1.5 m wide on all sides. Find the outside perimeter of the deck.

**Topic B**

10. Find the areas of the following figures.

   a) ![Diagram](3cm_5cm_1.5cm)

   b) ![Diagram](4in_5in_2in)

   c) ![Diagram](1.6m_4.5m)

11. Find the area (A) of the shaded area in the following figure.

   ![Diagram](7.3m_4.2m_8.4m)
12. A rectangular lawn measuring 24 m by 18 m has 3 circular flowerbeds cut from it. If the circular flowerbeds each have a diameter of 8 m, find the area of the grass remaining.

**Topic C**

13. Find the volumes \( V \) of the following figures.

   a) ![3.7 cm]

   b) ![3.3mm]

   c) ![r = 6.3 cm, h = 28.8 cm]

   d) ![h = 7 cm, r = 3.5 cm]

   e) ![5.3 cm, r = 3.2 cm]

14. A snowman is made of three balls of snow. One has a diameter of 28 cm, one of 18 cm, and one of 8 cm. What volume of snow does the snowman contain?

15. A conveyor belt unloading salt from a ship makes a conical pile 18 m high with a base diameter of 8 m. What is the volume of the salt in the pile?

16. A spherical balloon is filled with water and has a diameter of 30 cm. If the water was poured out into an empty tin can measuring 24 cm across and 28 cm high, would the water all fit?

17. The height of a cylindrical pail is 26 cm and the radius of the base is 10 cm. A ball with radius 6 cm is dropped in the pail. Find the volume of the region inside the pail but outside of the ball.
Topic D

18. Determine the LA of the rectangular solid.

19. Determine the SA of the rectangular solid.

20. Determine the lateral area and surface area of the following cylinder.

21. Determine the lateral area and total area of a cone whose diameter is 6.4 cm and slant height is 7.3 cm.

22. Determine the SA of a sphere whose diameter is 1.8 m.

23. A toy box measures 0.7 m long by 0.6 m wide and is 0.5 m high. What is the total area of plywood needed to build the box if it has no top?

24. A greenhouse is semi-cylindrical in shape.
   If a clear vinyl is used to cover the greenhouse (including the ends), how much vinyl is needed?
Unit 4
Measurement

Topic A: Metric system of measurement
- International system of units
- Metric conversion
- The unit factor method

Topic B: Metric units for area and volume
- Convert units of area and volume
- The relationship between mL, g and cm$^3$

Topic C: Imperial system
- The system of imperial units
- Imperial unit conversion

Topic D: Converting between metric and imperial units
- Imperial and metric conversions

Unit 4 Summary

Unit 4 Self - test
**Topic A: Metric System of Measurement**

**International System of Units**

**Metric system (SI – international system of units):** the most widely used system of measurement in the world. It is based on the basic units of meter, kilogram, second, etc.

**SI common units:**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>Unit symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>meter</td>
<td>m</td>
</tr>
<tr>
<td>Mass (or weight)</td>
<td>gram</td>
<td>kg</td>
</tr>
<tr>
<td>Volume</td>
<td>litre</td>
<td>L</td>
</tr>
<tr>
<td>Time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>Temperature</td>
<td>degree (Celsius)</td>
<td>°C</td>
</tr>
</tbody>
</table>

**Metric prefixes (SI prefixes):** large and small numbers are made by adding SI prefixes, which is based on multiples of 10.

**Key metric prefix:**

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol (abbreviation)</th>
<th>Power of 10</th>
<th>Multiple value</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>mega</td>
<td>M</td>
<td>$10^6$</td>
<td>1,000,000</td>
<td>1 Mm = 1,000,000 m</td>
</tr>
<tr>
<td>kilo-</td>
<td>k</td>
<td>$10^3$</td>
<td>1,000</td>
<td>1 km = 1,000 m</td>
</tr>
<tr>
<td>hecto-</td>
<td>h</td>
<td>$10^2$</td>
<td>100</td>
<td>1 hm = 100 m</td>
</tr>
<tr>
<td>deka-</td>
<td>da</td>
<td>$10^1$</td>
<td>10</td>
<td>1 dam = 10 m</td>
</tr>
<tr>
<td>meter/gram/liter</td>
<td>d</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>deci-</td>
<td>d</td>
<td>$10^{-1}$</td>
<td>0.1</td>
<td>1 m = 10 dm</td>
</tr>
<tr>
<td>centi-</td>
<td>c</td>
<td>$10^{-2}$</td>
<td>0.01</td>
<td>1 cm = 100 cm</td>
</tr>
<tr>
<td>milli-</td>
<td>m</td>
<td>$10^{-3}$</td>
<td>0.001</td>
<td>1 m = 1,000 mm</td>
</tr>
<tr>
<td>micro</td>
<td>μ</td>
<td>$10^{-6}$</td>
<td>0.000 001</td>
<td>1 m = 1,000,000 μm</td>
</tr>
</tbody>
</table>

**Metric prefix for length, weight and volume:**

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Length (m - meter)</th>
<th>Weight (g - gram)</th>
<th>Liquid volume (L - liter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>mega (M)</td>
<td>Mm (Megameter)</td>
<td>Mg (Megagram)</td>
<td>ML (Megaliter)</td>
</tr>
<tr>
<td>kilo (k)</td>
<td>km (Kilometer)</td>
<td>kg (Kilogram)</td>
<td>kL (Kiloliter)</td>
</tr>
<tr>
<td>hecto (h)</td>
<td>hm (hectometer)</td>
<td>hg (hectogram)</td>
<td>hL (hectoliter)</td>
</tr>
<tr>
<td>deka (da)</td>
<td>dam (dekameter)</td>
<td>dag (dekagram)</td>
<td>daL (dekaliter)</td>
</tr>
<tr>
<td>meter/gram/liter</td>
<td>m (meter)</td>
<td>g (gram)</td>
<td>L (liter)</td>
</tr>
<tr>
<td>deci (d)</td>
<td>dm (decimeter)</td>
<td>dg (decigram)</td>
<td>dL (deciliter)</td>
</tr>
<tr>
<td>centi (c)</td>
<td>cm (centimeter)</td>
<td>cg (centigram)</td>
<td>cL (centiliter)</td>
</tr>
<tr>
<td>milli (m)</td>
<td>mm (millimeter)</td>
<td>mg (milligram)</td>
<td>mL (milliliter)</td>
</tr>
<tr>
<td>micro (μ)</td>
<td>μm (micrometer)</td>
<td>μg (microgram)</td>
<td>μL (microliter)</td>
</tr>
</tbody>
</table>

Large

Small
Unit 4 Measurement

Metric Conversion

Metric conversion table:

<table>
<thead>
<tr>
<th>Value</th>
<th>1,000,000</th>
<th>1,000</th>
<th>100</th>
<th>10</th>
<th>1</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
<th>0.000 001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prefix</td>
<td>Mega</td>
<td>kilo</td>
<td>hecto</td>
<td>deka</td>
<td>meter (m)</td>
<td>gram (g)</td>
<td>liter (L)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>M</td>
<td>k</td>
<td>h</td>
<td>da</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Steps for metric conversion through decimal movement:

- Identify the number of places to move on the metric conversion table.
- Move the decimal point.
  - Convert a smaller unit to a larger unit: move the decimal point to the left.
  - Convert a larger unit to a smaller unit: move the decimal point to the right.

Example: 326 mm = (?) m
- Identify mm (millimeters) and m (meters) on the conversion table.
- Count places from mm to m: 3 places
- Move 3 decimal places:
  - (1 m = 1000 mm)

Example: 4.675 hg = (?) g
- Identify hg (hectograms) and g (grams) on the conversion table.
- Count places from hg to g: 2 places
- Move 2 decimal places:
  - (1 hg = 100 g)

Example: 30.5 mL = (?) kL
- Identify mL (milliliters) and kL (kiloliters) on the conversion table.
- Count places from mL to kL: 6 places
- Move 6 decimal places:
  - (1 kL = 1,000,000 mL)
The Unit Factor Method

Convert units using the unit factor method (or the factor-label method)

- Write the original term as a fraction (over 1).
  
  \[
  \text{Example: } 10 \text{g can be written as } \frac{10 \text{g}}{1}.
  \]

- Write the conversion formula as a fraction \( \frac{1}{()} \) or \( \frac{()}{1} \).
  
  \[
  \text{Example: } 1 \text{m} = 100 \text{cm} \text{ can be written as } \frac{1 \text{m}}{(100 \text{cm})} \text{ or } \frac{(100 \text{cm})}{1 \text{m}}.
  \]

(Put the desired or unknown unit on the top.)

- Multiply the original term by \( \frac{1}{()} \) or \( \frac{()}{1} \). (Cancel out the same units).

Metric conversion using the unit factor method:

Example: \( 1200 \text{ g} = (?) \text{ kg} \)

- Write the original term (the left side) as a fraction: \( 1200 \text{ g} = \frac{1200 \text{ g}}{1} \)

- Write the conversion formula as a fraction. \( 1 \text{ kg} = 1000 \text{ g} \): \( \frac{1 \text{ kg}}{(1000 \text{ g})} \) “kg” is the desired unit.

- Multiply: \( 1200 \text{ g} = \frac{1200 \text{ g}}{1} \cdot \frac{1 \text{ kg}}{(1000 \text{ g})} \)
  
  \[
  = \frac{1200 \text{ kg}}{1000}
  \]

  \[
  = 1.2 \text{ kg}
  \]

Example: \( 30 \text{ cm} = (?) \text{ mm} \)

- Write the original term (the left side) as a fraction: \( 30 \text{ cm} = \frac{30 \text{ cm}}{1} \)

- Write the conversion formula as a fraction. \( 1 \text{ cm} = 10 \text{ mm} \): \( \frac{10 \text{ mm}}{1 \text{ cm}} \) “mm” is the desired unit.

- Multiply: \( 30 \text{ cm} = \frac{30 \text{ cm}}{1} \cdot \frac{(10 \text{ mm})}{1 \text{ cm}} \)
  
  \[
  = \frac{30 \text{ cm}}{(30)(10) \text{ mm}}
  \]

  \[
  = \frac{1}{300 \text{ mm}}
  \]

Adding and subtracting SI measurements:

Example: \( 3 \text{ m} \quad \rightarrow \quad 3000 \text{ mm} \quad 1 \text{ m} = 1000 \text{ mm} \)

\[
\begin{array}{c}
3 \text{ m} \\
- 2000 \text{ mm}
\end{array}
\]

\[
\begin{array}{c}
3000 \text{ mm} \\
- 2000 \text{ mm}
\end{array}
\]

\[
\begin{array}{c}
1000 \text{ mm}
\end{array}
\]

Combine after converting to the same unit.

Example: \( 25 \text{ kg} \quad \rightarrow \quad 25000 \text{ g} \quad 1 \text{ kg} = 1000 \text{ g} \)

\[
\begin{array}{c}
25 \text{ kg} \\
+ 4 \text{ g}
\end{array}
\]

\[
\begin{array}{c}
25000 \text{ g} \\
+ 4 \text{ g}
\end{array}
\]

\[
\begin{array}{c}
25004 \text{ g}
\end{array}
\]
Topic B: Metric Units for Area and Volume

Convert Units of Area and Volume

Area unit conversion

- Area unit conversion: convert the length or distance twice.

Since the units of area are always expressed as square units (in m², cm², ft², yd², etc.)

**Example:** The area of a square is side squared \((A = s^2)\).

(Convert the unit of the side twice.)

- **Steps for area unit conversion:**

  **Steps**
  - Determine the number of decimal places it would move with ordinary units of length.
  - **Double** this number, and move that number of decimal places for units of area.

(Since area is in m², cm², ft², yd², etc.)

**Example:** Convert.

\[
0.03 \text{ km}^2 = ( ? ) \text{ m}^2 \\
0.03 \text{ km}^2 = 003000 \text{ m}^2 = 30000 \text{ m}^2
\]

Volume unit conversion

- Volume unit conversion: convert the length or distance three times.

Since the units of volume are always expressed as cubic units (in m³, cm³, ft³, yd³, etc.)

**Example:** The volume of a cube is side cubed \((V = s^3)\).

(Convert the unit of the side three times.)

- **Steps for volume unit conversion:**

  **Steps**
  - Determine the number of decimal places it would move with ordinary units of length.
  - **Triple** this number, and move that number of decimal places for units of volume.

(Since volume is in m³, cm³, ft³, yd³, etc.)

**Example:** Convert.

\[
5300 \text{ mm}^3 = ( ? ) \text{ cm}^3 \\
5300 \text{ mm}^3 = 5.3 \text{ cm}^3 \\
(5300 = 5300.)
\]
The Relationship between \( mL \), \( g \), and \( cm^3 \)

How are \( mL \), \( g \), and \( cm^3 \) related?

- Recall: millimeter = \( mL \), gram = \( g \), cubic centimeter = \( cm^3 \)
- A cube takes up 1 \( cm^3 \) of space \((1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 1\text{cm}^3)\).
- A cube holds 1 \( mL \) of water and has a mass of 1 gram at 4\( ^0 \text{C} \).

The relationship between \( mL \), \( g \) and \( cm^3 \) — formulas:

\[
1 \text{ cm}^3 = 1 \text{ mL} = 1 \text{ g}
\]

Or

\[
1 \text{ cm}^3 = 1 \text{ mL} \quad 1 \text{ mL} = 1 \text{ g} \quad 1 \text{ cm}^3 = 1 \text{ g}
\]

Example:

1) \( 16\text{cm}^3 = (\ ? \ ) \text{ g} \)

\[
16\text{cm}^3 = \frac{16}{1} \text{ g} \quad 1 \text{ cm}^3 = 1 \text{ g}
\]

2) \( 9 \text{ L} = (\ ? \ ) \text{ cm}^3 \)

\[
9 \text{ L} = 9000 \text{ mL} \quad 1 \text{ L} = 1000 \text{ mL} \quad 1 \text{ mL} = 1 \text{ cm}^3
\]

3) \( 35 \text{ cm}^3 = (\ ? \ ) \text{ cL} \)

\[
35\text{cm}^3 = 35 \text{ mL} \quad 1 \text{ cm}^3 = 1 \text{ mL} \quad 1 \text{ mL} = 1 \text{ cm}^3
\]

\[
= \frac{3.5}{1} \text{ cL} \quad \text{move 1 decimal place left.}
\]

\[
450 \text{ kg} = (\ ? \ ) \text{ L} \quad 1 \text{ kg} = 1000 \text{ g} \quad 1 \text{ g} = 1 \text{ mL} \quad 1 \text{ L} = 1000 \text{ mL}
\]

4) \( 450 \text{ kg} = 450,000 \text{ g} \)

\[
= 450,000 \text{ mL} \quad 1 \text{ km} = 1000 \text{ g} \quad 1 \text{ g} = 1 \text{ mL} \quad 1 \text{ L} = 1000 \text{ mL}
\]

\[
= \frac{450}{1} \text{ L}
\]

Example:

A swimming pool that measures 10 m by 8 m by 2 m. How many \textit{kiloliters} of water will it hold?

\[
V = wh = (8\text{m}) (10\text{m}) (2\text{m}) = 160 \text{ m}^3
\]

\[
160\text{m}^3 = 160,000,000 \text{ cm}^3 \quad 1 \text{ m} = 100 \text{ cm}, 3 \times 2 = 6, \text{ move 6 places right for volume.}
\]

\[
160,000,000 \text{ cm}^3 = 160,000,000 \text{ mL} \quad 1 \text{ mL} = 1 \text{ cm}^3
\]

\[
160,000,000 \text{ mL} = 160 \text{ kL} \quad 1 \text{ kL} = 1,000,000 \text{ mL}
\]

\[
160 \text{ m}^3 = 160 \text{ kL}
\]

The swimming pool will hold 160 kL of water.
Topic C: Imperial System

The System of Imperial Units

Imperial system units: a system of measurement units originally defined in England, including the foot, pound, quart, ounce, gallon, mile, yard, etc.

Length, weight, liquid volume and time:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>inch, foot, yard, mile, etc.</td>
</tr>
<tr>
<td>Weight</td>
<td>pound, ounce, ton, etc.</td>
</tr>
<tr>
<td>Liquid volume</td>
<td>fluid ounce, pint, quart, gallon, cup, teaspoon, etc.</td>
</tr>
<tr>
<td>Time</td>
<td>year, week, day, hour, minute, second, etc.</td>
</tr>
<tr>
<td>Temperature</td>
<td>degree / Fahrenheit (°F)</td>
</tr>
</tbody>
</table>

Imperial equivalents:

<table>
<thead>
<tr>
<th>Unit name</th>
<th>Symbol (abbreviation)</th>
<th>Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inch</td>
<td>in. or &quot;</td>
<td></td>
</tr>
<tr>
<td>foot</td>
<td>ft. or '</td>
<td>1 ft = 12 in</td>
</tr>
<tr>
<td>yard</td>
<td>yd.</td>
<td>1 yd = 3 ft</td>
</tr>
<tr>
<td>mile</td>
<td>mi.</td>
<td>1 mi = 5280 ft</td>
</tr>
<tr>
<td><strong>Weight</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ounce</td>
<td>oz.</td>
<td></td>
</tr>
<tr>
<td>pound</td>
<td>lb.</td>
<td>1 lb = 16 oz</td>
</tr>
<tr>
<td>ton</td>
<td>ton</td>
<td>1 ton = 2000 lb</td>
</tr>
<tr>
<td><strong>Liquid volume</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fluid ounce</td>
<td>fl oz.</td>
<td></td>
</tr>
<tr>
<td>pint</td>
<td>pt.</td>
<td>1 pt = 16 fl oz</td>
</tr>
<tr>
<td>quart</td>
<td>qt.</td>
<td>1 qt = 2 pt</td>
</tr>
<tr>
<td>gallon</td>
<td>gal.</td>
<td>1 gal = 4 qt</td>
</tr>
<tr>
<td>cup</td>
<td>c.</td>
<td>1 c = 8 fl oz</td>
</tr>
<tr>
<td>teaspoon</td>
<td>tsp.</td>
<td>3 tsp = 1 tbsp</td>
</tr>
<tr>
<td>tablespoon</td>
<td>tbsp.</td>
<td>16 tbsp = 1 c</td>
</tr>
<tr>
<td><strong>Time</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>second</td>
<td>s.</td>
<td>1 min. = 60 s</td>
</tr>
<tr>
<td>minute</td>
<td>min.</td>
<td>1 hr = 60 min = 3600 s</td>
</tr>
<tr>
<td>hour</td>
<td>hr.</td>
<td>1 d = 24 hr</td>
</tr>
<tr>
<td>day</td>
<td>d.</td>
<td>1 wk = 7 d</td>
</tr>
<tr>
<td>week</td>
<td>wk.</td>
<td>1 yr = 52 wk</td>
</tr>
<tr>
<td>year</td>
<td>yr.</td>
<td>1 yr = 365 d</td>
</tr>
</tbody>
</table>
Imperial Unit Conversion

Imperial conversion using the unit factor method:

- Write the original term as a fraction (over 1).
  
  Example: 10 g can be written as \[ \frac{10 \text{ g}}{1} \]

- Write the conversion formula as a fraction \( \frac{1}{()} \) or \( \frac{()}{1} \).
  
  Example: \( 1 \text{ ft} = 12 \text{ in} \) can be written as \( \frac{1 \text{ ft}}{12 \text{ in}} \) or \( \frac{12 \text{ in}}{1 \text{ ft}} \)

(Put the unknown or desired unit on the top.)

- Multiply the original term by \( \frac{1}{()} \) or \( \frac{()}{1} \). (Cancel out the same units).

Example: \( 4 \text{ ft} = (\ ?) \text{ in} \)

- Write the original term (the left side) as a fraction: \( 4 \text{ ft} = \frac{4 \text{ ft}}{1} \)

- Write the conversion formula as a fraction. \( 1 \text{ ft} = 12 \text{ in} \): \( \frac{12 \text{ in}}{1 \text{ ft}} \)

- Multiply: \( 4 \text{ ft} = \frac{4 \text{ ft}}{1} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = \frac{(4)(12 \text{ in})}{1} = 48 \text{ in} \)

Example: \( 20 \text{ qt} = (\ ?) \text{ pt} \)

- Write the original term as a fraction: \( 20 \text{ qt} = \frac{20 \text{ qt}}{1} \)

- Write the conversion formula as a fraction. \( 1 \text{ qt} = 2 \text{ pt} \): \( \frac{2 \text{ pt}}{1 \text{ qt}} \)

- Multiply: \( 20 \text{ qt} = \frac{20 \text{ qt}}{1} \cdot \frac{2 \text{ pt}}{1 \text{ qt}} = 40 \text{ pt} \)

Example: \( 8 \text{ mi} = (\ ?) \text{ yd} \)

- Write the original term as a fraction: \( 8 \text{ mi} = \frac{8 \text{ mi}}{1} \)

- Write the conversion formula as a fraction.
  
  \( 1 \text{ mi} = 5280 \text{ ft} \): \( \frac{5280 \text{ ft}}{1 \text{ mi}} \)

  \( 1 \text{ yd} = 3 \text{ ft} \): \( \frac{1 \text{ yd}}{3 \text{ ft}} \)

- Multiply: \( 8 \text{ mi} = \frac{8 \text{ mi}}{1} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{(8)(5280)(1 \text{ yd})}{3} = 14080 \text{ yd} \)
## Topic D: Converting between Metric and Imperial Units

### Imperial and Metric Conversion

#### Key imperial and metric unit conversions:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Metric to imperial</th>
<th>Imperial to metric</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 m ≈ 39 in</td>
<td>1 in ≈ 2.54 cm</td>
<td>inch: in. or “”</td>
<td></td>
</tr>
<tr>
<td>1 m ≈ 3.28 ft</td>
<td>1 ft ≈ 30.48 cm</td>
<td>foot: ft. or ’</td>
<td></td>
</tr>
<tr>
<td>1 m ≈ 1.09 yd</td>
<td>1 mi ≈ 1.61 km</td>
<td>yard: yd.</td>
<td></td>
</tr>
<tr>
<td>1 km ≈ 0.621 mi</td>
<td>1 yd ≈ 0.914 m</td>
<td>mile: mi.</td>
<td></td>
</tr>
<tr>
<td>1 kg ≈ 2.2 lb</td>
<td>1 oz ≈ 28.35 g</td>
<td>pound: lb.</td>
<td></td>
</tr>
<tr>
<td>1 ton ≈ 910 kg</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 L ≈ 0.264 gal</td>
<td>1 qt ≈ 0.946 L</td>
<td>gallon: gal.</td>
<td></td>
</tr>
<tr>
<td><strong>Volume</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 L ≈ 2.1 pt</td>
<td>1 gal ≈ 3.79 L</td>
<td>pint: pt.</td>
<td></td>
</tr>
<tr>
<td>1 L ≈ 1.06 qt</td>
<td>1 pt ≈ 470 mL</td>
<td>quart: qt.</td>
<td></td>
</tr>
<tr>
<td>1 mL = 0.2 tsp</td>
<td>1 tsp = 5 mL</td>
<td>teaspoon: tsp.</td>
<td></td>
</tr>
</tbody>
</table>

#### Imperial - metric unit conversion (the unit factor method):

- Write the original term as a fraction (over 1).
  
  e.g., 10 gal can be written as \( \frac{10 \text{ gal}}{1} \)

- Write the conversion formula as a fraction \( \frac{1}{()} \) or \( \frac{()}{1} \).
  
  e.g., 1 mL = 0.2 tsp can be written as \( \frac{1 \text{ mL}}{0.2 \text{ tsp}} \) or \( \frac{0.2 \text{ tsp}}{1 \text{ mL}} \)

  (Put the desired or unknown unit on the top.)

- Multiply the original term by \( \frac{1}{()} \) or \( \frac{()}{1} \). (Cancel out the same units).

**Example:**

2 ft = (?) m

- Write the original term (the left side) as a fraction:
  
  \( 2 \text{ ft} = \frac{2 \text{ ft}}{1} \)

- Write the conversion formula as a fraction. 1 m ≈ 3.28 ft:
  
  \( \frac{1 \text{ m}}{(3.28 \text{ ft})} \)

  “m” is the desired unit.

- Multiply:
  
  \( 2 \text{ ft} \times \frac{1 \text{ m}}{(3.28 \text{ ft})} \approx 0.61 \text{ m} \)
Example: 120 oz = (?) kg

- Write the original term (the left side) as a fraction: \( \frac{120 \text{ oz}}{1} \)

- Write the conversion formula as a fraction. 1 oz ≈ 28.35 g: \( \frac{28.35 \text{ g}}{1 \text{ oz}} \)  “g” is the desired unit.

- Multiply: \( 120 \text{ oz} \times \frac{28.35 \text{ g}}{1 \text{ oz}} = 3402 \text{ g} = \frac{3402 \text{ kg}}{1} \)  1 kg = 1000 g

Example: 250 mL = (?) tsp

- Original term to fraction: \( \frac{250 \text{ mL}}{1} \)

- Conversion formula: 1 tsp = 5 mL: \( \frac{1 \text{ tsp}}{5 \text{ mL}} \)  “tsp” is the desired unit.

- Multiply: \( 250 \text{ mL} \times \frac{1 \text{ tsp}}{5 \text{ mL}} = \frac{250 \text{ mL}}{1} \times \frac{1 \text{ tsp}}{5 \text{ mL}} = 50 \text{ tsp} \)

Example: 10560 yd = (?) mi

- Original term to fraction: \( \frac{10560 \text{ yd}}{1} \)

- Conversion formula: 3 ft = 1 yd: \( \frac{3 \text{ ft}}{1 \text{ yd}} \)  “ft” is the desired unit.

1 mi = 5280 ft: \( \frac{1 \text{ mi}}{5280 \text{ ft}} \)  “mi” is the desired unit.

- Multiply: \( 10560 \text{ yd} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} = \frac{10560 \text{ yd}}{1} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} = \frac{10560}{1} \times \frac{3}{1} \times \frac{1 \text{ mi}}{5280} = \frac{(10560)(3) \text{ mi}}{5280} = 6 \text{ mi} \)

Example: Two towns are 600 miles apart. How many kilometers separate them? \( \frac{600 \text{ mi}}{1} \)

- 600 miles = (?) km

- Original term to fraction: \( \frac{600 \text{ mi}}{1} \)

- Conversion formula: 1 km ≈ 0.6214 mi: \( \frac{1 \text{ km}}{(0.6214 \text{ mi})} \)  “km” is the desired unit.

- Multiply: \( 600 \text{ miles} \times \frac{1 \text{ km}}{(0.6214 \text{ mi})} = \frac{600 \text{ mi}}{1} \times \frac{1 \text{ km}}{(0.6214 \text{ mi})} \approx 965.6 \text{ km} \)

The distance between two towns is 965.6 km.
Unit 4: Summary

Measurement

Metric system (SI – international system of units): the most widely used system of measurement in the world. It is based on the basic units of meter, kilogram, second, etc.

Imperial system units: a system of measurement units originally defined in England, including the foot, pound, quart, ounce, gallon, mile, yard, etc.

Metric prefixes (SI prefixes): large and small numbers are made by adding SI prefixes, which is based on multiples of 10.

Steps for metric conversion through decimal movement:

- Identify the number of places to move on the metric conversion table.
- Move the decimal point.
  - Convert a smaller unit to a larger unit: move the decimal point to the left.
  - Convert a larger unit to a smaller unit: move the decimal point to the right.

Convert units using the unit factor method (or the factor-label method):

- Write the original term as a fraction (over 1).
  
  Example: 10g can be written as $\frac{10 \text{ g}}{1}$

- Write the conversion formula as a fraction $\frac{1}{()}$ or $\frac{()}{1}$.
  
  Example: $1 \text{ m} = 100 \text{ cm}$ can be written as $\frac{1 \text{ m}}{(100 \text{ cm})}$ or $\frac{(100 \text{ cm})}{1 \text{ m}}$

(Put the desired or unknown unit on the top.)

- Multiply the original term by $\frac{1}{()}$ or $\frac{()}{1}$. (Cancel out the same units).

Key metric prefix:

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol (abbreviation)</th>
<th>Power of 10</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>mega</td>
<td>M</td>
<td>$10^6$</td>
<td>1 Mm = 1,000,000 m</td>
</tr>
<tr>
<td>kilo-</td>
<td>k</td>
<td>$10^3$</td>
<td>1 km = 1,000 m</td>
</tr>
<tr>
<td>hecto-</td>
<td>h</td>
<td>$10^2$</td>
<td>1 hm = 100 m</td>
</tr>
<tr>
<td>deka-</td>
<td>da</td>
<td>$10^1$</td>
<td>1 dam = 10 m</td>
</tr>
<tr>
<td>meter/gram/liter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>deci-</td>
<td>d</td>
<td>$10^{-1}$</td>
<td>1 m = 10 dm</td>
</tr>
<tr>
<td>centi-</td>
<td>c</td>
<td>$10^{-2}$</td>
<td>1 m = 100 cm</td>
</tr>
<tr>
<td>milli-</td>
<td>m</td>
<td>$10^{-3}$</td>
<td>1 m = 1,000 mm</td>
</tr>
<tr>
<td>micro</td>
<td>$\mu$</td>
<td>$10^{-6}$</td>
<td>1 m = 1,000,000 $\mu$m</td>
</tr>
</tbody>
</table>
Metric conversion table:

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Value</th>
<th>1,000,000</th>
<th>1,000</th>
<th>100</th>
<th>10</th>
<th>1</th>
<th>.</th>
<th>.1</th>
<th>.01</th>
<th>0.001</th>
<th>0.000 001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>Larger</td>
<td>M</td>
<td>k</td>
<td>h</td>
<td>da</td>
<td>meter (m)</td>
<td>gram (g)</td>
<td>liter (L)</td>
<td>.</td>
<td>d</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>Small</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Steps for area unit conversion:
- Determine the number of decimal places it would move with ordinary units of length.
- Double this number, and move that number of decimal places for units of area.

Steps for volume unit conversion:
- Determine the number of decimal places it would move with ordinary units of length.
- Triple this number, and move that number of decimal places for units of volume.

The relationship between mL, g and cm³ — formulas:
- A cube holds 1 mL of water and has a mass of 1 gram at 40 C.
- \( 1 \text{ cm}^3 = 1 \text{ mL} = 1 \text{ g} \)
- Or \( 1 \text{ cm}^3 = 1 \text{ mL} \) \( 1 \text{ mL} = 1 \text{ g} \) \( 1 \text{ cm}^3 = 1 \text{ g} \)
Unit 4: Self - Test

Measurement

Topic A

1. Convert each measurement using the metric conversion table.
   a) \(439 \text{ mm} = (?) \text{ m}\)
   b) \(2.236 \text{ hg} = (?) \text{ g}\)
   c) \(48.3 \text{ mL} = (?) \text{ kL}\)
   d) \(2.5 \text{ kg} = (?) \text{ hg}\)

2. Convert each measurement using the unit factor method.
   a) \(7230 \text{ g} = (?) \text{ kg}\)
   b) \(52 \text{ cm} = (?) \text{ mm}\)
   c) \(3.4 \text{ dL} = (?) \text{ L}\)
   d) \(52 \text{ daL} = (?) \text{ cL}\)

3. Combine.
   a) \(7 \text{ m} - 3000 \text{ mm} = (?) \text{ mm}\)
   b) \(63 \text{ kg} + 6 \text{ g} = (?) \text{ g}\)
   c) \(0.72 \text{ L} + 4.58 \text{ L} - 10\text{ mL} = (?) \text{ mL}\)
   d) \(25.3 \text{ km} + 357 \text{ dam} = (?) \text{ km}\)

Topic B

   a) \(7400 \text{ cm}^2 = (?) \text{ m}^2\)
   b) \(0.09 \text{ km}^2 = (?) \text{ m}^2\)
   c) \(5\text{ m}^3 = (?) \text{ cm}^3\)
   d) \(567 \text{ mm}^3 = (?) \text{ cm}^3\)

5. Complete.
a) A cube holds 1 mL of water and has a mass of 1 gram at ( ) °C.
b) 38 cm³ = ( ) g
c) 5 L = ( ) cm³
d) 27 cm³ = ( ? ) cL
e) 76 cm³ of water at 4°C has a mass of ( ) g.
f) 18 L of water has a volume of ___________ cm³.
g) 257 kg = ( ? ) L
h) A fish box that measures 45 cm by 35 cm by 25 cm. How many kiloliters of water will it hold?

**Topic C**

6. Convert the following imperial system units.
   a) 9 ft to inches
   b) 47 qt to pints
   c) 4 mi to yards
   d) 9276 pounds to tons

**Topic D**

7. Convert.
   a) 8 ft. to meters
   b) 268 oz. to kilograms
   c) 465 mL to tsp
   d) 15840 yd. to miles
   e) Two towns are 450 miles apart. How many kilometers separate them?
Unit 5
The Real Number System

Topic A: Rational and irrational numbers
- Real numbers

Topic B: Properties of addition and multiplication
- Properties of addition
- Properties of multiplication
- Properties of addition & multiplication

Topic C: Signed numbers and absolute value
- Signed numbers
- Absolute value

Topic D: Operations with signed numbers
- Adding and subtracting signed numbers
- Multiplying signed numbers
- Dividing signed numbers

Unit 5 Summary

Unit 5: Self - test
Topic A: Rational and Irrational Numbers

Real Numbers

Natural numbers: the numbers used for counting. 1, 2, 3, 4, 5, 6 …

Whole numbers: the natural numbers plus 0. 0, 1, 2, 3, 4, 5, 6 …

Integers: all the whole numbers and their negatives. … -4, -3, -2, -1, 0, 1, 2, 3, 4 …

Rational number: a number that can be expressed as a fraction of two integers \( \frac{a}{b} \).

Examples of rational numbers:

\[
\frac{3}{4}, \quad \frac{2}{3} (= \frac{14}{7}), \quad 11 (= \frac{11}{1}), \quad 0 (= \frac{0}{1}), \quad 0.52 (= \frac{52}{100}), \quad -4.5 (= \frac{-9}{2}), \quad \sqrt{4} (=2)
\]

Rational numbers can be expressed as terminating decimals or repeating decimals.

Example:

\[
\frac{3}{4} = 0.75 \quad \text{A terminating decimal.}
\]

\[
\frac{2}{3} = 0.66666… = 0.\overline{6} \quad \text{A repeating decimal.}
\]

\[
0.232323… = 0.\overline{23} \quad \text{A repeating decimal.}
\]

Irrational number: a number that cannot be represented by the fractions of two integers.

Examples of irrational numbers: \( \pi \), \( \sqrt{3} \), \( \sqrt{19} \), \( 5\sqrt{13} \)

Irrational numbers cannot be expressed as terminating decimals or repeating decimals.

\[
\pi \approx 3.14159265358979323… \quad \text{A non-terminating and non-repeating decimal.}
\]

\[
\sqrt{3} \approx 1.73205… \quad \text{A non-terminating and non-repeating decimal.}
\]

Real numbers (\( R \)): rational numbers plus irrational numbers.

The real number system:
**Topic B: Properties of Addition and Multiplication**

### Properties of Addition

**Commutative property:** changing the order of the numbers does not change the sum (order does not matter).

\[ a + b = b + a \]

**Example:** \[ 2 + 3 = 3 + 2 \]

**Associative property:** regrouping the numbers does not change the sum (it does not matter where you put the parenthesis).

\[ (a + b) + c = a + (b + c) \]

**Example:** \[ (2 + 1) + 3 = 2 + (1 + 3) \]

**Additive identity property:** the sum of any number and zero leaves that number unchanged.

\[ a + 0 = a \]

**Example:** \[ 100 + 0 = 100 \]

**Closure property of addition:** the sum of any two real numbers equals another real number.

**Example:** If 3 and 8 are real numbers, then \[ 3 + 8 = 11 \] is another real number.

**Additive inverse property:** the sum of any real number and its negative is always a zero.

\[ -a + a = 0 \]

**Example:** \[ 7 + (-7) = 0 \]

A summary of properties of addition:

<table>
<thead>
<tr>
<th>Additive Properties</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Commutative property</strong> (switch order)</td>
<td>[ a + b = b + a ]</td>
</tr>
<tr>
<td><strong>Associative property</strong> (switch parentheses)</td>
<td>[ (a + b) + c = a + (b + c) ]</td>
</tr>
<tr>
<td><strong>Identity property</strong></td>
<td>[ a + 0 = a ]</td>
</tr>
<tr>
<td><strong>Closure property</strong></td>
<td>If ( a ) and ( b ) are real numbers, then ( a + b ) is a real number.</td>
</tr>
<tr>
<td><strong>Inverse property</strong></td>
<td>[ -a + a = 0 ]</td>
</tr>
</tbody>
</table>

**Example:** Name the properties.

1) \[ 7x + 0 = 7x \]

**Answer:** Identity property

2) \[ (97 + 22) + 3 = (97 + 3) + 22 \]

**Answer:** Commutative property (switch order)

3) \[ (3 + 11x) + 7x = 3 + (11x + 7x) \]

**Answer:** Associative property (switch parentheses)

4) \[ (4y + 3) + [- (4y + 3)] = 0 \]

**Answer:** Inverse property of addition
Properties of Multiplication

**Commutative property:** changing the order of the numbers does not change the product (order does not matter).

\[ a \cdot b = b \cdot a \]

Example:
\[ 2 \cdot 6 = 6 \cdot 2 \]
\[ 12 = 12 \]

**Associative property:** regrouping the numbers does not change the product (it does not matter where you put the parenthesis).

\[ (a \cdot b) \cdot c = a \cdot (b \cdot c) \]

Example:
\[ (2 \cdot 4) \cdot 3 = 2 \cdot (4 \cdot 3) \]
\[ 24 = 24 \]

**Multiplicative identity property:** a number does not change when it is multiplied by 1.

Example:
\[ 9 \cdot 1 = 9 \]
\[ a \cdot 1 = a \]

**Distributive property:** multiply the number outside the parenthesis by each of the numbers inside the parenthesis.

\[ a \cdot (b + c) = ab + ac \] or \[ a \cdot (b - c) = ab - ac \]

Example:
\[ 2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4 \]
\[ 14 = 14 \]
\[ 5 \cdot (6 - 3) = 5 \cdot 6 - 5 \cdot 3 \]
\[ 15 = 15 \]

**Multiplicative property of zero:** any number multiplied by zero always equals zero.

Example:
\[ 100 \cdot 0 = 0 \]
\[ a \cdot 0 = 0 \]

**Closure property of multiplication:** the product of any two real numbers equals another real number.

Example: If 5 and 4 are real numbers, then \( 5 \cdot 4 = 20 \) is another real number.

**Multiplicative inverse property:** the product of any nonzero real number and its reciprocal is always one.

\[ a \cdot \frac{1}{a} = 1 \]

Example:
1) \[ 9 \cdot \frac{1}{9} = 1 \]
2) \[ (12x) \cdot \left( \frac{1}{12x} \right) = 1 \]

Recall reciprocal: Reciprocal = \[ \frac{1}{\text{number}} \]

For example, the reciprocal of \( \frac{1}{4} \) is \( \frac{4}{1} \) its reciprocal.
A summary of properties of multiplication:

<table>
<thead>
<tr>
<th>Multiplicative properties</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Commutative property</strong></td>
<td>$a \cdot b = b \cdot a$</td>
</tr>
<tr>
<td>(Switch order)</td>
<td>$2 \cdot 3 = 3 \cdot 2$</td>
</tr>
<tr>
<td><strong>Associative property</strong></td>
<td>$(a \cdot b) \cdot c = a \cdot (b \cdot c)$</td>
</tr>
<tr>
<td>(Switch parentheses)</td>
<td>$(2 \cdot 1) \cdot 3 = 2 \cdot (1 \cdot 3)$</td>
</tr>
<tr>
<td><strong>Identity property of 1</strong></td>
<td>$a \cdot 1 = a$</td>
</tr>
<tr>
<td></td>
<td>$100 \cdot 1 = 100$</td>
</tr>
<tr>
<td><strong>Closure property</strong></td>
<td>If $a$ and $b$ are real numbers, then $ab$ is a real number.</td>
</tr>
<tr>
<td></td>
<td>$3$ and $4$ are real numbers, so $3 \cdot 4 = 12$ is a real number</td>
</tr>
<tr>
<td><strong>Distributive property</strong></td>
<td>$a \cdot (b + c) = a \cdot b + a \cdot c$</td>
</tr>
<tr>
<td></td>
<td>$2 \cdot (3 + 4) = 2 \cdot 3 + 2 \cdot 4$</td>
</tr>
<tr>
<td></td>
<td>$a \cdot (b - c) = a \cdot b - a \cdot c$</td>
</tr>
<tr>
<td></td>
<td>$3 \cdot (4 - 2) = 3 \cdot 4 - 3 \cdot 2$</td>
</tr>
<tr>
<td><strong>Property of zero</strong></td>
<td>$a \cdot 0 = 0$</td>
</tr>
<tr>
<td></td>
<td>$35 \cdot 0 = 0$</td>
</tr>
<tr>
<td><strong>Inverse property</strong></td>
<td>$a \cdot \frac{1}{a} = 1$</td>
</tr>
<tr>
<td></td>
<td>$5 \cdot \frac{1}{5} = 1$</td>
</tr>
</tbody>
</table>

**Example:** Name the properties

1) $(3y) \cdot (5y) = (5 \cdot 3) \cdot (y \cdot y)$
   
   $= 15y^2$

2) $(9x) \cdot x^2 = 9 \cdot (x \cdot x^2)$

   $= 9x^3$

3) $\frac{1}{5} \cdot (10x - 15) = \frac{1}{5} \cdot 10x - \frac{1}{5} \cdot 15$
   
   $= 2x - 3$

4) $-(7 + 3x) \cdot \frac{1}{-(7+3x)} = 1$

5) $(2x - 3y) \cdot x = 2x^2 - 3xy$

6) $\frac{1}{4x} \cdot 0 = 0$

7) $(1000 \cdot 8) \cdot 9 = 1000 \cdot (8 \cdot 9)$

   $= 1000 \cdot 72 = 72000$

**Answer**

- Commutative property of multiplication
- Associative property of multiplication
- Distributive property of multiplication
- Inverse property of multiplication
- Distributive property
- Multiplicative property of zero
- Associative property of multiplication
## Properties of Addition & Multiplication

### Properties of Addition and Multiplication:

<table>
<thead>
<tr>
<th>Name</th>
<th>Additive properties</th>
<th>Multiplicative properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative property</td>
<td>(a + b = b + a)</td>
<td>(a \cdot b = b \cdot a)</td>
</tr>
<tr>
<td>Associative property</td>
<td>((a + b) + c = a + (b + c))</td>
<td>((a \cdot b) \cdot c = a \cdot (b \cdot c))</td>
</tr>
<tr>
<td>Identity property</td>
<td>(a + 0 = a)</td>
<td>(a \cdot 1 = a)</td>
</tr>
<tr>
<td>Closure property</td>
<td>If (a) and (b) are real numbers, then (a + b) is a real number.</td>
<td>If (a) and (b) are real numbers, then (a \cdot b) is a real number.</td>
</tr>
<tr>
<td>Inverse property</td>
<td>(-a + a = 0)</td>
<td>(a \cdot \frac{1}{a} = 1)</td>
</tr>
<tr>
<td>Distributive property</td>
<td>(a \cdot (b + c) = ab + ac)</td>
<td></td>
</tr>
<tr>
<td>Property of zero</td>
<td>(a \cdot 0 = 0)</td>
<td></td>
</tr>
</tbody>
</table>

### Example:

Regroup and simplify the calculations using properties.

1) \((43 + 1998) + 2 = ?\)

\[
43 + (1998 + 2) = 2043 \\
\text{Associative property of addition}
\]

2) \((7 \cdot 1000) \cdot 9 = ?\)

\[
(7 \cdot 9) \cdot 1000 = 63,000 \\
\text{Commutative property of multiplication}
\]

### Example:

Solving the problems in two ways.

1) \(3 (4 + 2) = ?\)

   a) \(3 \cdot 6 = 18\)
   
   b) \(3 \cdot 4 + 3 \cdot 2 = 18\)  \text{Distributive property}

2) \(\frac{1}{2} \left(\frac{1}{2} + \frac{2}{3}\right) = ?\)

   a) \(\frac{1}{2} \left(\frac{1}{2} + \frac{5}{3}\right) = \frac{1}{2} \left(\frac{3}{6} + \frac{10}{6}\right) = \frac{1}{2} \cdot \frac{13}{6} = \frac{13}{12} = \frac{1}{12} + \frac{1}{12}\)

   b) \(\frac{1}{2} \left(\frac{1}{2} + \frac{5}{3}\right) = \frac{1}{2} \left(\frac{1}{2} \right) + \frac{1}{2} \left(\frac{5}{3}\right)\)  \text{Distributive property}

\[
= \frac{1}{4} + \frac{5}{6} = \frac{3}{12} + \frac{10}{12} = \frac{13}{12} = \frac{1}{12} + \frac{1}{12}
\]
Topic C:  Signed Numbers and Absolute Value

Signed Numbers: a positive number is written with a plus sign (or without sign) in front and a negative number is written with a minus sign in front.

Example: Positive number: +5 (or 5), 7x, 4y^2
Negative number: -3, -2, -9x

Positive and negative numbers in real life:

<table>
<thead>
<tr>
<th>Meaning</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td></td>
</tr>
<tr>
<td>+ 0°C: above 0 degree</td>
<td>+20°C</td>
</tr>
<tr>
<td>- 0°C: below 0 degree</td>
<td>-5°C</td>
</tr>
<tr>
<td>Money</td>
<td></td>
</tr>
<tr>
<td>+ $: gain or own</td>
<td>Own: +$10000</td>
</tr>
<tr>
<td>- $: loss or owe</td>
<td>Owe: -$500</td>
</tr>
<tr>
<td>Sports</td>
<td></td>
</tr>
<tr>
<td>+ points: gain</td>
<td>Gain 3 points: +3</td>
</tr>
<tr>
<td>- points: loss</td>
<td>Lost 2 points: -2</td>
</tr>
</tbody>
</table>

Positive and negative numbers: positive numbers are greater than zero; negative numbers are less than zero.

The real number line: a straight line on which every point corresponds to a real number.

Example: Put the following numbers on the real number line.

\[
\frac{3}{4}, \quad -2 \frac{1}{3}, \quad -0.67, \quad \sqrt{3} \approx 1.732, \quad \pi \approx 3.1416
\]

The number on the right is greater than the number on the left on the number line.

Example: \(-5 < -3, \quad -1 < 4, \quad 0 > -2, \quad 2 > \frac{1}{3}, \quad \frac{4}{5} < -\frac{2}{5}\)

Example: Arrange the following numbers from the smallest to the largest number.

a) \(-17, \quad 3, \quad -3, \quad -6, \quad 11, \quad 0\)

\[-17 < -6 < -3 < 0 < 3 < 11\]

b) \(-\frac{1}{2}, \quad \frac{2}{3}, \quad -\frac{1}{4}, \quad 2 \frac{2}{3}\)

\[-\frac{1}{2} = -0.5, \quad \frac{2}{3} \approx 0.67, \quad -\frac{1}{4} = -0.25, \quad 2 \frac{2}{3} = \frac{8}{3} \approx 2.67\]

\(-0.5 < -0.25 < 0.67 < 2.67\)

\[-\frac{1}{2} < -\frac{1}{4} < \frac{2}{3} < 2 \frac{2}{3}\]
Absolute Value

**Absolute value**: geometrically, it is the distance of a number \( x \) from zero on the number line. It is symbolized “\(|x|\)”.

**Example**:  
\(|5|\) is 3 units away from 0.  
\(|18|\) is 18 units away from 0.

**No negatives for absolute value**: the distance is always positive, and absolute value is distance, so the absolute value is never negative.

**Example**:  
\(|2|\) is 2 units away from 0.  
\(|-2|\) is also 2 units away from 0.

**Example**:

a) \(|-8| = 8\)  
b) \(|12 - 2| = 10\)  
c) \(|0.8 - 0.6| = 0.2\)  
d) \(-|-5| = -(5) = -5\)  
e) \(-|-6^2| = -(36) = -36\)

**Order of operations**:

<table>
<thead>
<tr>
<th>Order of operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear the brackets or parentheses and absolute values (innermost first).</td>
</tr>
<tr>
<td>Calculator exponents (power) and radicals.</td>
</tr>
<tr>
<td>Perform multiplication or division (from left-to-right).</td>
</tr>
<tr>
<td>Perform addition or subtraction (from left-to-right).</td>
</tr>
</tbody>
</table>

**Example**:  
1) \(3\ [7 - 4 + (10 - 2)] = 3\ [7 - 4 + 8]\)  
\[= 3\ [3 + 8]\]  
\[= 3 \cdot 11\]  
\[= 33\]

2) \(\frac{|-8|}{2^2} - (4 - 3) = \frac{8}{2^2} - 1\)  
\[= \frac{8}{4} - 1\]  
\[= 2 - 1\]  
\[= 1\]
Adding and Subtracting Signed Numbers

Adding signed numbers

- Add two numbers with the same sign: add their values and keep their common sign.

  **Example:**
  1) \(5 + 4 = 9\)  
  2) \((-6) + (-2) = -8\)  
  3) \(-\frac{1}{2} + (-1\frac{1}{2}) = -\frac{1}{2} + (-\frac{3}{2}) = -\frac{4}{2} = -2\)

- Add two numbers with different signs: subtract their values and keep the sign of the larger absolute value.

  **Example:**
  1) \(2 + (-5) = -3\)  
  2) \((-3) + 7 = 4\)  
  3) \(3.2 + (-0.2) = 3\)

Subtracting signed numbers

- Subtract a number by adding its opposite (additive inverse), i.e. \(a - b = a + (-b)\)

  (Change the sign of \(b\) and then follow the rules for adding signed numbers.)

  **Example:**
  1) \((-3) - (-4) = (-3) + (4) = 1\)  
  2) \((-7) - 2 = (-7) + (-2) = -9\)  
  3) \(-\frac{1}{3} - \frac{2}{3} = -\frac{1}{3} + (-\frac{2}{3}) = -\frac{3}{3} = -1\)

- Opposite (or additive inverse): the opposite of a number (two numbers whose sum is 0).

  **Example:**
  1) The additive inverse of \(7\) is \(-7\)  
  2) The additive inverse of \(-\frac{2}{5}\) is \(\frac{2}{5}\)
Multiplying Signed Numbers

Multiplying two numbers with the same sign: the product is positive.

Example: \(4 \cdot 5 = 20\)
\((-3) \cdot (-5) = 15\)

Multiplying two numbers with different signs: the product is negative.

Example: \((-5) \cdot (6) = -30\)
\((0.3) \cdot (-3) = -0.9\)
\((-4)^2 = 16\)

Multiplying by -1: \(-1 \cdot a = -a\)

Example: \(-1 \cdot 6x = -6x\)
\(-4^2 = -1 \cdot 4^2 = -16\)

Signs of multiplication:

<table>
<thead>
<tr>
<th>Multiplication</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive (\times) Positive = Positive</td>
<td>((+) \times (+) = (+))</td>
</tr>
<tr>
<td>Negative (\times) Positive = Negative</td>
<td>((-) \times (+) = (-))</td>
</tr>
<tr>
<td>Positive (\times) Negative = Negative</td>
<td>((+) \times (-) = (-))</td>
</tr>
<tr>
<td>Negative (\times) Negative = Positive</td>
<td>((-) \times (-) = (+))</td>
</tr>
</tbody>
</table>

Multiplying two or more numbers:

- If the two signs are the same, the result is positive.
  \((-3) \cdot (-4) = 12\)
- If the two signs are different, the result is negative.
  \((-0.5) \cdot (0.6) = -0.3\)
- The product of an even number of negative numbers is always positive.
  \((-4) \cdot (-2) \cdot (-5) \cdot (-1) = 40\)
- The product of an odd number of negative numbers is always negative.
  \((-1)^7 = -1\)

Evaluating expressions:

Example: Evaluate \(a^4 - b + c\) if \(a = -1\), \(b = -2\), \(c = 4\).
\[a^4 - b + c = (-1)^4 - (-2) + 4 = 1 + 2 + 4 = 7\]

Substitute \(a\) for -1, \(b\) for -2 (add parentheses), and \(c\) for 4.
Dividing Signed Numbers

Dividing signed numbers

- Dividing two numbers with the same sign: the quotient is positive.
  
  Example:  
  1) \(-9 \div (-3) = \frac{3}{1}\)  
  2) \(\frac{1.8}{2} = 0.9\)  
  3) \(-\frac{8}{4} \div \left(-\frac{1}{4}\right) = -\frac{8}{4} \times \left(-\frac{4}{1}\right) = 8\)

- Dividing two numbers with different signs: the quotient is negative.
  
  Example:  
  1) \(8 \div (-2) = -4\)  
  2) \(-\frac{49}{7} = -7\)  
  3) \(\frac{3}{9} + \left(-\frac{6}{3}\right) = \frac{3}{9} + \left(-\frac{3}{2}\right) = \frac{-1}{6}\)

Signs of division:

<table>
<thead>
<tr>
<th>Division</th>
<th>Sign</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive ÷ Positive = Positive</td>
<td>+</td>
<td>(\frac{28}{7} = 4)</td>
</tr>
<tr>
<td>Negative ÷ Positive = Negative</td>
<td>-</td>
<td>(-\frac{9}{3} = -3)</td>
</tr>
<tr>
<td>Positive ÷ Negative = Negative</td>
<td>+</td>
<td>(\frac{4.9}{-0.7} = -7)</td>
</tr>
<tr>
<td>Negative ÷ Negative = Positive</td>
<td>-</td>
<td>(-\frac{72}{-8} = 9)</td>
</tr>
</tbody>
</table>

Properties of zero:

- The number 0 divided by any nonzero number is zero. 
- A number divided by 0 is undefined (not allowed).

Evaluating expressions:

Example: Evaluate \(a^2 - \frac{a}{abc}\) if \(a = -2,\ b = 1,\ c = (-1),\ and\ d = 0\).

\[
a^2 - \frac{a}{abc} + \frac{d}{c} = (-2)^2 - \frac{-2}{(-2)(1)(-1)} + \frac{0}{-1} = 4 - \frac{-2}{2} + 0 = \frac{5}{2}.
\]
The Real Number System

The real number system:

<table>
<thead>
<tr>
<th>Real Numbers</th>
<th>Irrational Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational Numbers: ( \frac{3}{5}, -4.27, 0.\overline{6} )</td>
<td>Irrational Numbers: ( \sqrt{7}, \pi, \ldots )</td>
</tr>
<tr>
<td>Integers: ( \ldots -2, -1, 0, 1, 2, \ldots )</td>
<td></td>
</tr>
<tr>
<td>Whole Numbers: ( 0, 1, 2, 3 \ldots )</td>
<td></td>
</tr>
<tr>
<td>Natural Numbers: ( 1, 2, 3 \ldots )</td>
<td></td>
</tr>
</tbody>
</table>

Properties of addition and multiplication:

<table>
<thead>
<tr>
<th>Name</th>
<th>Additive properties</th>
<th>Multiplicative properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative property</td>
<td>( a + b = b + a )</td>
<td>( a \cdot b = b \cdot a )</td>
</tr>
<tr>
<td>Associative property</td>
<td>( (a + b) + c = a + (b + c) )</td>
<td>( (a \cdot b) \cdot c = a \cdot (b \cdot c) )</td>
</tr>
<tr>
<td>Identity property</td>
<td>( a + 0 = a )</td>
<td>( a \cdot 1 = a )</td>
</tr>
<tr>
<td>Closure property</td>
<td>If ( a ) and ( b ) are real numbers, then ( a + b ) is a real number.</td>
<td>If ( a ) and ( b ) are real numbers, then ( a \cdot b ) is a real number.</td>
</tr>
<tr>
<td>Inverse property</td>
<td>( -a + a = 0 )</td>
<td>( a \cdot \frac{1}{a} = 1 )</td>
</tr>
<tr>
<td>Distributive property</td>
<td>( a (b + c) = ab + ac )</td>
<td>( a (b - c) = ab - ac )</td>
</tr>
<tr>
<td>Property of zero</td>
<td></td>
<td>( a \cdot 0 = 0 )</td>
</tr>
</tbody>
</table>

Signed number: a positive number is written with a plus sign (or without sign) in front and a negative number is written with a minus sign in front.

Positive and negative numbers: positive numbers are greater than zero; negative numbers are less than zero.

The real number line: a straight line on which every point corresponds to a real number.

The number on the right is greater than the number on the left on the number line.

Absolute value: geometrically, it is the distance of a number \( x \) from zero on the number line. It is symbolized “\(|x|\)”.
No negatives for absolute value: the distance is always positive, and absolute value is distance, so the absolute value is never negative.

Order of operations with absolute value:

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</tr>
<tr>
<td>Perform addition or subtraction (from left-to-right).</td>
</tr>
</tbody>
</table>

Signed numbers summary:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adding signed numbers</td>
<td>• Add two numbers with the <em>same</em> sign:</td>
</tr>
<tr>
<td></td>
<td>Add their values, and keep their common sign.</td>
</tr>
<tr>
<td></td>
<td>• Add two numbers with <em>different</em> signs:</td>
</tr>
<tr>
<td></td>
<td>Subtract their values, and keep the sign of the larger number.</td>
</tr>
<tr>
<td>Subtracting signed numbers</td>
<td>Subtract a number by adding its opposite.</td>
</tr>
<tr>
<td>Multiplying signed numbers</td>
<td>(+)(+) = (+), (-)(-) = (+), (-)(+) = (-), (+)(-) = (-)</td>
</tr>
<tr>
<td>Dividing signed numbers</td>
<td>$\frac{1}{+} = +$, $\frac{1}{-} = +$, $\frac{-}{+} = -, \frac{-}{-} = -$</td>
</tr>
</tbody>
</table>

Multiplying two or more numbers:

- If the two signs are the same, the result is positive.
- If the two signs are different, the result is negative.
- The product of an *even* number of negative numbers is always *positive*.
- The product of an *odd* number of negative numbers is always *negative*.

Opposite (or additive inverse): the opposite of a number.

Properties of zero

- The number 0 divided by any nonzero number is zero. $\frac{0}{A} = 0$
- A number divided by 0 is undefined (not allowed). $\frac{A}{0}$ is undefined.
Unit 5: Self-Test

The Real Number System

Topic A

1. Give two examples of rational numbers that are not integers.

2. Given the set of numbers:

   -3, 4.7, 0, 8, \( \frac{3}{5} \), 2.56, 5.4259..., \( \pi \), \( \sqrt{5} \)

   Determine which of the numbers above are

   a) natural numbers?
   b) integers?
   c) rational numbers?
   d) irrational numbers?

Topic B

3. Name the properties.
   a) \( 12a + 0 = 12a \)
   b) \( (3x + 11y) + 7 = 7 + (3x + 11y) \)
   c) \( (4 + x) + 11 = 4 + (x + 11) \)
   d) \( (6a + 5) + [-(6a + 5)] = 0 \)
   e) \( 7(3y + 4) = 7 \cdot 3y + 7 \cdot 4 = 21y + 28 \)
   f) \( (0.5a) b = 0.5 (a b) \)
   g) \( (4x) (7y) = (4 \cdot 7) (x y) \)
   h) \( -(8y) \cdot \frac{1}{-(8y)} = 1 \)
   i) \( (4 - 7y) 3 = 12 - 21y \)
   j) \( \frac{1}{23+7x} \cdot 0 = 0 \)
   k) \( (199 + 36) + 1 = (199 + 1) + 36 \)
   l) \( (1000 \cdot 8) \cdot 9 = 1000 (8 \cdot 9) \)
4. Regroup and simplify the calculations using properties.
   a) $12 + (45 + 88)$
   b) $9 (1000 \cdot 8)$
   c) $3 + (2997 + 56)$

5. Use the distributive property to write an equivalent expression without parentheses.
   a) $4y (y + 0.3)$
   b) $(2 - 3y^2) 5$
   c) $\frac{1}{3} \left(\frac{2}{3} - \frac{1}{2} x\right)$

**Topic C**

6. Compare these numbers using either $<$ or $>$.
   a) $6$ $8$
   b) $0$ $-6$
   c) $-4$ $-2$
   d) $-\frac{3}{7}$ $\frac{1}{7}$
   e) $-0.6$ $-0.8$
   f) $1\frac{1}{2}$ $\frac{3}{8}$

7. Arrange the following numbers from the smallest to the largest number (using $<$ to order them).
   a) $8$, $-9$, $-4$, $23$, $0$, $-17$
   b) $0.05$, $-8$, $\frac{2}{5}$, $\frac{3}{5}$, $-3.24$
   c) $-\frac{1}{3}$, $\frac{2}{5}$, $-\frac{1}{7}$, $\frac{3}{4}$

8. Preform the indicated operation.
   a) $|-67|$
   b) $|35 - 14|$
   c) $|-0.45 + 0|$
   d) $-|7^2|$
   e) $|\ -\frac{1}{8} |$

9. Preform the indicated operation.
   a) $4 [7 - 3 + (30 - 5)]$
   b) $\frac{|-9|}{3^2} + (27 - 3)$
10. Preform the indicated operation.
   a) 13 + 24
   b) (-7) + (-8)
   c) $\frac{-1}{5} + (-2\frac{2}{5})$
   d) 9 + (-4)
   e) (-25) + 12
   f) 8.4 + (-0.9)
   g) (-7) - (6)
   h) (-5) - (-7)
   i) $-\frac{3}{7} - \frac{2}{7}$
   j) $|\frac{1}{7} - 1\frac{3}{4}|$
   k) -45 ÷ (-9)
   l) $-\frac{3.6}{6}$
   m) $-\frac{9}{5} ÷ \left(-\frac{1}{15}\right)$
   n) -72 ÷ 9
   o) $\frac{0}{1789}$
   p) $\frac{3.78}{0}$

11. Write the additive inverse (opposite) of each number.
   a) -45
   b) $\frac{5}{8}$
   c) -1

12. If $x = -2$, $y = 5$, $z = 4$ and $w = 0$, evaluate each of the following.
   a) $zy + x^3$
   b) $x^2 - 2xy + y^2 + \frac{w}{3xyz}$
   c) $(x + y)(x - y) - 5z$
   d) $4\left(\frac{2xy}{3w}\right)$
Unit 6
Polynomials

Topic A: Introduction to polynomials

- Polynomials
- Degree of a polynomial
- Combine like terms
- Removing parentheses

Topic B: Multiplying and dividing polynomials

- Multiplying and dividing monomials
- Multiplying / dividing polynomials by monomials
- FOIL method to multiply binomials

Unit 6 Summary

Unit R Self-test
**Topic A: Introduction to Polynomials**

**Polynomials**

**Basic algebraic terms:**

<table>
<thead>
<tr>
<th>Algebraic term</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
</table>
| **Algebraic expression** | A mathematical phrase that contains numbers, variables (letters), and arithmetic operations (+, −, ×, ÷, etc.). | $3x - 4$  
$5a^2 - b + 3$  
$12y^3 + 7y^2 - 5y + \frac{2}{3}$ |
| **Constant**           | A number on its own.                                                         | $2y + 5$  
constant: 5 |
| **Coefficient**        | The number in front of a variable.                                           | $-9x^2$  
coefficient: -9  

$x$ coefficient: 1  

$(x = 1 \cdot x)$ |
| **Term**               | A term can be a constant, a variable, or the product of a number and variable. (Terms are separated by a plus or minus sign.) | $2x^3 + 7x^2 - 9y - 8$  
Terms:  

$2x^3$, $7x^2$, $-9y$, $-8$ |
| **Like terms**         | The terms that have the same variables and exponents (differ only in their coefficients). | $2x$ and $-7x$  

$-4y^2$ and $9y^2$  

$0.5pq^2$ and $\frac{2}{3}pq^2$ |

**Polynomial:** an algebraic expression that contains one or more terms.  
The prefix “poly-” means many.

**Example:** $7x$, $5ax - 9b$, $6x^2 - 5x + \frac{2}{3}$, $7a^2 + 8b + ab - 5$

**There are special names for polynomials that have one, two, or three terms:**

- **Monomial:** an algebraic expression that contains only one term.
  
**Example:** $9x$, $4xy^2$, $0.8mn^2$, $\frac{1}{3}a^2b$  
The prefix “mono” means one.

- **Binomial:** an algebraic expression that contains two terms.
  
The prefix “bino-” means two.

**Example:** $7x + 9$, $9t^2 - 2t$, $0.3y + \frac{1}{3}$

- **Trinomial:** an algebraic expression that contains three terms.
  
**Example:** $ax^2 + bx + c$, $-4qp^2 + 3q + 5$  
The prefix “tri-” means three.

**Polynomials in ascending or descending order:** a polynomial can be arranged in ascending or descending order.

- **Descending order:** the exponents of variables are arranged from largest to smallest number.
  
**Example:** $5a^3 - 3a^2 + a + 1$  
The exponents of $a$ decrease from left to right.  

$19y^4 + 31y^3 - y^2 + 2y - \frac{2}{3}$  
The exponents of $y$ decrease from left to right.

- **Ascending order:** the exponents of variables are arranged from smallest to largest number.
  
**Example:** $2 - 0.3x + 4.5x^2 - 7x^3$  
The exponents of $x$ increase from left to right.  

$7 + \frac{3}{7}w + 4w^2 - 8w^3 + w^4$  
The exponents of $w$ increase from left to right.
Degree of a Polynomial

Classification of polynomial: polynomials are classified according to their number of terms and degrees.

Degree of a term:

- The degree of a term with one variable: the exponent of its variable.
  
  Example: \(9x^3\)  
  The degree of the term: 3

- The degree of a term with more variables: the sum of the exponents of its variables.
  
  Example: \(-8a^2 b^3 c^6\)  
  The degree of the term: 11 \((2 + 3 + 6 = 11)\)

- More examples:

<table>
<thead>
<tr>
<th>Monomial</th>
<th>Degree</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4x)</td>
<td>1</td>
<td>(x = x^1) (x has an exponent of 1.)</td>
</tr>
<tr>
<td>(7xy^3)</td>
<td>4</td>
<td>(1 + 3 = 4)</td>
</tr>
<tr>
<td>(-\frac{3}{5}x^2y^3z)</td>
<td>7</td>
<td>(2 + 4 + 1 = 7) ((z = z^1))</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>(13 = 13 \cdot 1 = 13 \cdot x^0 = 13) ((x^0 = 1))</td>
</tr>
</tbody>
</table>

Degree of a polynomial: the highest degree of any individual term in it.

Examples:

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Degree</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>(7x^8 + 5x^5 + 8)</td>
<td>8</td>
<td>The highest exponent of the term is (7x^8).</td>
</tr>
<tr>
<td>(3a^2 + 4a^2b^2 + 7a^4b^5c^2)</td>
<td>11</td>
<td>The highest degree of the term is (7a^4b^5c^2).</td>
</tr>
</tbody>
</table>

Example: Arrange polynomials in descending order and identify the degrees and coefficients.

a) \(5 + 2a - 4a^2 + a^3\)

  Descending order: \(a^3 - 4a^2 + 2a + 5\)

  Coefficients: \(1, -4, 2, 3\)

  Degree of the polynomial: 3

b) \(-2xy + 9x^3 + 5x^5y + \frac{3}{4} + 7x^2 - \frac{1}{2}x^4\)

  Descending order: \(5x^5y - \frac{1}{2}x^4 + 9x^3 + 7x^2 - 2xy + \frac{3}{4}\)

  Coefficients: \(5, -\frac{1}{2}, 9, 7, -2\)

  Degree of the polynomial: 6

  \(y = y^1\)
Like terms: terms that have the same variables and exponents (the coefficients can be different).

Examples:

<table>
<thead>
<tr>
<th>Example</th>
<th>Like or unlike terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>7y and -9y</td>
<td>Like terms</td>
</tr>
<tr>
<td>6a², -32a², and -a²</td>
<td>Like terms</td>
</tr>
<tr>
<td>0.3x²y and -4.8x²y</td>
<td>Like terms</td>
</tr>
<tr>
<td>-(\frac{2}{7}u^2v^3) and (\frac{3}{5}u^2v^3)</td>
<td>Like terms</td>
</tr>
<tr>
<td>-8y and 78x</td>
<td>Unlike terms</td>
</tr>
<tr>
<td>6m³ and -9m²</td>
<td>Unlike terms</td>
</tr>
<tr>
<td>-9u³w² and -9w³u²</td>
<td>Unlike terms</td>
</tr>
</tbody>
</table>

Combine like terms: add or subtract their coefficients and keep the same variables and exponents.

Note: unlike terms cannot be combined.

Example: Combine like terms.

a) \(3a + 7b - 9a + 15b = (3a - 9a) + (7b + 15b)\)
   = \(-6a + 22b\)
   - Regroup like terms.
   - Combine like terms.

b) \(2y^2 - 4x + 3x - 5y^2 = (2y^2 - 5y^2) + (-4x + 3x)\)
   = \(-3y^2 - 1x\)
   - Regroup like terms.
   - Combine like terms.

Or underline like terms and without regrouping.

\[8xy^2 - x^2y + 4x^2y - 6xy^2\]
   = \[8xy^2 - x^2y + 4x^2y - 6xy^2\]
   = \[2xy^2 + 3x^2y\]
   - Combine like terms.

\[2(2m + 3n) + 3(m - 4n) = 4m + 6n + 3m - 12n\]
   = \[7m - 6n\]
   - Distributive property.
   - Combine like terms.

\[8v + 4(2v - u^2) + 3(u^2 + v) = 8v + 8v - 4u^2 + 3u^2 + 3v\]
   = \(-u^2 + 19v\)
   - Distributive property.
   - Combine like terms.
If the sign preceding the parentheses is positive (+), do not change the sign of terms inside the parentheses, just remove the parentheses.

Example: \((x - 5) = x - 5\)

If the sign preceding the parentheses is negative (-), remove the parentheses and the negative sign (in front of parentheses), and change the sign of each term inside the parentheses.

Example: \(- (x - 7) = -x + 7\)

Remove parentheses:

<table>
<thead>
<tr>
<th>Algebraic expression</th>
<th>Remove parentheses</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>((ax + b))</td>
<td>(ax + b)</td>
<td>((5x + 2) = 5x + 2)</td>
</tr>
<tr>
<td>((ax - b))</td>
<td>(ax - b)</td>
<td>((9y - 4) = 9y - 4)</td>
</tr>
<tr>
<td>(- (ax + b))</td>
<td>(-ax - b)</td>
<td>(- \left(\frac{3}{4} \times 7\right) = -\frac{3}{4} \times 7)</td>
</tr>
<tr>
<td>(- (ax - b))</td>
<td>(-ax + b)</td>
<td>(- (0.5b - 2.4) = -0.5b + 2.4)</td>
</tr>
</tbody>
</table>

Example: Simplify.

a) \(9x^2 + 7 - (2x^2 - 2) = 9x^2 + 7 - 2x^2 + 2\) \(= 7x^2 + 9\)  
   Remove parentheses. Combine like terms.

b) \((-8y + 5z) - 4(y - 7z) = -8y + 5z - 4y + 28z\) \(= -12y + 33z\)  
   Remove parentheses. Combine like terms.

c) \(- (3a^2 + 4a - 4) + 3(4a^2 - 6a + 7)\) \(= -3a^2 - 4a + 4 + 12a^2 - 18a + 21\) \(= 9a^2 - 22a + 25\)  
   Remove parentheses. Distributive property. Combine like terms.

d) \(-5(u^2 - 3u) + 3(2u - 4) - (5 - 3u + 4u^2)\) \(= -5u^2 + 15u + 6u - 12 - 5 + 3u - 4u^2\) \(= -9u^2 + 24u - 17\)  
   Distributive property. Remove parentheses. Combine like terms.

e) \((p - q - 4cd) - 3(-pq + 5cd) = 8pq - 32cd + 3pq - 15cd\) \(= 11pq - 47cd\)  
   Distributive property. Combine like terms.
Topic B: Multiplying and Dividing Polynomials

Multiplying and Dividing Monomials

Basic rules of exponents:

<table>
<thead>
<tr>
<th>Name</th>
<th>Rule</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product of like bases</td>
<td>( a^m a^n = a^{m+n} )</td>
<td>( 2^1 \cdot 2^2 = 2^{1+2} = 2^3 ) (Since ( 2^3 = (2 \cdot 2 \cdot 2) (2 \cdot 2) = 2^4 ))</td>
</tr>
<tr>
<td>(The same base)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quotient of like bases</td>
<td>( \frac{a^m}{a^n} = a^{m-n} )</td>
<td>( \frac{x^5}{x^3} = x^{5-3} = x^2 ) (Since ( \frac{x^5}{x^3} = \frac{x \times x \times x}{x \times x} = x^2 ))</td>
</tr>
<tr>
<td>(The same base)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative exponent</td>
<td>( a^{-n} = \frac{1}{a^n} )</td>
<td>( 3^{-2} = \frac{1}{3^2} = \frac{1}{9} = 0.11 ) (Since ( 3^{-1} = \frac{1}{3} = 1 \div 3, \ 3^{-2} = \frac{1}{3^2} = \frac{1}{9} = 0.11 ))</td>
</tr>
</tbody>
</table>

Example: Simplify the following.

a) \( x^4 \cdot 3 = x^{4+3} = x^7 \)
b) \( y^{-6} = y^{-6-3} = y^{-9} = \frac{1}{y^9} \)

Multiplying monomials (one term):

- Regroup coefficients and variables.
- Multiply coefficients (the numbers in front of the variable).
- Multiply variables (add exponents with the same base, apply \( a^m \cdot a^n = a^{m+n} \)).

Example: 1) \( (-4x^4 \cdot y^2) \cdot (7x^3 \cdot y^2) = (-4 \cdot 7) (x^4 \cdot x^3) (y^2 \cdot y^2) \)

\[ = -28 x^{4+3} y^{2+2} = -28 x^7 y^4 \]

2) \( \left( \frac{3}{4} a^2 b^3 c^2 \right) \left( \frac{4}{6} a b^2 c^2 \right) = \left( \frac{3}{4} \cdot \frac{4}{6} \right) (a^2 a) (b^3 b^2) (c^2 c^2) \)

\[ = \frac{1}{2} a^3 b^5 c^4 \]

Dividing monomials:

- Divide coefficients.
- Divide variables (subtract exponents with the same base, apply \( \frac{a^m}{a^n} = a^{m-n} \)).

Example: 1) \( \frac{4a^5}{16a^2} = \left( \frac{4}{16} \right) \left( \frac{a^5}{a^2} \right) \)

\[ = \frac{1}{4} a^{5-2} = \frac{1}{4} a^3 \]

2) \( \frac{t^2}{t^7} = t^{2-7} = t^{-5} = \frac{1}{t^5} \)

3) \( \frac{-12x^2 y^5}{4x^3 y^5} = \left( \frac{-12}{4} \right) \left( \frac{x^2}{x^3} \right) \left( \frac{y^5}{y^5} \right) \)

\[ = -3x^{2-3}y^{5-5} = -3x^{-1}y^0 = \frac{-3}{x} \]

\( x^{-1} = \frac{1}{x}, \ 1 \times^0 = 1 \)
### Multiplying / Dividing Polynomials by Monomials

**Multiplying a monomial and a polynomial:**

- Use the distributive property: \( a (b + c) = ab + ac \)
- Multiply coefficients and add exponents with the same base. Apply \( a^m \cdot a^n = a^{m+n} \)

**Examples:**

1) \( 3x^3 (5x^2 - 2x) = (3x^3) (5x^2) - (3x^3) (2x) \)

Use the distributive property: \( a (b + c) = ab + ac \)

\[= (3 \cdot 5) (x^3 x^2) - (3 \cdot 2) (x^3 x^1)\]

Regroup \( x = x^1 \)

\[= 15 (x^{3+2}) - 6 (x^{3+1})\]

Multiply the coefficients & add the exponents.

\[= 15x^5 - 6x^4\]

2) \( 5ab^2 (2a^2b + ab^2 - a) \)

Distribute.

\[= (5ab^2) (2a^2b) + (5ab^2) (ab^2) + (5ab^2) (-a)\]

Multiply the coefficients and add exponents.

\[= (5 \cdot 2) (a^{1+2} b^{2+1}) + (5a^{1+1} b^{2+2}) - (5a^{1+1} b^2)\]

\[= 10a^3b^3 + 5a^2b^4 - 5a^2b^2\]

**Dividing a polynomial by a monomial**

- Split the polynomial into several parts.
- Divide a monomial by a monomial. Apply \( \frac{a^m}{a^n} = a^{m-n} \)

**Example:**

\[
\frac{12x^2 + 4x - 2}{4x}
\]

**Steps** | **Solution**
--- | ---
- Split the polynomial into three parts: \( \frac{12x^2 + 4x - 2}{4x} = \frac{12x^2}{4x} + \frac{4x}{4x} - \frac{2}{4x} \)
- Divide a monomial by a monomial: \( \frac{3x + 1}{2x} = \frac{3x}{2x} + \frac{1}{2x} \) \( a^m \div a^n = a^{m-n} \)
FOIL Method to Multiply Binomials

The **FOIL method**: an easy way to find the product of two binomials (two terms).

\[(a + b) (c + d) = ac + ad + bc + bd\]

<table>
<thead>
<tr>
<th></th>
<th>FOIL Method</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>F - First terms</td>
<td>first term \times first term</td>
<td>((a + b) (c + d))</td>
</tr>
<tr>
<td>O - Outer terms</td>
<td>outside term \times outside term</td>
<td>((a + b) (c + d))</td>
</tr>
<tr>
<td>I - Inner terms</td>
<td>inside term \times inside term</td>
<td>((a + b) (c + d))</td>
</tr>
<tr>
<td>L - Last terms</td>
<td>last term \times last term</td>
<td>((a + b) (c + d))</td>
</tr>
</tbody>
</table>

**FOIL method**

### Example: Multiplying binomials (2 terms × 2 terms)

**Example**: Multiply.

1) \((2x + 3) (5x – 6) = 2x \cdot 5x + 2x (-6) + 3 \cdot 5x + 3 (-6)\)

\[= 10x^2 - 12x + 15x - 18\]

\[= 10x^2 + 2x - 18\]

The FOIL method.

2) \((3r – t) (5r + t^2) = 3r \cdot 5r + 3r \cdot t^2 - t \cdot 5r - t \cdot t^2\)

\[= 15r^2 + 3rt^2 - 5rt - t^3\]

\[a^n \cdot a^m = a^{n+m}\]

Combine like terms.

3) \((xy^2 + y) (2x^2y + x) = xy^2 \cdot 2x^2y + xy^2 \cdot x + y \cdot 2x^2y + y \cdot x\)

\[= 2x^3y^3 + x^2y^2 + 2x^2y^2 + xy\]

\[= 2x^3y^3 + 3x^2y^2 + xy\]

\[a^n \cdot a^m = a^{n+m}\]

Combine like terms.

4) \((a - \frac{1}{3}) (a - \frac{1}{3}) = a^2 - \frac{1}{3}a - \frac{1}{3}a + (- \frac{1}{3})(- \frac{1}{3})\)

\[= a^2 - (\frac{2}{3}a + \frac{1}{9})\]

\[a^n \cdot a^m = a^{n+m}\]

Combine like terms.
Unit 6:  Summary

Polynomials

Basic algebraic terms:

<table>
<thead>
<tr>
<th>Algebraic term</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic expression</td>
<td>A mathematical phrase that contains numbers, variables (letters), and arithmetic operations (+, −, ×, ÷, etc.).</td>
<td>3x − 4, 5a² − b + 3</td>
</tr>
<tr>
<td>Constant</td>
<td>A number on its own.</td>
<td>2y + 5 constant: 5</td>
</tr>
<tr>
<td>Coefficient</td>
<td>The number in front of a variable.</td>
<td>−9x² coefficient: -9</td>
</tr>
<tr>
<td>Term</td>
<td>A term can be a constant, a variable, or the product of a number and variable. Terms are separated by a plus or minus sign.</td>
<td>7a² − 6b + 8 Terms: 7a², -6b, 8</td>
</tr>
<tr>
<td>Like terms</td>
<td>The terms that have the same variables and exponents (differ only in their coefficients).</td>
<td>2x and -7x -4y² and 9y²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monomial (one term)</td>
<td>0.67x</td>
</tr>
<tr>
<td>Binomial (two terms)</td>
<td>4x − 2/3</td>
</tr>
<tr>
<td>Trinomial (three terms)</td>
<td>2a² − ab + 5</td>
</tr>
<tr>
<td>Polynomial (one or more terms)</td>
<td>2xy, 4x³ + 11, −2x³/3 + x − 5y + 4</td>
</tr>
</tbody>
</table>

Descending order: the exponents of variables are arranged from largest to smallest number.

Ascending order: the exponents of variables are arranged from smallest to largest number.

Degree of a term/polynomial:

- The degree of a term with one variable: the exponent of its variable.
- The degree of a term with more variables: the sum of the exponents of its variables.
- Degree of a polynomial: the highest degree of any individual term in it.

Like terms: terms that have the same variables and exponents (the coefficients can be different.)

Combine like terms: add or subtract their numerical coefficients and keep the same variables and exponents.

Remove parentheses:

- If the sign preceding the parentheses is positive (+), do not change the sign of terms inside the parentheses, just remove the parentheses.
• If the sign preceding the parentheses is negative (-), remove the parentheses and the negative sign (in front of parentheses), and change the sign of terms inside the parentheses.

**Basic rules of exponents:**

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<tr>
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<th>Rule</th>
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</tr>
</thead>
<tbody>
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<td>Product of like bases</td>
<td>$a^m \cdot a^n = a^{m+n}$ ((a \neq 0))</td>
<td>$2^3 \cdot 2^2 = 2^5 = 32$</td>
</tr>
<tr>
<td>Quotient of like bases</td>
<td>$\frac{a^m}{a^n} = a^{m-n}$ ((a \neq 0))</td>
<td>$\frac{y^3}{y^2} = y^{3-2} = y^1 = y$</td>
</tr>
<tr>
<td>Negative exponent $a^{-n}$</td>
<td>$a^{-n} = \frac{1}{a^n}$ ((a \neq 0))</td>
<td>$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$</td>
</tr>
</tbody>
</table>

**Multiply monomials (one term):**

- Multiply coefficients.
- Multiply variables (add exponents with the same base, apply $a^m \cdot a^n = a^{m+n}$).

**Dividing monomials:**

- Divide coefficients.
- Divide variables (subtract exponents with the same base, apply $\frac{a^m}{a^n} = a^{m-n}$).

**Multiplying a monomial and a polynomial:**

- Use the distributive property: $a \ (b + c) = ab + ac$
- Multiply coefficients and add exponents with the same base. Apply $a^m \cdot a^n = a^{m+n}$

**Dividing a polynomial by a monomial**

- Split the polynomial into several parts.
- Divide a monomial by a monomial. Apply $\frac{a^m}{a^n} = a^{m-n}$

**The FOIL method:**

<table>
<thead>
<tr>
<th>((a + b)\ (c + d) = ac + ad + bc + bd)</th>
<th>F - First terms</th>
<th>O - Outer terms</th>
<th>I - Inner terms</th>
<th>L - Last terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>first term $\times$ first term</td>
<td>(a + b) \ (c + d)</td>
<td>(a + b) \ (c + d)</td>
<td>(a + b) \ (c + d)</td>
<td>(a + b) \ (c + d)</td>
</tr>
</tbody>
</table>
Topic A

1. Identify the terms of each polynomial.
   a) \(5x^3 - 8x^2 + 2x\)
   b) \(-\frac{2}{3}y^4 + 9a^2 + a - 1\)

2. Identify the coefficients and the degree of the polynomials.
   a) \(2a^3 - 7a^2b^3 + 9b + 11\)
   b) \(-8xy^5 - \frac{2}{3}y^4 + 11x^2y^3 + 4y^2 - 23y + \frac{5}{6}\)

3. Identify each polynomial as a monomial, binomial, or trinomial.
   a) \(3x^2 - 7x\)
   b) \(-29xy^3\)
   c) \(8mn^2 + 7m - 45\)

4. Arrange polynomials in descending order.
   a) \(3 + 8x - 23x^2 + 15x^3\)
   b) \(-3y^3 - 45y^2 + 4y + \frac{2}{3}y^4\)

5. Combine like terms.
   a) \(7x + 10y - 8x + 9y\)
   b) \(12a^2 - 33b + 2b - 6a^2\)
   c) \(12uv^2 - 5u^2v + 15u^2v - 8uv^2\)
   d) \(5(4t - 6r) + 3(t + 7r)\)
   e) \(13n + 5(6n - m^2) + 7(2m^2 + 3n)\)

   a) \(15a^2 + 9 - (5a^2 - 4)\)
   b) \((-13x + 9y) - 6(x - 5y)\)
c) \(- (7z^2 + 6z - 15) + 2(7z^2 - 5z + 8)\)
d) \(-11(y^2 - 3y) + 4(2y - 5) - (13 - 6y + 9y^2)\)
e) \(5(ab - 2xy) - 6(-2ab + 3xy)\)

**Topic B**

7. Simplify the following.
   a) \(a^3 a^6\)
   b) \(\frac{x^{-4}}{x^7}\)
   c) \(\frac{t^3}{t^9}\)
   d) \((-6a^3 b^5)(7a^4 b^6)\)
   e) \(\left(\frac{5}{6}x^3 y^4 z^5\right) \left(\frac{2}{10}x y^3 z^4\right)\)
   f) \(\frac{6y^8}{36y^3}\)
   g) \(\frac{-81m^3 n^9}{9m^4 n^9}\)

8. Perform the indicated operation.
   a) \(-4x^3 (3x^4 - 7x)\)
   b) \(9a^3 b (3ab^2 + 2a^2 b^2 - a)\)
   c) \(\frac{35a^2 + 5a - 4}{5a}\)
   d) \((5y - 7) (8y + 9)\)
   e) \((7r - 2t) (3r + 4t^2)\)
   f) \((2ab^2 + 3b) (5a^2 b + 3a)\)
   g) \((x - \frac{1}{3}) (x - \frac{2}{3})\)
Unit 7
Equations

Topic A: Properties of equations

- Introduction to equations
- Solving one-step equations
- Properties of equality

Topic B: Solving equations

- Solving multi-step equations
- Equation solving strategy
- Equations involving decimals / fractions

Topic C: One solution, no solutions, infinite solutions

- Types of equations

Topic D: Writing and solving equations

- Number problems
- Consecutive integers:
- Mixed problems

Unit 7 Summary

Unit 7 Self-test
Topic A: Properties of Equations

**Introduction to Equations**

**Equation:** a mathematical sentence that contains two expressions and separated by an equal sign (both sides of the equation have the same value).

**Example:** 4 + 3 = 7, 9x - 4 = 5, 2y - \( \frac{1}{3} \) = y

**To solve an equation** is the process of finding a particular value for the variable in the equation that makes the equation true (left side = right side or LS = RS).

**Example:** For the equation x + 4 = 5

only x = 1 can make it true, since 1 + 4 = 5 (LS = RS)

**Solution of an equation:** the value of the variable in the equation that makes the equation true.

**Example:** For the equation x + 4 = 5, x = 1 is the solution.

**Examples:** Indicate whether each of the given number is a solution to the given equation.

1) 2: 4x - 3 = 5, 4 - 3 = 5, 5 = 5, Yes Replace x with 2.

2) 15: \( \frac{-3}{15} \) y = -3, \( \frac{-3}{15} \) (15) = -3, -3 = -3, Yes Replace y with 15.

3) \( \frac{1}{2} \): 8t = 3, 8 (\( \frac{1}{2} \)) = 3, 4 ≠ 3, No Replace t with \( \frac{1}{2} \).

**An equation behaves like a pair of balanced scales.** The scales remain balanced when the same weight is put on to or taken away from each side. Always do the same thing on both sides to keep an equation true.

Left side = Right side (LS = RS)  
Left side ≠ Right side (LS ≠ RS)
Solving One-Step Equations

To solve a one-step addition equation: \( x + a = b \)

Isolate the variable “\( x \)” by subtracting the same number \( a \) from each side of the equation (to get rid of the constant \( a \) on the left side of the equal sign so that the letter \( x \) is on its own).

**Example:** Solve \( x + 7 = 9 \)

\[
x + 7 - 7 = 9 - 7
\]

\( x = 2 \)

or \( x + 7 = 9 \)

Solution: \( x = 2 \)

Check: substitute the solution into the equation to verify that is true.

(Left side = Right side).

\[
x + 7 = 9
\]

? \( 2 + 7 = 9, \ 9 = 9 \) LS = RS (correct) Replace \( x \) with 2.

**Example:** Solve \( u + \frac{2}{5} = \frac{3}{5} \)

\[
u + \frac{2}{5} - \frac{2}{5} = \frac{3}{5} - \frac{2}{5}
\]

\( u = \frac{1}{5} \)

or \( u + \frac{2}{5} = \frac{3}{5} \)

Solution: \( u = \frac{1}{5} \)

Check: \( u + \frac{2}{5} = \frac{3}{5} \)

Replace \( u \) with \( \frac{1}{5} \).

\[
\frac{1}{5} + \frac{2}{5} = \frac{3}{5}, \quad \frac{3}{5} = \frac{3}{5}
\]

LS = RS (correct)
To solve a one-step subtraction equation: \( x - a = b \)

Isolate the variable by adding the same number \( a \) to each side of the equation.

**Example:** Solve \( x - 5 = 8 \)

\[
x - 5 + 5 = 8 + 5 \quad \text{Add 5 to both sides.}
\]

Solution: \( x = 13 \)

To solve a one-step multiplication equation: \( a \cdot x = b \)

Isolate the variable “\( x \)” by dividing the same number \( a \) from each side of the equation.

**Example:** Solve \( 6x = 42 \)

\[
\frac{6x}{6} = \frac{42}{6} \quad \text{Divide both sides by 6.}
\]

Solution: \( x = 7 \)

**Example:** Solve \( \frac{4y}{5} = \frac{4}{15} \)

\[
\frac{4y}{5} \div \frac{4}{5} = \frac{4}{15} \div \frac{4}{5} \quad \text{Divide both sides by} \ \frac{4}{5}.
\]

\[
\frac{4y}{5} \cdot \frac{5}{4} = \frac{4}{15} \cdot \frac{5}{4} \quad \text{Multiply both sides by} \ \frac{5}{4}.
\]

Solution: \( y = \frac{1}{3} \)

To solve a one-step division equation: \( \frac{x}{a} = b \)

Isolate the variable by multiplying the same number \( a \) to each side of the equation.

**Example:** Solve \( \frac{x}{7} = 6 \)

\[
\frac{x}{7} \cdot 7 = 6 \cdot 7 \quad \text{Multiply both sides by 7.}
\]

Solution: \( x = 42 \)

**Example:** Solve \( \frac{1}{5} y = 8 \)

\[
\frac{1}{5} (-5) y = 8 (-5) \quad \text{Multiply both sides by} \ -5.
\]

Solution: \( y = -40 \)
**Properties of Equality**

**Basic rules for solving one-step equations:**

- Add, subtract, multiply or divide the same quantity to both sides of an equation can result in a valid equation.
- Remember to always do the same thing to both sides of the equation (balance).

**Properties for solving equations:**

<table>
<thead>
<tr>
<th>Properties</th>
<th>Equality</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
<td>( A = B ) ( A + C = B + C )</td>
<td>Solve ( x - 6 = 3 ) ( x - 6 + 6 = 3 + 6 ) ( x = 9 )</td>
</tr>
<tr>
<td><strong>Subtraction</strong></td>
<td>( A = B ) ( A - C = B - C )</td>
<td>Solve ( y + 5 = -8 ) ( y + 5 - 5 = -8 - 5 ) ( y = -13 )</td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
<td>( A = B ) ( A \cdot C = B \cdot C )</td>
<td>Solve ( \frac{m}{9} = 2 ) ( 9 \cdot \frac{m}{9} = 2 \cdot 9 ) ( m = 18 )</td>
</tr>
<tr>
<td><strong>Division</strong></td>
<td>( A = B ) ( \frac{A}{C} = \frac{B}{C} ) ( C \neq 0 )</td>
<td>Solve ( \frac{3n}{3} = \frac{-15}{3} ) ( n = -5 )</td>
</tr>
</tbody>
</table>

**Example:** Solve the following equations.

1) \(-9 + x = 5\) \(\Rightarrow -9 + x + 9 = 5 + 9\) \(x = 14\)  
   Check: \(-9 + 14 = 5\)  
   Replace \(x\) with 14.

2) \(\frac{2}{5} t = -\frac{1}{5}\) \(\Rightarrow y + \frac{2}{5} = -\frac{1}{5} - \frac{2}{5}\) \(y = -\frac{3}{5}\)  
   Property of subtraction.

3) \(-\frac{1}{6} x = 7\) \(\Rightarrow -6 \cdot \frac{1}{6} x = 7(-\frac{6}{1})\) \(x = -\frac{42}{6}\)  
   Property of multiplication.

4) \(-5x = 30\) \(\Rightarrow -\frac{5x}{-5} = \frac{30}{-5}\) \(x = 6\)  
   Property of division.

5) \(0.7y = -0.63\) \(\Rightarrow \frac{0.7y}{0.7} = \frac{-0.63}{0.7}\) \(y = -0.9\)  
   Property of division.

6) \(y - 3\frac{2}{5} = 2\frac{3}{10}\) \(\Rightarrow y - 3\frac{2}{5} + 3\frac{2}{5} = 2\frac{3}{10} + 3\frac{2}{5}\) \(y = 2\frac{3}{10} + 3\frac{4}{10}\) \(\Rightarrow y = 5\frac{7}{10}\)  
   Property of addition.

The LCD = 10
**Topic B: Solving Equations**

**Solving Multi-Step Equations**

**Multi-step equation:** an equation that requires more than one step to solve it.

**Steps for solving multi-step equations:**

- Simplify the equation and remove parentheses if necessary.
- Combine like terms on each side of the equation.
- Collect the variable (letter) terms on one side of the equation and the numerical terms (numbers) on the other side.
- Isolate the variable and find the solution: make the coefficient of the variable (number in front of the variable) equal to one.
- Check: substitute the solution back into the equation to verify that it is true (LS = RS).

**Example:** Solve \(9x + 6 = 12\)

- **Simplify:** \(3x + 2 = 4\) Divide each term by 3.
- **Combine like terms:** \(3x + 2 - 2 = 4 - 2\) Subtract 2 from both sides.
- \(3x = 2\)

<table>
<thead>
<tr>
<th>Variable term</th>
<th>Constant term</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3x)</td>
<td>(2)</td>
</tr>
</tbody>
</table>

- **Isolate the variable** \(\frac{3x}{3} = \frac{2}{3}\) Divide both sides by 3.
- **Solution:** \(x = \frac{2}{3}\)

- **Check:** \(9x + 6 = 12\) Original equation.
  
  \[? \quad \checkmark\]
  
  \[9 \cdot \frac{2}{3} + 6 = 12\]
  
  \[12 = 12\]
  
  Replace \(x\) with \(\frac{2}{3}\).

**Example:** Solve \(13t - 10 = 3\)

\[
13t - 10 + 10 = 3 + 10 \\
13t = 13 \\
\frac{13t}{13} = \frac{13}{13} \\
t = 1
\]

Solution.

**Example:** Solve \(2(x - 4) + 5x + 3 = 3(2 - 3x)\).

\[
2x - 8 + 5x + 3 = 6 - 9x \\
7x - 5 = 6 - 9x \\
7x - 5 + 5 = 6 - 9x + 5 \\
7x = 11 - 9x \\
7x + 9x = 11 - 9x + 9x \\
16x = 11 \\
\]

\[x = \frac{11}{16}\]

Divide both sides by 16.
Equations Solving Strategy

Procedure for solving equations

<table>
<thead>
<tr>
<th>Equation solving strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Clear the fractions or decimals if necessary.</td>
</tr>
<tr>
<td>• Simplify and remove parentheses if necessary.</td>
</tr>
<tr>
<td>• Combine like terms on each side of the equation.</td>
</tr>
<tr>
<td>• Collect the variable terms on one side of the equation and the constants on the other side.</td>
</tr>
<tr>
<td>• Isolate the variable (to get the variable alone on one side of the equation).</td>
</tr>
<tr>
<td>• Check the solution with the original equation.</td>
</tr>
</tbody>
</table>

Steps for solving equations:

<table>
<thead>
<tr>
<th>Steps</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Eliminate the denominators if the equation has fractions.</td>
<td>Solve $\frac{1}{5}(y + 10) = 3y - \frac{9}{5}y$</td>
</tr>
<tr>
<td>• Remove parentheses.</td>
<td>$5 \cdot \frac{1}{5}(y + 10) = 5(3y) - 5 \left(\frac{9}{5}y\right)$</td>
</tr>
<tr>
<td>• Combine like terms.</td>
<td>Multiply each term by 5.</td>
</tr>
<tr>
<td>• Collect variable terms on one side and the constants on the other side.</td>
<td>$y + 10 = 15y - 9y$</td>
</tr>
<tr>
<td>• Isolate the variable.</td>
<td>$y + 10 = 6y$</td>
</tr>
<tr>
<td></td>
<td>$y + 10 - 10 = 6y - 10$</td>
</tr>
<tr>
<td></td>
<td>$y = 6y - 10$ Subtract 10 from both sides.</td>
</tr>
<tr>
<td></td>
<td>$y - 6y = 6y - 10 - 6y$ Subtract 6y from both sides.</td>
</tr>
<tr>
<td></td>
<td>$-5y = -10$ Divide both sides by -5.</td>
</tr>
<tr>
<td></td>
<td>$y = \frac{-10}{-5}$</td>
</tr>
<tr>
<td></td>
<td>$y = \frac{-10}{-5} \Rightarrow y = 2$</td>
</tr>
<tr>
<td>• Check with the original equation.</td>
<td>$\frac{1}{5}(2 + 10) = 3 \cdot 2 - \frac{9}{5} \cdot 2$ Replace y with 2.</td>
</tr>
<tr>
<td></td>
<td>$5 \cdot \frac{1}{5}(2 + 10) = 5 \cdot 3 \cdot 2 - 5 \cdot \frac{9}{5} \cdot 2$ Multiply each term by 5.</td>
</tr>
<tr>
<td></td>
<td>$(2 + 10) = 30 - 18$</td>
</tr>
<tr>
<td></td>
<td>$12 = 12$ $\sqrt{LS = RS}$ (correct)</td>
</tr>
</tbody>
</table>
Equations Involving Decimals / Fractions

Equations involving decimals

Tip: Multiply every term of both sides of the equation by a multiple of 10 (10, 100, 1000, etc.) to clear the decimals (based on the number with the largest number of decimal places in the equation).

Steps

- Multiply each term by 100 to clear the decimal.
- Collect the variable terms on one side of the equation and the constants on the other side.
- Isolate the variable.

Example: Solve $0.4y + 0.08 = 0.016$

\[
1000(0.4y) + 1000(0.08) = 1000(0.016)
\]

\[
400y + 80 = 16
\]

\[
400y = -64
\]

\[
y = -0.16
\]

Equations involving fractions

Steps

- Multiply each term by the LCD.
- Collect the variable terms on one side of the equation and the constants on the other side.
- Isolate the variable.

Example: Solve \( \frac{t}{3} + \frac{3}{4} = -\frac{t}{2} - \frac{1}{3} \).

\[
12 \cdot \frac{t}{3} + 12 \cdot \frac{3}{4} = 12\left(-\frac{t}{2}\right) - 12 \cdot \frac{1}{3}
\]

\[
\frac{3}{1} \cdot \frac{3}{2} + \frac{3}{2} = \frac{3}{1} \cdot \frac{3}{1}
\]

\[
\text{LCD} = 2 \times 3 \times 2 = 12
\]

\[
4t + 9 = -6t - 4
\]

\[
t = -\frac{13}{10} = -1 \frac{3}{10}
\]

Divide both sides by 10.
Types of Equations

**Types of equations:** a mathematical equation can be a contradiction, an identity, or a conditional equation.

**Contradiction equation:** an equation which is never true, regardless of the value of the variable, and thus has no solution.

**Example:**

\[
3(x + 1) - 3x = -7 \\
\text{Distribute property.} \\
3x + 3 - 3x = -7 \\
\text{Combine like terms.} \\
3 = -7 \\
\text{False, } 3 \neq -7 \\
\text{No solution} \\
\]

There are no real numbers that can make this equation true.

**Note:** If the resulting equation is a false statement with no variables, it is a contradiction equation.

**Identity equation:** an equation which is always true for every value of the variable and thus has an infinite number of solutions (the solution is all real numbers).

**Example:**

\[
12x - 3(2 + 4x) = -6 \\
\text{Distribute property.} \\
12x - 6 - 12x = -6 \\
\text{Combine like terms.} \\
-6 = -6 \\
\text{The solution is all real numbers.} \\
\]

The equation is always true no matter what value is substituted for the variable.

**Note:** If the resulting equation is a true statement and with no variables, it is an identity equation.

**Conditional equation:** an equation is true only for the certain value of the variable (one solution).

**Example:**

\[
2x - 3 = -7x \\
\text{Add } 7x \text{ to both sides.} \\
9x = 3 \\
\text{Add 3 to both sides.} \\
x = \frac{1}{3} \\
\text{Divide both sides by 9.} \\
\]

If \( x = \frac{1}{3} \), the equation is true, otherwise, the equation is false.
Summary: types of equations

<table>
<thead>
<tr>
<th>Types of equations</th>
<th>Characteristic</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contradiction equation</td>
<td>Always false</td>
<td>No solution</td>
</tr>
<tr>
<td>Identity equation</td>
<td>Always true</td>
<td>All real numbers</td>
</tr>
<tr>
<td>Conditional equation</td>
<td>It is true only for the certain value.</td>
<td>One solution</td>
</tr>
</tbody>
</table>

**Example:** Determine each equation as a contradiction, an identity, or a conditional equation.

1) \[4x - (3 - x) = 5(x - 1)\]
   Remove parentheses.
   
   \[4x - 3 + x = 5x - 5\]
   Combine like terms.
   
   \[5x - 3 = 5x - 5\]
   
   \[-5x - 3 - 5x = 5x - 5 - 5x\]
   Subtract 5x from both sides.
   
   \[-3 = -5\]
   No solution – contradiction equation
   The resulting equation is a false statement with no variables.

2) \[\frac{y}{2} + 2(y - 3) = 2 - 3y\]
   Multiply each term by 2.
   
   \[\cdot \frac{y}{2} + 2 \cdot 2(y - 3) = 2 \cdot 2 - 2(3y)\]
   Remove parentheses.
   
   \[y + 4y - 12 = 4 - 6y\]
   Combine like terms.
   
   \[5y - 12 + 12 = 4 - 6y + 12\]
   Add 12 to both sides.
   
   \[5y = 16 - 6y\]
   
   \[5y + 6y = 16 - 6y + 6y\]
   Add 6y to both sides.
   
   \[11y = 16\]
   Divide both sides by 11.
   
   \[y \approx 1.455\]
   One solution – conditional equation

3) \[4t - 3(t + 4) = t - 12\]
   Distribute property.
   
   \[4t - 3t - 12 = t - 12\]
   Combine like terms.
   
   \[t - 12 = t - 12\]
   Add 12 to both sides.
   
   \[t + 12 = t + 12\]
   Subtract t from both sides.
   
   \[-t = -t\]
   \[0 = 0\]
   All real numbers – identity equation
   The resulting equation is a true statement with no variables.
**Topic D: Writing and Solving Equations**

### Number Problems

**Number problems - examples**

<table>
<thead>
<tr>
<th>English phrase</th>
<th>Algebraic expression / equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seven more than the difference of a number and four.</td>
<td>$(x - 4) + 7$</td>
</tr>
<tr>
<td>The quotient of five and the product of six and a number.</td>
<td>$\frac{5}{6x}$</td>
</tr>
<tr>
<td>The product of nine and a number, decreased by five.</td>
<td>$9x - 5$</td>
</tr>
<tr>
<td>Ten less than three times two numbers is seven more than their sum.</td>
<td>$3xy - 10 = x + y + 7$</td>
</tr>
<tr>
<td>The sum of the squares of two numbers is nine less than their product.</td>
<td>$x^2 + y^2 = xy - 9$</td>
</tr>
<tr>
<td>Two more than the quotient of $11x$ by 5 is seven times that number.</td>
<td>$2 + \frac{11x}{5} = 7x$</td>
</tr>
</tbody>
</table>

Let $x$ = a number, $y$ = a number

### Steps for solving word problems:

**Procedure for solving word problems**

- Organize the facts given from the problem.
- Identify and label the unknown quantity (let $x$ = unknown).
- Draw a diagram if it will make the problem clearer.
- Convert words into a mathematical equation.
- Solve the equation and find the solution(s).
- Check the solution with the original equation (check it back into the problem – is it logical? if necessary).

### Example: The product of nine and a number is twenty-seven. Determine the value of this number.

- Organize the facts and assign the unknown quantity:
  
  **Facts**
  
  The product of 9 and $x$ is 27

  **Unknown**
  
  Let $x$ = number

- Write an equation: $9 \cdot x = 27$ or $9x = 27$

- Solve the equation: $\frac{9x}{9} = \frac{27}{9}$  
  Divide both sides by 9.

  $x = 3$

- Check: $9 \cdot 3 = 27$  
  Replace $x$ with 3.

  $27 = 27$  
  LS = RS (correct)

**Answer:** The value of the number is 3.
Example: Eight less than two times a number is five less than the number divided by two.

Find the number.

- Organize the facts: \(-8\) \(2x\) \(-5\) \(\frac{x}{2}\)
- Equation: \(2x - 8 = \frac{x}{2} - 5\)
  \[2(2x) - 2 \cdot 8 = 2 \left(\frac{x}{2}\right) - 2 \cdot 5\]
  \[4x - 16 = x - 10\]
  \[3x = 6\]
  \(\frac{x}{2} = 2\)
- Solution: \(x = x = 2\)
- Check: \(2(2) - 8 = \frac{2}{2} - 5\)
  \[2 - 8 = 1 - 5\]
  \[-6 = -4\]
  \(\sqrt{\text{LS} = \text{RS}}\ \text{(correct)}\)
- Answer: The number is 2.

Example: There are three numbers, the first is four less than three times the second, and the third is two more than the first. The sum of these three numbers is fifteen.

Find each number.

- Organize the facts:

<table>
<thead>
<tr>
<th>Number</th>
<th>Words</th>
<th>Algebraic expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd number</td>
<td>Let 2nd number = (x)</td>
<td>(x)</td>
</tr>
<tr>
<td>1st number</td>
<td>4 less than 3 times the 2nd number</td>
<td>(3x - 4)</td>
</tr>
<tr>
<td>3rd number</td>
<td>2 more than the 1st number</td>
<td>((3x - 4) + 2)</td>
</tr>
<tr>
<td>Sum</td>
<td>The sum of three numbers is 15</td>
<td>(1^{\text{st}} # + 2^{\text{nd}} # + 3^{\text{rd}} # = 15)</td>
</tr>
</tbody>
</table>

- Equation: \((3x - 4) + x + [(3x - 4) + 2] = 15\)
  \[3x - 4 + x + 3x - 2 = 15\]
  \[7x - 6 = 15\]
  \[7x = 21\]
  \(x = 3\)
- Solution: \(x = 3\)
- Check: \(5 + 3 + 7 = 15\) Yes!
Consecutive Integers

Consecutive integers:

<table>
<thead>
<tr>
<th>English phrase</th>
<th>Algebraic expression</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two consecutive integers</td>
<td>$x, \ x + 1$</td>
<td>If $x = 1, \ x + 1 = 2$</td>
</tr>
<tr>
<td>Three consecutive integers</td>
<td>$x, \ x + 1, \ x + 2$</td>
<td>If $x = 1, \ x + 1 = 2, \ x + 2 = 3$</td>
</tr>
<tr>
<td>Two consecutive odd integers</td>
<td>$x, \ x + 2$</td>
<td>If $x = 1, \ x + 2 = 3$</td>
</tr>
<tr>
<td>Three consecutive odd integers</td>
<td>$x, \ x + 2, \ x + 4$</td>
<td>If $x = 1, \ x + 2 = 3, \ x + 4 = 5$</td>
</tr>
<tr>
<td>Two consecutive even integers</td>
<td>$x, \ x + 2$</td>
<td>If $x = 2, \ x + 2 = 4$</td>
</tr>
<tr>
<td>Three consecutive even integers</td>
<td>$x, \ x + 2, \ x + 4$</td>
<td>If $x = 2, \ x + 2 = 4, \ x + 4 = 6$</td>
</tr>
</tbody>
</table>

Examples:

<table>
<thead>
<tr>
<th>English phrase</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The difference of two consecutive integers is one.</td>
<td>$(x + 1) - x = 1$</td>
</tr>
<tr>
<td>The sum of three consecutive odd integers is nine.</td>
<td>$x + (x + 2) + (x + 4) = 9$</td>
</tr>
<tr>
<td>The product of two consecutive even integers is eight.</td>
<td>$x(x + 2) = 8$</td>
</tr>
<tr>
<td>Three consecutive even integers whose sum is twelve.</td>
<td>$x + (x + 2) + (x + 4) = 12$</td>
</tr>
</tbody>
</table>

Example: The sum of three consecutive odd integers is twenty-one, find each number.

- Organize the facts:

<table>
<thead>
<tr>
<th>1st consecutive odd number</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd consecutive odd number</td>
<td>$x + 2$</td>
</tr>
<tr>
<td>3rd consecutive odd number</td>
<td>$x + 4$</td>
</tr>
</tbody>
</table>

- Write an equation: $x + (x + 2) + (x + 4) = 21$ Combine like terms.
- Solve the unknown: $3x + 6 = 21$ Subtract 6 from both sides.
  \[
  3x = 15 \quad \text{Divide both sides by 3.}
  \]
  \[
  x = 5
  \]

<table>
<thead>
<tr>
<th>1st consecutive even number</th>
<th>$x = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd consecutive even number</td>
<td>$x + 2 = 5 + 2 = 7$</td>
</tr>
<tr>
<td>3rd consecutive even number</td>
<td>$x + 4 = 5 + 4 = 9$</td>
</tr>
</tbody>
</table>

- Check: $5, \ 7, \ 9$ = consecutive odd integers Yes!
  \[
  5 + (5 + 2) + (5 + 4) = 21 \quad \text{Replace } x \text{ with 5.}
  \]
  \[
  5 + 7 + 9 = 21 \quad \sqrt{21 = 12} \quad \text{LS = RS (correct)}
  \]
- State the answer: $x = 5, \ x + 2 = 7, \ x + 4 = 9$
Example: The second angle of a triangle is twelve times as large as the first. The third angle is five degrees more than the second angle. Find the measure of each angle.

1st angle \( x \)
2nd angle \( 12x \)
3rd angle \( 12x + 5^0 \)

- Equation \( x + 12x + (12x + 5^0) = 180^0 \)
  \[ 25x + 5^0 = 180^0 \]
  \[ 25x = 175^0 \]
- Solve: \( x = \frac{175}{25} = 7^0 \)
- The answer:

<table>
<thead>
<tr>
<th>1st angle</th>
<th>2nd angle</th>
<th>3rd angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 7^0 )</td>
<td>( 12x = 12(7) = 84^0 )</td>
<td>( 12x + 5^0 = 12(7^0) + 5^0 = 89^0 )</td>
</tr>
</tbody>
</table>

- Check: \( 7^0 + 84^0 + 89^0 = 180^0 \)
  \[ \sqrt{180^0} = 180^0 \]
  Yes!

Example: The perimeter of a rectangle is 164 meters. The width is 13 meters shorter than the length. Find the dimensions (width and length).

- List the facts and sign the unknown quantity:

<table>
<thead>
<tr>
<th>Facts</th>
<th>Perimeter</th>
<th>( P = 164 ) m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unknown</td>
<td>Let ( l = \text{length}, ) width = ( l - 13 )</td>
<td></td>
</tr>
</tbody>
</table>

- Equation: \( 2l + 2(l - 13) = 164 \)
  \[ 4l - 26 = 164 \]
  \[ 4l = 190 \]
  Divide both sides by 4.
  Length: \( l = 47.5 \) m
- Find the width: \( w = l - 13 \)
  \[ w = 47.5 - 13 \]
  \[ w = 34.5 \]
  Width: \( w = 34.5 \) m
### Formulas

<table>
<thead>
<tr>
<th></th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original price</td>
<td>Original price = Sale price + Discount</td>
</tr>
<tr>
<td>Discount</td>
<td>Discount = Discount rate × Original price</td>
</tr>
<tr>
<td>Sale price</td>
<td>Sale price = Original price – Discount</td>
</tr>
</tbody>
</table>

#### Example
After a 35% reduction, a women’s jacket is on sale for $30.55. What is the discount? What was the original price?

- **Organize the facts:**

<table>
<thead>
<tr>
<th>Sale price</th>
<th>$30.55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>35%</td>
</tr>
<tr>
<td>Unknown</td>
<td>Let $x$ = original price</td>
</tr>
</tbody>
</table>

- **Discount:**
  \[
  \text{Discount} = \text{Discount rate} \times \text{Original price} \\
  = (35\%) \times \text{x}
  \]

- **Equation:**
  \[
  \text{Original price} = \text{Sale price} + \text{Discount} \\
  x = 30.55 + 35\% \times x
  \]

- **Solve:**
  \[
  x - 0.35x = 30.55 \\
  0.65x = 30.55 \\
  \frac{0.65x}{0.65} = \frac{30.55}{0.65} \\
  x = \frac{30.55}{0.65} = 47
  \]

- **State the answer:**
  The original price was $47.

#### Example
A $159.99 instant pot is labeled "30% off". What is the sale price?

<table>
<thead>
<tr>
<th>Original price</th>
<th>$159.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>30%</td>
</tr>
<tr>
<td>Unknown</td>
<td>Let $x$ = sale price</td>
</tr>
</tbody>
</table>

- **Equation:**
  \[
  \text{Sale price} = \text{Original price} – \text{Discount} \\
  x = 159.99 – (30\%) \times (159.99) \\
  x = 159.99 – (0.3 \times 159.99) \\
  x \approx 111.99
  \]

- **State the answer:**
  The sale price is $111.99.
**Unit 7: Summary**

**Equations**

**Equation:** a mathematical sentence that contains two expressions and separated by an equal sign.

**To solve an equation** is the process of finding a particular value for the variable in the equation that makes the equation true.

**Solution of an equation:** the value of the variable in the equation that makes the equation true.

**An equation behaves like a pair of balanced scales.** The scales remain balanced when the same weight is put on to or taken away from each side. Always do the same thing on both sides to keep an equation true (LS = RS).

**Basic rules for solving one-step equations:**
- Add, subtract, multiply or divide the same quantity to both sides of an equation can result in a valid equation.
- Remember to always do the same thing to both sides of the equation (balance).

**Properties for solving equations:**

<table>
<thead>
<tr>
<th>Properties</th>
<th>Equality</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong> property of equality</td>
<td>$A = B$ \quad $A + C = B + C$</td>
<td>Solve $x - 6 = 3$ \quad $x - 6 + 6 = 3 + 6$ \quad $x = 9$</td>
</tr>
<tr>
<td><strong>Subtraction</strong> property of equality</td>
<td>$A = B$ \quad $A - C = B - C$</td>
<td>Solve $y + 5 = -8$ \quad $y + 5 - 5 = -8 - 5$ \quad $y = -13$</td>
</tr>
<tr>
<td><strong>Multiplication</strong> property of equality</td>
<td>$A = B$ \quad $A \cdot C = B \cdot C$</td>
<td>Solve $\frac{m}{9} = 2$ \quad $9 \cdot \frac{m}{9} = 2 \cdot 9$ \quad $m = 18$</td>
</tr>
<tr>
<td><strong>Division</strong> property of equality</td>
<td>$A = B$ \quad $A \div C = B \div C$ \quad ($C \neq 0$)</td>
<td>Solve $3n = -15$ \quad $\frac{3n}{3} = -15 \div 3$ \quad $n = -5$</td>
</tr>
</tbody>
</table>

**Steps for solving equations:**

<table>
<thead>
<tr>
<th><strong>Equation solving strategy</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>- Clear the fractions or decimals if necessary.</td>
</tr>
<tr>
<td>- Simplify and remove parentheses if necessary.</td>
</tr>
<tr>
<td>- Combine like terms on each side of the equation.</td>
</tr>
<tr>
<td>- Collect the variable terms on one side of the equation and the constants on the other side.</td>
</tr>
<tr>
<td>- Isolate the variable.</td>
</tr>
<tr>
<td>- Check the solution with the original equation.</td>
</tr>
</tbody>
</table>
Types of equations:

<table>
<thead>
<tr>
<th>Types of equations</th>
<th>Characteristic</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contradiction equation</td>
<td>Always false</td>
<td>No solution</td>
</tr>
<tr>
<td>Identity equation</td>
<td>Always true</td>
<td>All real numbers</td>
</tr>
<tr>
<td>Conditional equation</td>
<td>It is true only for the certain value.</td>
<td>One solution</td>
</tr>
</tbody>
</table>

- If the resulting equation is a false statement with no variables, it is a contradiction equation.
- If the resulting equation is a true statement and with no variables, it is an identity equation.

Steps for solving word problems:

**Procedure for solving word problems**

- **Organize** the facts given from the problem.
- Identify and **label** the unknown quantity (let $x = \text{unknown}$).
- Draw a diagram if it will make the problem clearer.
- Convert words into a mathematical **equation**.
- **Solve** the equation and find the solution(s).
- **Check** the solution with the original equation (check it back into the problem – is it logical? if necessary).

Consecutive integers:

<table>
<thead>
<tr>
<th>English phrase</th>
<th>Algebraic expression</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three consecutive integers</td>
<td>$x, \ x + 1, \ x + 2$</td>
<td>If $x = 1, \ x + 1 = 2, \ x + 2 = 3$</td>
</tr>
<tr>
<td>Three consecutive odd integers</td>
<td>$x, \ x + 2, \ x + 4$</td>
<td>If $x = 1, \ x + 2 = 3, \ x + 4 = 5$</td>
</tr>
<tr>
<td>Three consecutive even integers</td>
<td>$x, \ x + 2, \ x + 4$</td>
<td>If $x = 2, \ x + 2 = 4, \ x + 4 = 6$</td>
</tr>
</tbody>
</table>

**Formulas**

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original price</td>
<td>Original price = Sale price + Discount</td>
</tr>
<tr>
<td>Discount</td>
<td>Discount = Discount rate $\times$ Original price</td>
</tr>
<tr>
<td>Sale price</td>
<td>Sale price = Original price − Discount</td>
</tr>
</tbody>
</table>
Unit 7: Self-Test

Topic A

1. Indicate whether each of the given number is a solution to the given equation.
   a) \(2: \ 9x - 7 = 11\)
   b) \(17: \ -\frac{5}{17} \ y = -9\)
   c) \(\frac{2}{3}: \ 9m = 6\)

2. Solve the following equations.
   a) \(x - 7 = 12\)
   b) \(y + \frac{3}{8} = \frac{5}{8}\)
   c) \(m - 6 = 17\)
   d) \(9t = 72\)
   e) \(\frac{3x}{2} = \frac{9}{16}\)
   f) \(\frac{y}{13} = -4\)
   g) \(-21 + x = 7\)
   h) \(y + \frac{4}{9} = -\frac{3}{9}\)
   i) \(-\frac{4}{14} \ x = -2\)
   j) \(-19 \ t = 38\)
   k) \(0.8y = -0.64\)
   l) \(x - 4\frac{2}{3} = 3\frac{2}{9}\)

Topic B

3. Solve the following equations.
   a) \(14t + 5 = 8\)
b) \(7m - 23 = 40\)
c) \(7(x - 3) + 3x - 5 = 2(5 - 4x)\)
d) \(\frac{1}{7}(y + 12) = 4y - \frac{3}{7}y\)
e) \(0.63x - 0.29 = -3.56x\)
f) \(0.5t + 0.05 = 0.025\)
g) \(\frac{x}{4} + \frac{2}{5} = -\frac{x}{2} - \frac{1}{5}\)

**Topic C**

4. Determine each equation as a contradiction, an identity, or a conditional equation.
   a) \(5(y + 2) - 5y = -8\)
   b) \(8x - 4(3 + 2x) = -12\)
   c) \(7t - 9 = -3t\)
   d) \(5y - (4 - y) = 6(y - 2)\)
   e) \(\frac{x}{3} + 3(x - 4) = 5 - 8x\)
   f) \(7m - 5(m + 3) = 2m - 15\)

**Topic D**

5. Write each of the following as an algebraic expression.
   a) Nine more than the difference of a number and seven.
   b) The quotient seven and the product of nine and a number.
   c) The product of eleven and a number, decreased by eight.

6. Write each of the following as an algebraic expression or equation.
   a) Thirteen less than four times two numbers is six more than their sum.
   b) The sum of the squares of two numbers is twenty-six less than their product.
   c) Five more than the quotient of 5x by 23 is eleven times that number.
   d) The difference of two consecutive integers is one.
   e) The sum of three consecutive odd integers is fifteen.
   f) The product of two consecutive even integers is forty-eight.
7. Solve each problem by writing and solving an equation.

a) The product of seven and a number is forty-two. Determine the value of this number.

b) Three less than four times a number is nine less than the number divided by four. Find the number.

c) There are three numbers, the first is three less than five times the second, and the third is four more than the first. The sum of these three numbers is twenty. Find each number.

d) The sum of three consecutive odd integers is twenty-seven, find each number.

e) The second angle of a triangle is seven times as large as the first. The third angle is thirty degrees more than the second angle. Find the measure of each angle.

f) The perimeter of a rectangle is 128 meters. The width is 8 meters shorter than the length. Find the dimensions (width and length).

g) After a 20% reduction, a TV is on sale for $199.99. What is the discount? What was the original price?

h) A $379.99 laptop is labeled "10% off". What is the sale price?
Unit 8
Formulas

Topic A: Substitution into formulas
- Geometry formulas
- Substituting into formulas

Topic B: Solving formulas
- Solving for a specific variable
- More examples for solving formulas

Topic C: Pythagorean theorem
- Pythagorean theorem
- Applications of the Pythagorean theorem

Unit 8 Summary

Unit 8 Self-test
### Geometry Formulas

**Formula:** an equation that contains more than one variable and is used to solve practical problems in everyday life.

**Geometry formulas review:**

- \( s \) – side, \( P \) – perimeter, \( C \) – circumference, \( A \) – area, \( V \) – volume

<table>
<thead>
<tr>
<th>Name of the figure</th>
<th>Formula</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangle</strong></td>
<td>( P = 2w + 2l ) ( A = wl ) ((w = \text{width}, \ l = \text{length}))</td>
<td><img src="image" alt="Rectangle" /></td>
</tr>
<tr>
<td><strong>Parallelogram</strong></td>
<td>( P = 2a + 2b ) ( A = bh ) ((a \text{ and } b = \text{sides}, \ h = \text{height}))</td>
<td><img src="image" alt="Parallelogram" /></td>
</tr>
<tr>
<td><strong>Circle</strong></td>
<td>( C = \pi d = 2\pi r ) ( A = \pi r^2 ) ((r = \text{radius}, \ d = \text{diameter}))</td>
<td><img src="image" alt="Circle" /></td>
</tr>
<tr>
<td><strong>Triangle</strong></td>
<td>( &lt; A + &lt; B + &lt; C = 180^\circ ) ( A = \frac{1}{2}bh ) ((b = \text{base}, \ h = \text{height}))</td>
<td><img src="image" alt="Triangle" /></td>
</tr>
<tr>
<td><strong>Trapezoid</strong></td>
<td>( A = \frac{1}{2}h(b + B) ) ((b = \text{top base}, \ B = \text{bottom base}, \ h = \text{height}))</td>
<td><img src="image" alt="Trapezoid" /></td>
</tr>
<tr>
<td><strong>Cube</strong></td>
<td>( V = s^3 ) ((s = \text{side}))</td>
<td><img src="image" alt="Cube" /></td>
</tr>
<tr>
<td><strong>Rectangular solid</strong></td>
<td>( V = whl ) ((w = \text{width}, \ l = \text{length}, \ h = \text{height}))</td>
<td><img src="image" alt="Rectangular solid" /></td>
</tr>
<tr>
<td><strong>Cylinder</strong></td>
<td>( V = \pi r^2h ) ((r = \text{radius}, \ h = \text{height}))</td>
<td><img src="image" alt="Cylinder" /></td>
</tr>
<tr>
<td><strong>Sphere</strong></td>
<td>( V = \frac{4}{3}\pi r^3 ) ((r = \text{radius}))</td>
<td><img src="image" alt="Sphere" /></td>
</tr>
<tr>
<td><strong>Cone</strong></td>
<td>( V = \frac{1}{3}\pi r^2h ) ((r = \text{radius}, \ h = \text{height}))</td>
<td><img src="image" alt="Cone" /></td>
</tr>
</tbody>
</table>
Substituting into Formulas

- **Examples of formula:**

<table>
<thead>
<tr>
<th>Application</th>
<th>Formula</th>
<th>Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>( d = v \ t )</td>
<td>( d ) – distance ( v ) – velocity ( t ) – time</td>
</tr>
<tr>
<td>Simple interest</td>
<td>( I = P \ r \ t )</td>
<td>( I ) – interest ( P ) – principle ( r ) – interest rate (%) ( t ) – time (years)</td>
</tr>
<tr>
<td>Compound interest</td>
<td>( B = P \ (100% + r )^t )</td>
<td>( B ) – balance ( P ) – principle ( r ) – interest rate (%) ( t ) – time (years)</td>
</tr>
<tr>
<td>Percent increase</td>
<td>( \frac{N - O}{O} )</td>
<td>( N ) – new value ( O ) – original value</td>
</tr>
<tr>
<td>Percent decrease</td>
<td>( \frac{O - N}{O} )</td>
<td>( N ) – new value ( O ) – original value</td>
</tr>
<tr>
<td>Sale price and discount</td>
<td>( S = P - d \ P ) ( D = d \ P )</td>
<td>( S ) – sale price ( P ) – price (original or regular price) ( d ) – discount rate ( D ) – discount</td>
</tr>
<tr>
<td>Original price and markup</td>
<td>( P = C + m \ C ) ( M = m \ C )</td>
<td>( P ) – price (original or selling price) ( C ) – cost ( m ) – markup rate ( M ) – markup</td>
</tr>
<tr>
<td>Intelligence quotient (I.Q.)</td>
<td>( I = \frac{100m}{c} )</td>
<td>( I ) – I.Q. ( m ) – mental age ( c ) – chronological age</td>
</tr>
<tr>
<td>Cost of running electrical appliances</td>
<td>( C = \frac{Wtr}{1000} )</td>
<td>( C ) – Cost (in cents) ( W ) – power in watts (watts used) ( t ) – time (hours) ( r ) – rate (per kilowatt-hour)</td>
</tr>
</tbody>
</table>

**Substitution into formula:** "substitution" means replacing numbers with variables (letters).

**Example:** Find the IQ of a 10-year-old girl with a mental age of 12.

- **Formula:** \( I = \frac{100m}{c} \)
- **Facts:** \( m = 12 \) years, \( c = 10 \) years
- **Substituting:**

\[
I = \frac{100m}{c} = \frac{100 (12 \text{ y})}{10 \text{ y}} = \frac{1200}{10} = 120
\]

Substitute \( m \) for 12 y and \( c \) for 10 y.

The 10-year-old girl has an IQ of 120.
**Example:** Find the distance travelled by a train which has a velocity of 83 km per hour for 3 hours.

- **Formula:** \( d = v \times t \)
- **Facts:** \( v = 83 \text{ km/h} \), \( t = 3 \text{ h} \)
- **Substituting:** 
  \[
  d = (83 \text{ km/h}) \times (3 \text{ h}) = 249 \text{ km}
  \]
  
  The distance is 249 km.

**Example:** Find the volume of a cylinder with a radius of 2.3 cm and a height of 4.2 cm.

- **Formula:** \( V = \pi r^2 h \)
- **Facts:** \( r = 2.3 \text{ cm} \), \( h = 4.2 \text{ cm} \)
- **Substituting:** 
  \[
  V = \pi (2.3 \text{ cm})^2 \times (4.2 \text{ cm}) \\
  V \approx 69.8 \text{ cm}^3
  \]
  
  The volume of the cylinder is 69.8 cm\(^3\).

**Example:** Find the area of a triangle with a base of 12 ft and a height of 34 ft.

- **Formula:** \( A = \frac{1}{2}bh \)
- **Facts:** \( b = 12 \text{ ft} \), \( h = 34 \text{ ft} \)
- **Substituting:** 
  \[
  A = \frac{1}{2} \times 12 \text{ ft} \times 34 \text{ ft} = 204 \text{ ft}^2
  \]
  
  The area of the triangle is 204 ft\(^2\).

**Example:** An electric stove top burner runs for 2 hours and uses 750 watts of electricity at a cost of 10 cents per kilowatt-hour. What is the total cost of running the stove top burner?

- **Formula:** \( C = \frac{Wtr}{1000} \)
- **Facts:** \( t = 2 \text{ h} \), \( W = 750 \text{ w} \), \( r = 10\text{¢} / \text{kwh} \)
- **Substituting:** 
  \[
  C = \frac{750 \text{w} \times 2 \text{h} \times 10\text{¢} / \text{kwh}}{1000} = \frac{15 \text{¢}}{1000} = 15 \text{¢}
  \]

  The cost of running the stove top burner is 15 cents.
Topic B: Solving Formulas

To solve for a variable in a formula: isolate the unknown or desired variable so that it is by itself on one side of the equals sign and all the other terms are on the other side.

- Use the same process as you would for regular linear equations, the only difference is that you will be working with more variables.
- Remember to always do the same thing to both sides of the formula (add, subtract, multiply or divide the same variable or number to both sides of a formula).

Rearrange the formula so that the unknown or desired variable is by itself on one side of the equals sign. You can reverse the sides of the formula if you want.

Example: Solve each formula for the given variable.

1) Solve $d = rt$ for $t$.

$$\frac{d}{r} = \frac{rt}{r}$$

$$\frac{d}{r} = t \quad \text{or} \quad t = \frac{d}{r}$$

Tip: solve a formula for a given letter by isolating the given letter on one side of the formula.

2) Solve $I = Prt$ for $r$ and $P$.

$r$:

$$\frac{I}{Pt} = \frac{Prt}{Pt}$$

$$\frac{I}{Pt} = r \quad \text{or} \quad r = \frac{I}{Pt}$$

$P$:

$$\frac{I}{rt} = \frac{Prt}{rt}$$

$$\frac{I}{rt} = P \quad \text{or} \quad P = \frac{I}{rt}$$

3) Solve $P = 2w + 2l$ for $w$.

$$P - 2l = 2w + 2l - 2l$$

$$P - 2l = 2w$$

$$\frac{P-2l}{2} = \frac{2w}{2}$$

$$\frac{P-2l}{2} = w \quad \text{or} \quad w = \frac{P-2l}{2}$$

Reverse the sides of the formula.
More Examples for Solving Formulas

Example: Solve each formula for the given variable.

1) a) Solve \( F = \frac{9}{5}C + 32 \) for \( C \).

Solution:

\[
F - 32 = \frac{9}{5}C + 32 - 32
\]

\[
F - 32 = \frac{9}{5}C
\]

\[
\frac{5}{9}(F - 32) = \frac{5}{9} \cdot \frac{9}{5}C
\]

\[
\frac{5}{9}(F - 32) = C \quad \text{or} \quad C = \frac{5}{9}(F - 32)
\]

b) If \( F = 68 \), \( C = ? \)

\[
C = \frac{5}{9}(68 - 32)
\]

\[
C = 20
\]

2) Solve \( P = C + m \, C \) for \( C \).

\[
P = C \,(1 + m)
\]

\[
\frac{P}{1+m} = C \,(1+m)\]

\[
\frac{P}{1+m} = C \quad \text{or} \quad C = \frac{P}{1+m}
\]

3) Solve \( p = 35 \, q^2 + s \, q \) for \( s \).

\[
p - 35 \, q^2 = 35 \, q^2 + s \, q - 35 \, q^2
\]

\[
p - 35 \, q^2 = s \, q
\]

\[
\frac{p-35q^2}{q} = \frac{sq}{q}
\]

\[
s = \frac{p-35q^2}{q}
\]

4) Solve \( x = \frac{y-z}{t} \) for \( y \).

\[
x \, t = \frac{y-z}{t} \cdot t
\]

\[
x \, t + z = y - z + z
\]

\[
y = x \, t + z
\]
**Topic C: Pythagorean Theorem**

**Right triangle:** a triangle containing a 90° angle.

**Pythagorean theorem:** a relation among the three sides of a right triangle which states that the square of the hypotenuse is equal to the sum of the squares of the other two sides (legs).

\[
\text{hypotenuse}^2 = \text{leg}_1^2 + \text{leg}_2^2
\]

\[
\text{hypotenuse} = \sqrt{\text{leg}_1^2 + \text{leg}_2^2}, \quad \text{leg}_1 = \sqrt{\text{hypotenuse}^2 - \text{leg}_2^2}
\]

Using the Pythagorean theorem can find the length of the missing side in a right triangle.

\[
c^2 = a^2 + b^2
\]

\[
b = \sqrt{c^2 - a^2}, \quad a = \sqrt{c^2 - b^2}
\]

- \(c\) is the longest side of the triangle (hypotenuse).
- Other two sides (legs) of the triangle \(a\) and \(b\) can be exchanged.

**Example:** Find the missing side of the following triangles.

\[
a = 3\text{cm}
\]

\[
b = 4\text{cm}
\]

\[
c = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = 5\text{ cm}
\]

\[
\text{hypotenuse} = \sqrt{\text{leg}_1^2 + \text{leg}_2^2}
\]

\[
x = ?
\]

\[
x = \sqrt{5^2 - 3^2} = 4\text{ ft}
\]

\[
\text{arm} = \sqrt{\text{hypotenuse}^2 - \text{arm}^2}
\]
Applications of the Pythagorean Theorem

**Example:** Find the distance of the diagonal across the rectangle.

\[ x = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{5}{6}\right)^2} \approx 1.067 \text{ m} \]

The distance of the diagonal is 1.067 m.

**Example:** What is the length of one leg of a right triangle whose hypotenuse measures 5.36 cm and the other leg measures 3.24 cm?

\[ x = \sqrt{5.36^2 - 3.24^2} \approx 4.27 \text{ cm} \]

The length of one leg is 4.27 cm.

**Example:** A plane leaves the Vancouver airport and flies 245 km west, then 350 km north. How far is the plane from the airport?

\[ c = \sqrt{245^2 + 350^2} \approx 427.23 \text{ km} \]

The distance of the plane from the airport is 427.23 km.

**Example:** A kite at the end of a 55 m line is 26 m behind the runner. How high is the kite?

\[ x = \sqrt{55^2 - 26^2} \approx 48.47 \text{ m} \]

The height of the kite is 48.47 m.
**Unit 8: Summary**

**Formulas**

**Formula:** an equation that contains more than one variable and is used to solve practical problems in everyday life.

**Geometry formulas review:**


<table>
<thead>
<tr>
<th>Name of the figure</th>
<th>Formula</th>
<th>Figure</th>
</tr>
</thead>
</table>
| **Rectangle**      | $P = 2w + 2l$  
                      $A = w\cdot l$  
                      ($w$ = width, $l$ = length) | ![Rectangle](w l) |
| **Parallelogram**  | $P = 2a + 2b$  
                      $A = b\cdot h$  
                      ($a$ and $b$ = sides, $h$ = height) | ![Parallelogram](h a b) |
| **Circle**         | $C = \pi d = 2\pi r$  
                      $A = \pi r^2$  
                      ($r$ = radius, $d$ = diameter) | ![Circle](r d) |
| **Triangle**       | $\angle A + \angle B + \angle C = 180^\circ$  
                      $A = \frac{1}{2}bh$  
                      ($b$ = base, $h$ = height) | ![Triangle](B h) |
| **Trapezoid**      | $A = \frac{1}{2}h(b + B)$  
                      ($b$ = top base, $B$ = bottom base, $h$ = height) | ![Trapezoid](h) |
| **Cube**           | $V = s^3$  
                      ($s$ = side) | ![Cube](s) |
| **Rectangular solid** | $V = w\cdot l\cdot h$  
                      ($w$ = width, $l$ = length, $h$ = height) | ![Rectangular solid](l w) |
| **Cylinder**       | $V = \pi r^2h$  
                      ($r$ = radius, $h$ = height) | ![Cylinder](r h) |
| **Sphere**         | $V = \frac{4}{3}\pi r^3$  
                      ($r$ = radius) | ![Sphere](r) |
| **Cone**           | $V = \frac{1}{3}\pi r^2h$  
                      ($r$ = radius, $h$ = height) | ![Cone](h r) |
**Substitution into formula:** "substitution" means replacing numbers with variables (letters).

**Examples of formula:**

<table>
<thead>
<tr>
<th>Application</th>
<th>Formula</th>
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<tbody>
<tr>
<td>Distance</td>
<td>( d = v t )</td>
<td>( d ) – distance</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( v ) – velocity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( t ) – time</td>
</tr>
<tr>
<td>Simple interest</td>
<td>( I = P r t )</td>
<td>( I ) – interest</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P ) – principle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r ) – interest rate (%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( t ) – time (years)</td>
</tr>
<tr>
<td>Compound interest</td>
<td>( B = P (100% + r)^t )</td>
<td>( B ) – balance</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P ) – principal</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r ) – interest rate (%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( t ) – time (years)</td>
</tr>
<tr>
<td>Percent increase</td>
<td>( \frac{N - O}{O} )</td>
<td>( N ) – new value</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( O ) – original value</td>
</tr>
<tr>
<td>Percent decrease</td>
<td>( \frac{O - N}{O} )</td>
<td>( N ) – new value</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( O ) – original value</td>
</tr>
<tr>
<td>Sale price and Discount</td>
<td>( S = P - dP )</td>
<td>( S ) – sale price</td>
</tr>
<tr>
<td></td>
<td>( D = dP )</td>
<td>( P ) – price (original or regular price)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( d ) – discount rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( D ) – discount</td>
</tr>
<tr>
<td>Original price and Markup</td>
<td>( P = C + mC )</td>
<td>( P ) – price (original or selling price)</td>
</tr>
<tr>
<td></td>
<td>( M = mC )</td>
<td>( C ) – cost</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( m ) – markup rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( M ) – markup</td>
</tr>
<tr>
<td>Intelligence quotient (I.Q.)</td>
<td>( I = \frac{100m}{c} )</td>
<td>( I ) – I.Q.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( m ) – mental age</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( c ) – chronological age</td>
</tr>
<tr>
<td>Cost of running electrical appliances</td>
<td>( C = \frac{Wtr}{1000} )</td>
<td>( C ) – Cost (in cents)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( W ) – power in watts (watts used)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( t ) – time (hours)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( r ) – rate (per kilowatt-hour)</td>
</tr>
</tbody>
</table>

**Pythagorean theorem:** a relation among the three sides of a right triangle which states that the square of the hypotenuse is equal to the sum of the squares of the other two sides (legs).

\[
\begin{align*}
  a & \quad \triangle \quad c \\
  b & \quad \sqrt{a^2 + b^2} \\
  c & \quad \text{is the longest side of the triangle (hypotenuses).}
\end{align*}
\]

**Using the Pythagorean theorem can find the length of the missing side in a right triangle.**

- \( c \) is the longest side of the triangle (hypotenuses).
- Other two sides (legs) of the triangle \( a \) and \( b \) can be exchanged.
Unit 8: Self-Test

Topic A

1. Find the IQ of a 70-year-old man with a mental age of 85.
2. Find the distance travelled by a train which has a velocity of 78 km per hour for 2.5 hours.
3. Steve rides his bicycle at a speed of 11 miles per hour. He goes on a 22-mile bike ride. How many hours does this ride take?
4. Find the volume of a cone with a radius of 4.6 cm and a height of 8.4 cm.
5. Find the area and perimeter of a rectangle with a width of 11 cm and a length of 35 cm.
6. Find the area of a triangle with a base of 24 ft and a height of 58 ft.
7. The diameter of a circle is 4.8 ft. Find the circumference and area of the circle.
8. Ann invests $15,000 at an annual interest rate of 0.75%. How much simple interest will she earn by the end of 3 years?
9. An electric stove top burner runs for 2.5 hours and uses 800 watts of electricity per hour at a cost of 9 cents per kilowatt-hour. What is the total cost of running the stove top burner?

Topic B

10. Solve each formula for the given variable.
   a) Solve \( d = r t \) for \( r \).
   b) Solve \( I = P r t \) for \( t \).
   c) Solve \( P = 2w + 2l \) for \( l \).
d) Solve \( C = \frac{5}{9} (F - 32) \) for \( F \).

If \( C = 24 \), \( F = ? \)

e) Solve \( P = C + m C \) for \( m \).

f) Solve \( x = 35 y^2 + z y \) for \( z \).

g) Solve \( A = \frac{1}{2} bh^2 \) for \( b \).

h) Solve \( x = \frac{y - z}{t} \) for \( z \).

i) Solve \( w = \frac{\pi r^2 h}{35} \) for \( h \).

j) Solve \( x = y - (2z + 3) w \) for \( w \),

if \( x = 2, \ y = 3, \ z = 4 \).

**Topic C**

11. Find the missing side of the following triangles.

```
7cm
\( x = ? \)
```

12. Find the distance of the diagonal across the rectangle.

```
\( \frac{2}{5} \) m
\( \frac{6}{7} \) m
```

13. What is the length of one leg of a right triangle whose hypotenuse measures 21.34 ft and the other leg measures 15.27 ft?

14. A plane leaves the Calgary airport and flies 134 km east, then 250 km south. How far is the plane from the airport?

15. A kite at the end of a 89 ft line is 57 ft behind the runner. How high is the kite?
Unit 9
Ratio, Proportion, and Percent

Topic A: Ratio and rate
- Ratio
- Rate

Topic B: Proportion
- Solving proportion

Topic C: Percent
- Percent review
- Solving percent problems

Topic D: Similar triangles
- Similar triangles
- Solving similar triangles

Unit 9 Summary

Unit 9 Self-test
**Topic A: Ratio and Rate**

**Ratio**
- Ratio: a relationship between two numbers, expressed as the quotient with the *same unit* in the denominator and the numerator.
- Express a ratio: there are three ways to write a ratio.
  
  The ratio of \( a \) and \( b \) is: \( a \) to \( b \) or \( a : b \) or \( \frac{a}{b} \)

**Example:** Write the ratio of 5 cents to 9 cents.

\[
\text{5 to 9 or 5 : 9 or } \frac{5}{9}
\]

- Write a ratio in lowest terms (simplify):
  - Write the ratio in a fractional form.
  - Simplify and drop the units if given (as they cancel each other out).

**Example:**

\[
4 : 28 = \frac{4}{28} = \frac{1}{7}
\]

\[
\frac{0.75 \text{ meters}}{0.25 \text{ meters}} = \frac{75}{25} = \frac{3}{1} = 3
\]

**Grade and pitch**
- Grade (or slope, pitch, incline etc.): the slope of a straight line is the rate of change in height over a distance. It is a measure of the “steepness” “or incline” of a line.

- **The grade or slope formula:**

  \[
  \text{Grade or slope} = \frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{\text{rise}}{\text{run}}
  \]

**Example:** Determine the grade (%) of a road that has a length of 75 m and a vertical height of 3 m.

\[
\text{Grade} = \frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{3 \text{ m}}{75 \text{ m}} = 0.04 = 4\%
\]
Rate

Rate: a ratio of two quantities with different units.

Example: teachers to students; money to time; distance to time, etc.

\[
\frac{2 \text{ teachers}}{83 \text{ students}}, \quad \frac{24 \text{ dollars}}{3 \text{ hours}}, \quad \frac{85 \text{ miles}}{2 \text{ hours}}
\]

Write a rate in lowest terms (simplify the rate):

Example: 80 kilometres per 320 minutes:

\[
\frac{80 \text{ km}}{320 \text{ min}} = \frac{1 \text{ km}}{4 \text{ min}}
\]

Unit rate: a rate in which the number in the second term (denominator) is 1.

Example: 15 dollars per hours:

\[
\frac{\$15}{1 \text{ h}} = \$15 \text{ per h}
\]

Some unit rates:
- Miles (or kilometres) per hour (or minute).
- Cost (dollars/cents) per item or quantity.
- Earnings (dollars) per hour (or week).

Unit price and the best buy.

Example: Find the best buy.

12 eggs for $3.19; 18 eggs for $4.91; 30 eggs for $7.13.

\[
s_\frac{3.19}{12 \text{ eggs}} \approx \$0.266 \text{ per egg}
\]

\[
s_\frac{4.91}{18 \text{ eggs}} \approx \$0.273 \text{ per egg}
\]

\[
s_\frac{7.13}{30 \text{ eggs}} \approx \$0.238 \text{ per egg}
\]

So the best buy is 30 eggs for $7.13 (the lowest price). 0.238 < 0.266 < 0.273
**Topic B: Proportion**

**Proportion**: an equation with a ratio (or rate) on two sides \( \frac{a}{b} = \frac{c}{d} \), in which the two ratios are equal.

**Example**: Write the following sentence as a proportion.

3 printers is to 18 computers as 2 printers is to 12 computers.

Review ratio, rate and proportion:

<table>
<thead>
<tr>
<th>Representation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio</td>
<td>5 to 9  or 5:9 or ( \frac{5 \text{ km}}{9 \text{ km}} )</td>
</tr>
<tr>
<td>Rate</td>
<td>3 to 7  or 3:7 or ( \frac{3 \text{ cm}}{7 \text{ km}} )</td>
</tr>
<tr>
<td>Proportion</td>
<td>( \frac{a}{b} = \frac{c}{d} )</td>
</tr>
</tbody>
</table>

**Note**: the units for both numerators must match and the units for both denominators must match.

| Example: | \( \frac{\text{in}}{\text{ft}} = \frac{\text{in}}{\text{ft}} \), \( \frac{\text{minutes}}{\text{hours}} = \frac{\text{minutes}}{\text{hours}} \) |

**Solving a proportion:**

- Cross multiply: multiply along two diagonals. \( \frac{a}{b} = \frac{c}{d} \)
- Solve for the unknown.

**Application**

**Example**: 4 liters of milk cost $4.38, how much do 2 liters cost?

- Facts and unknown:
  
<table>
<thead>
<tr>
<th>4 L milk</th>
<th>2 L milk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.38</td>
<td>$x</td>
</tr>
</tbody>
</table>

  \[ \frac{4 \text{ L}}{4.38} = \frac{2 \text{ L}}{x} \] The same units.

- Proportion:
  
  \[ \frac{4 \text{ L}}{4.38} = \frac{2 \text{ L}}{x} \] The same units.

- Cross multiply:
  
  \[ \frac{4 \text{ L}}{4.38} = \frac{2 \text{ L}}{x} \]

  \[ (4)(x) = (2)(4.38) \]
- Solve for \( x \):
\[
\frac{4x}{4} = \frac{2(4.38)}{4}
\]
\[
x = \frac{(2)(4.38)}{4} = 2.19
\]
2 liters of milk cost $2.19.

- Check:
\[
\frac{\text{4L}}{\$4.38} = \frac{\text{2L}}{\$2.19}
\]
\[
? \quad (4)(2.19) = (2)(4.38)
\]
\[
\sqrt{8.76} = 8.76 \quad \text{Correct!} \quad (\text{LS} = \text{RS})
\]

**Example:** Tom’s height is 1.75 meters, and his shadow is 1.09 meters long. A building’s shadow is 10 meters long at the same time. How high is the building?

- Facts and unknown:

<table>
<thead>
<tr>
<th>Tom’s height = 1.75 m</th>
<th>Building’s height ((x) = ?)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tom’s shadow = 1.09 m</td>
<td>Building’s shadow = 10 m</td>
</tr>
</tbody>
</table>

- Proportion:
\[
\frac{1.75 \text{ m}}{1.09 \text{ m}} = \frac{x \text{ m}}{10 \text{ m}}
\]

- Cross multiply:
\[
\frac{1.75 \text{ m}}{1.09 \text{ m}} = \frac{x \text{ m}}{10 \text{ m}}
\]
\[
(1.75) (10) = (1.09) (x)
\]

- Solve for \( x \):
\[
\frac{(1.75)(10)}{1.09} = \frac{(1.09)x}{1.09}
\]
\[
x = \frac{(1.75)(10)}{1.09} \approx 16.055 \quad \text{The building’s height is} \ 16.055 \text{ m}.
\]

- Check:
\[
\frac{1.75 \text{ m}}{1.09 \text{ m}} = \frac{16.055 \text{ m}}{10 \text{ m}}
\]
\[
? \quad (1.75) (10) = (16.055) (1.09)
\]
\[
\sqrt{17.5} = 17.5 \quad \text{Correct!} \quad (\text{LS} = \text{RS})
\]

**Example:** If 15 mL of medicine must be mixed with 180 mL of water, how many mL of medicine must be mixed in 230 mL of water?

- Proportion:
\[
\frac{15 \text{ mL}}{180 \text{ mL}} = \frac{x \text{ mL}}{230 \text{ mL}}
\]

- Cross multiply:
\[
\frac{15 \text{ mL}}{180 \text{ mL}} = \frac{x \text{ mL}}{230 \text{ mL}}
\]

- Solve for \( x \):
\[
x = \frac{(15 \text{ mL})(230 \text{ mL})}{180 \text{ mL}} \approx 19.17 \text{ mL}
\]
\[
19.17 \text{ mL} \text{ of medicine must be mixed in} \ 230 \text{ mL of water}.
\]
Topic C: Percent

Percent (%): one part per hundred, or per one hundred.

Review - converting between percent, decimal and fraction:

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Steps</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent → Decimal</td>
<td>Move the decimal point two places to the left, then remove %.</td>
<td>31% = 31. % = 0.31</td>
</tr>
<tr>
<td>Decimal → Percent</td>
<td>Move the decimal point two places to the right, then insert %.</td>
<td>0.317 = 0.317 = 31.7 %</td>
</tr>
<tr>
<td>Percent → Fraction</td>
<td>Remove %, divide by 100, then simplify.</td>
<td>15% = 15 / 100 = 3 / 20</td>
</tr>
<tr>
<td>Fraction → percent</td>
<td>Divide, move the decimal point two places to the right, then insert %.</td>
<td>1 / 4 = 1 ÷ 4 = 0.25 = 25 %</td>
</tr>
</tbody>
</table>
| Decimal → Fraction  | Convert the decimal to a percent, then convert the percent to a fraction. | 0.35 = 35% = 35 / 100 = 7 / 20 |%

Two methods to solve percent problems

- Percent proportion method
- Translation (translate the words into mathematical symbols.)

Percent proportion method:

With the word “is”

\[
\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}
\]

or

\[
\frac{\text{"is" number}}{\text{"of" number}} = \frac{\%}{100}
\]

With the word “of”

Step

- Identify the part, whole, and percent.
- Set up the proportion equation.
- Solve for unknown (x).

Example

8 percent of what number is 4?

\[
\frac{4}{x} = \frac{8}{100} \quad \text{or} \quad \frac{\text{Part}}{\text{Whole}} = \frac{\%}{100}
\]

\[
x = \frac{(4)(100)}{8} = 50, \quad \frac{\text{Part}}{\text{Whole}} = \frac{\%}{100}
\]

\[
x = 50
\]
Translation method (translate the words into mathematical symbols):

Translation:
- What → \( x \) : the word “what” represents an unknown quantity \( x \).
- Is → \( = \) : the word “is” represents an equal sign.
- of → \( \times \) : the word “of” represents a multiplication sign.
- \% to decimal: always change the percent to a decimal.

Example:
1) What is 15\% of 80?
   \[ x = 0.15 \times 80 \]
   \[ x = (0.15)(80) = 12 \]
2) What percent of 90 is 45?
   \[ x\% \times 90 = 45 \]
   \[ x\% = \frac{45}{90} = 0.5 = 50\% \]
   Divide both sides by 90.
3) 12 is 8\% of what number?
   \[ 12 = 0.08 \times x \]
   \[ x = \frac{12}{0.08} = 150 \]
   Divide both sides by 0.08.

- Percent increase or decrease:

<table>
<thead>
<tr>
<th>Application</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent increase</td>
<td>( \text{Percent increase} = \frac{\text{New value} - \text{Original value}}{\text{Original value}}, \quad x = \frac{N-O}{O} )</td>
</tr>
<tr>
<td>Percent decrease</td>
<td>( \text{Percent decrease} = \frac{\text{Original value} - \text{New value}}{\text{Original value}}, \quad x = \frac{O-N}{O} )</td>
</tr>
</tbody>
</table>

Example: A product increased production from 1500 last month to 1650 this month. Find the percent increase.
- New value (N): 1650
- Original value (O): 1500
- Percent increase: \( x = \frac{N-O}{O} = \frac{1650-1500}{1500} = 0.1 = 10\% \)
  A 10\% increase.

Example: A product was reduced from $33 to $29. What percent reduction is this?

Percent decrease: \( x = \frac{O-N}{O} = \frac{33-29}{33} \approx 0.12 = 12\% \)
  A 12\% decrease.
**Topic D: Similar Triangles**

**Similar triangles:** triangles that have the same shape and proportions, but may have different sizes.

The symbol “\( \triangle \)” is used for triangle; the symbol “\(<\)” is used for angle.

**Sides and angles in a triangle \( \triangle \):**
- Sides are labeled with lower case letters.
- Angles (\(<\)) are labeled with uppercase letters.

**Corresponding (matching) angles and corresponding sides** of two similar triangles:

\[
\triangle ABC \sim \triangle XYZ
\]

- The corresponding angles of two similar triangles are equal.
  \(< A = < X \quad < B = < Y \quad < C = < Z\)

- The corresponding sides of two similar triangles are proportional in length.
  - Side \( a \) corresponds to side \( x \).
  - Side \( b \) corresponds to side \( y \).
  - Side \( c \) corresponds to side \( z \).

**The formula for similar triangles:**

\[
\frac{a}{x} = \frac{b}{y} = \frac{c}{z}
\]

This includes three proportions:

\[
\frac{a}{x} = \frac{b}{y} \quad \frac{a}{x} = \frac{c}{z} \quad \frac{b}{y} = \frac{c}{z}
\]
**Solving Similar Triangles**

**Example:** Find the value of the missing side in the following figures (the two triangles are similar).

1) \[ \frac{b}{y} = \frac{c}{z} \quad \text{or} \quad \frac{b}{6m} = \frac{3m}{5m} \]

\[ b = \frac{(3m)(6m)}{5m} = 3.6m \]

2) \[ \frac{b}{y} = \frac{c}{z} \quad \text{or} \quad \frac{b}{6m} = \frac{5m}{3m} \]

\[ b = \frac{(5m)(6m)}{3m} = 10m \]

3) \[ \frac{a}{4cm} = \frac{6cm}{7cm} \]

\[ a = \frac{(4cm)(6cm)}{7cm} \approx 3.43 \text{ cm} \]
Unit 9: Summary

Ratio, Proportion, and Percent

Ratio, rate and proportion:

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<tr>
<td><strong>Ratio</strong></td>
<td>$a \text{ to } b \text{ or } a:b \text{ or } \frac{a}{b}$</td>
<td>with the same unit. $5 \text{ to } 9 \text{ or } 5:9 \text{ or } \frac{5 \text{ m}}{9 \text{ m}}$</td>
</tr>
<tr>
<td><strong>Rate</strong></td>
<td>$a \text{ to } b \text{ or } a:b \text{ or } \frac{a}{b}$</td>
<td>with different units. $3 \text{ to } 7 \text{ or } 3:7 \text{ or } \frac{3 \text{ cm}}{7 \text{ m}}$</td>
</tr>
<tr>
<td><strong>Proportion</strong></td>
<td>$\frac{a}{b} = \frac{c}{d}$</td>
<td>an equation with a ratio/rate on each side. $\frac{x \text{ m}}{5 \text{ km}} = \frac{3 \text{ m}}{8 \text{ km}}, \frac{x \text{ m}}{7 \text{ m}} = \frac{2 \text{ m}}{5 \text{ m}}$</td>
</tr>
</tbody>
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**Note:** the units for both numerators must match and the units for both denominators must match.

**Unit rate:** A rate in which the number in the second term (denominator) is 1.

**Solving a proportion:**

- Cross multiply: multiply along two diagonals. $\frac{a}{b} \times \frac{c}{d}$
- Solve for the unknown.

**Percent (%)**: one part per hundred, or per one hundred.

**Converting between percent, decimal and fraction:**

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<td>Remove %, divide by 100, then simplify.</td>
<td>$15% = \frac{15}{100} = \frac{3}{20}$</td>
</tr>
<tr>
<td>Fraction → Percent</td>
<td>Divide, move the decimal point two places to the right, then insert %.</td>
<td>$\frac{1}{4} = 1 \div 4 = 0.25 = 25%$</td>
</tr>
</tbody>
</table>
### Decimal → Fraction

| Decimal → Fraction | Convert the decimal to a percent, then convert the percent to a fraction. | 0.35 = 35% = $\frac{35}{100} = \frac{7}{20}$ |

### Grade and pitch

- Grade (or slope, pitch, incline etc.): the slope of a straight line is the rate of change in height over a distance. It is a measure of the “steepness” or incline” of a line.

- **The grade or slope formula:**

  \[
  \text{Grade or slope} = \frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{\text{rise}}{\text{run}}
  \]

### Two methods to solve percent problems

- Percent proportion method

- Translation (translate the words into math symbols.)

#### Percent proportion method:

With the word “is”

\[
\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}
\]

or

\[
\frac{\text{“is” number}}{\text{“of” number}} = \frac{\text{%}}{100}
\]

With the word “of”

#### Translation method (translate the words into mathematical symbols):

- What \( x \): the word “what” represents an unknown quantity \( x \).

- Is \( = \): the word “is” represents an equal sign.

- of \( \times \): the word “of” represents a multiplication sign.

- % to decimal: always change the percent to a decimal.

### Percent increase or decrease:

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<tr>
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<th>Formula</th>
</tr>
</thead>
</table>
| Percent increase  | \[
\text{Percent increase} = \frac{\text{New value} - \text{Original value}}{\text{Original value}},
\]

\[
\text{Percent increase} = x = \frac{O-N}{O}
\]

| Percent decrease  | \[
\text{Percent decrease} = \frac{\text{Original value} - \text{New value}}{\text{Original value}},
\]

\[
\text{Percent decrease} = x = \frac{O-N}{O}
\]

### Grade and pitch

- Grade (or slope, pitch, incline etc.): the slope of a straight line is the rate of change in height over a distance. It is a measure of the “steepness” or incline” of a line.

- **The grade or slope formula:**

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  \]

### Two methods to solve percent problems

- Percent proportion method

- Translation (translate the words into math symbols.)

#### Percent proportion method:

With the word “is”

\[
\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percent}}{100}
\]

or

\[
\frac{\text{“is” number}}{\text{“of” number}} = \frac{\text{%}}{100}
\]

With the word “of”

#### Translation method (translate the words into mathematical symbols):

- What \( x \): the word “what” represents an unknown quantity \( x \).

- Is \( = \): the word “is” represents an equal sign.

- of \( \times \): the word “of” represents a multiplication sign.

- % to decimal: always change the percent to a decimal.

### Percent increase or decrease:

<table>
<thead>
<tr>
<th>Application</th>
<th>Formula</th>
</tr>
</thead>
</table>
| Percent increase  | \[
\text{Percent increase} = \frac{\text{New value} - \text{Original value}}{\text{Original value}},
\]

\[
\text{Percent increase} = x = \frac{O-N}{O}
\]

| Percent decrease  | \[
\text{Percent decrease} = \frac{\text{Original value} - \text{New value}}{\text{Original value}},
\]

\[
\text{Percent decrease} = x = \frac{O-N}{O}
\]
The symbol “\(\triangle\)” is used for triangle; the symbol “\(<\)” is used for angle.

**Similar (\(\sim\)) triangles:** triangles that have the same shape and proportions, but may be of different sizes.

**Sides and angles in a triangle:**
- Sides are labeled with lower case letters.
- Angles (<) are labeled with uppercase letters.

**Corresponding angles and corresponding (matching) sides:**

\[ \triangle ABC \sim \triangle XYZ \]

- The corresponding angles of two similar triangles are equal.
  \(<A = <X \quad <B = <Y \quad <C = <Z\)

- The corresponding sides of two similar triangles are proportional in length.
  - Side \(a\) corresponds to side \(x\).
  - Side \(b\) corresponds to side \(y\).
  - Side \(c\) corresponds to side \(z\).

**Solve similar triangles:**

\[
\frac{a}{x} = \frac{b}{y} = \frac{c}{z}
\]

This includes three proportions:

\[
\begin{align*}
\frac{a}{x} &= \frac{b}{y} \\
\frac{a}{x} &= \frac{c}{z} \\
\frac{b}{y} &= \frac{c}{z}
\end{align*}
\]
Unit 9: Self-Test

Topic A

1. Write the following as a ratio or rate in lowest terms.
   a) 15 nickels to 45 nickels.
   b) 24 kilometers to 88 kilometers.
   c) 350 people for 1500 tickets. \(\frac{350}{1500}\)
   d) 0.33 centimetres to 0.93 centimetres.
   e) 160 kilometres per 740 minutes.

2. Determine the grade (%) of a road that has a length of 2,500 m and a vertical height of 3.5 m.

3. What is the grade (%) of a river that drops 9 meters over a distance of 720 meters?

4. A train travelled 459 km in 6 hours. What is the unit rate? \(\frac{459}{6}\) km/h

5. A 4 L bottle of milk sells for $4.47. A 2 L bottle of the same milk sells for $3.43. What is the best buy?

6. An 8-pound bag of apples costs $7.49. A 6-pound bag of the same apples costs $5.99. What is the best buy?

Topic B

7. Write the following sentence as a proportion.
   a) 5 teachers is to 110 students as 15 teachers is to 330 students. \(\frac{5}{110} = \frac{15}{330}\)
   b) 24 hours is to 1,940 kilometers as 12 hours is to 985 kilometers.

8. 4 liters of juice cost $7.38, how much do 2 liters cost?
9. Todd’s height is 5.44 feet, and his shadow is 8.5 feet long. A building’s shadow is 25 feet long at the same time. How high is the building?

10. Sarah earns $4,500 in 30 days. How much does she earn in 120 days? \[ \frac{15 \times 12}{30} \]

**Topic C**

11. What is 45% of 260?

12. 36 is 12% of what number?

13. A product increased production from 2,800 last year to 3,920 this year. Find the percent increase.

14. A product was reduced from $199 to $159. What percent reduction is this?

15. Find the value of the missing side in the following figures (the two triangles are similar).

a) \[ \begin{align*} c &= 4 \text{ cm} \\
      a &= ? \\
   \end{align*} \quad \begin{align*} z &= 7.4 \text{ cm} \\
      y &= 9.6 \text{ cm} \\
   x &= 5.55 \text{ cm} \\
\end{align*} \]

b) \[ \begin{align*} c &= 7 \text{ m} \\
      z &= 5 \text{ m} \\
      b &= ? \\
      y &= 8 \text{ m} \\
\end{align*} \]

c) \[ \begin{align*} x &= ? \\
       6 \text{ cm} \\
\end{align*} \quad \begin{align*} 8 \text{ cm} \\
       7 \text{ cm} \\
\end{align*} \]
Unit 10

Trigonometry

Topic A: Angles and triangles
- Angles
- Triangles
- Find the missing measurement

Topic B: Trigonometric functions
- Sides and angles
- Trigonometric functions
- Sine, cosine, and tangent

Topic C: Solving right triangles
- Trigonometry using a calculator
- Solving triangles
- Angles of depression and elevation
- Applications of trigonometry

Unit 10 Summary

Unit 10 Self-test
Topic A: Angles and Triangles

Angles

**Angle:** two rays (sides) that have a common point (the vertex).

\[
\angle B = \angle ABC = \angle CBA
\]

The angle \( B \) in the figure above could be called \( \angle B \) or \( \angle ABC \) or \( \angle CBA \).

An angle can vary from 0 to 360 degrees (360°).

Classifying angles:

<table>
<thead>
<tr>
<th>Angle</th>
<th>Definition</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Straight angle</strong></td>
<td>An angle of exactly 180 degrees.</td>
<td><img src="image" alt="Straight angle" /></td>
</tr>
<tr>
<td><strong>Right angle</strong></td>
<td>An angle of exactly 90 degrees.</td>
<td><img src="image" alt="Right angle" /></td>
</tr>
<tr>
<td><strong>Acute angle</strong></td>
<td>An angle between 0 and 90 degrees. (Less than 90°)</td>
<td><img src="image" alt="Acute angle" /></td>
</tr>
<tr>
<td><strong>Obtuse angle</strong></td>
<td>An angle between 90 and 180 degrees.</td>
<td><img src="image" alt="Obtuse angle" /></td>
</tr>
<tr>
<td><strong>Reflex angle</strong></td>
<td>An angle between 180 and 360 degrees.</td>
<td><img src="image" alt="Reflex angle" /></td>
</tr>
<tr>
<td><strong>Complementary angles</strong></td>
<td>Two angles whose sum is exactly 90 degrees.</td>
<td><img src="image" alt="Complementary angles" /></td>
</tr>
<tr>
<td><strong>Supplementary angles</strong></td>
<td>Two angles whose sum is exactly 180 degrees.</td>
<td><img src="image" alt="Supplementary angles" /></td>
</tr>
<tr>
<td><strong>Vertical angles</strong></td>
<td>Two angles formed by the intersection</td>
<td><img src="image" alt="Vertical angles" /></td>
</tr>
<tr>
<td></td>
<td>of two straight lines.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \angle A ) and ( \angle B ) are vertical angles.</td>
<td></td>
</tr>
</tbody>
</table>
Example: Label each of the following angles.

1) \[ \begin{align*} &\angle 21^\circ \quad \text{Acute angles.} \\
&\angle 70^\circ \end{align*} \]

2) \[ \begin{align*} &\angle 112^\circ \quad \text{Obtuse angles.} \\
&\angle 130^\circ \end{align*} \]

3) \[ \begin{align*} &\text{Obtuse angles.} \quad \text{Reflex angle.} \end{align*} \]

Example: What is the complementary angle to 38 degrees?

\[ \begin{align*} \angle x + 38^\circ &= 90^\circ \\
\therefore \angle x &= 90^\circ - 38^\circ = 52^\circ \end{align*} \]

Example: What is the supplementary angle to 137 degrees?

\[ \begin{align*} \angle x + 137^\circ &= 180^\circ \\
\therefore \angle x &= 180^\circ - 137^\circ = 43^\circ \end{align*} \]

Example: What is the size of the angle \( x \)?

\[ \begin{align*} \angle x &= 110^\circ - 85^\circ = 25^\circ \end{align*} \]

Example 1) Two angles \( A \) and 45 degrees that add together to measure 180 degrees are said to be\( \underline{\text{ supplementary}} \)?

2) What is the size of angle \( A \) and \( B \)?

\[ \begin{align*} \angle A + 45^\circ &= 180^\circ, \quad \angle A = 180^\circ - 45^\circ, \quad \angle A = 135^\circ \\
\angle A + \angle B &= 180^\circ, \quad \angle B = 180^\circ - \angle A, \quad \angle B = 45^\circ \end{align*} \]

How to use a protractor:

- Place the protractor so that the center hole is over the angle’s vertex.
- Line up the base line of the protractor with one of the sides of the angle.
- Read the angle over the the second side of the angle.
Classify triangles:

<table>
<thead>
<tr>
<th>Name of triangle</th>
<th>Definition</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral triangle</td>
<td>A triangle that has three equal sides and three equal angles.</td>
<td><img src="image" alt="Equilateral triangle" /></td>
</tr>
<tr>
<td></td>
<td>(a = b = c), (&lt; A = &lt; B = &lt; C = 60^\circ)</td>
<td></td>
</tr>
<tr>
<td>Isosceles triangle</td>
<td>A triangle that has two equal sides and two equal angles.</td>
<td><img src="image" alt="Isosceles triangle" /></td>
</tr>
<tr>
<td></td>
<td>(a = b), (&lt; A = &lt; B)</td>
<td></td>
</tr>
<tr>
<td>Acute triangle</td>
<td>A triangle that has three acute angles ((&lt; 90^\circ)).</td>
<td><img src="image" alt="Acute triangle" /></td>
</tr>
<tr>
<td>Right triangle</td>
<td>A triangle that has a right angle ((= 90^\circ)). (A right angle is usually marked on the figure as a small square.)</td>
<td><img src="image" alt="Right triangle" /></td>
</tr>
<tr>
<td>Obtuse triangle</td>
<td>A triangle that has an obtuse angle ((&gt; 90^\circ)).</td>
<td><img src="image" alt="Obtuse triangle" /></td>
</tr>
<tr>
<td>Scalene triangle</td>
<td>A triangle that has three unequal sides.</td>
<td><img src="image" alt="Scalene triangle" /></td>
</tr>
</tbody>
</table>

Angles in a triangle: the sum of the three angles in a triangle is always \(180^\circ\).

\[\angle A + \angle B + \angle C = 180^\circ\]

Example: What is the size of angle \(C\) in the following figure?

\[102^\circ + 50^\circ + \angle C = 180^\circ\]

\[\angle C = 180^\circ - (102^\circ + 50^\circ) = 28^\circ\]

Example: What is the size of angle \(C\), \(D\) and the side \(b\) in the following figure?

\[61^\circ + 58^\circ + \angle C = 180^\circ\]

\[\angle C = 180^\circ - (61^\circ + 58^\circ) = 61^\circ\]

\[\angle D = 180^\circ - \angle C = 180^\circ - 61^\circ = 119^\circ\]

\[b = 2 \text{ cm}\] (An isosceles triangle)

Example: Match the following triangles to the letter with the best definition.

- _____Scalene triangle  a. has three equal sides  c.
- _____Equilateral triangle b. has two equal sides  a.
- _____Isosceles triangle c. has three unequal sides  b.
Find the Missing Measurement

Example: Find the missing measurement and then name the kind of triangle.

1) \[ B = 180^0 - (60^0 + 60^0) \]
\[ = 60^0 \]
It is an equilateral triangle.
(An acute triangle: \( 60^0 < 90^0 \).)

2) \[ B = 180^0 - (45^0 + 45^0) \]
\[ = 90^0 \]
It is an isosceles triangle.
(An right triangle: it has a \( 90^0 \) angle.)

3) \[ a = 3.5 \text{ cm} \]
It is an isosceles triangle.

4) \[ y = 43 \text{ m} \]
It is an isosceles triangle.
(An acute triangle: \( 65^0 < 90^0 \))
**Topic B: Trigonometric Functions**

**Sides and Angles**

**Trigonometry:** the study of the relationships between sides and angles of right triangles and trigonometric functions.

**Right triangle review:** a triangle that has a $90^\circ$ angle (right-angled triangle).

**Sides and angles:**
- $< C$ is a right angle ($90^\circ$).
- Sides are labeled with lower case letters (or two capital letters).
  - **Example:** The side $a$ or $BC$, the side $b$ or $AB$, the side $c$ or $AC$.
- Angles are labeled with uppercase letters.
  - **Example:** $< A$, $< B$, $< C$
- Side $a$ will be the side opposite angle $A$; side $b$ will be the side opposite angle $B$; and side $c$ will be the side opposite angle $C$.

**Hypotenuses, adjacent, and opposite:**
- The longest side of the triangle is the hypotenuses (the side opposite the $90^\circ$ angle).
- “Opposite” and “adjacent” refer to sides that are opposite or adjacent to the two acute angles ($< A$ and $< B$) of the triangle.
- Adjacent side: the side next to the acute angle.
- Opposite side: the side opposite the acute angle.
  - (An acute angle $< 90^\circ$.)

**Example:** Fill in the blanks in each of the following

1) Side $EG$ (or $f$) is _________ angle $F$.  opposite
2) Side $FG$ (or $e$) is _________ angle $F$.  adjacent
3) Side $EF$ (or $g$) is the _________.    hypotenuse
4) Side $EG$ (or $f$) is _________ angle $E$.  adjacent
5) Side $FG$ (or $e$) is _________ angle $E$.  opposite
6) Side $EG$ is opposite to angle ______.  $F$
Trigonometric Functions (of right triangles):

- There are six trigonometric functions (or ratios): sine (sin), cosine (cos), tangent (tan), secant (sec), cosecant (csc), and cotangent (cot).
- The lengths of the sides are used to define the trigonometric functions (or ratios).

Sine, cosine, and tangent (three main trigonometric functions):

- The sine of the angle \( \theta \) = \( \frac{\text{the length of the opposite side}}{\text{the length of the hypotenuse}} \)

\[
\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}
\]

- The cosine of the angle \( \theta \) = \( \frac{\text{the length of the adjacent side}}{\text{the length of the hypotenuse}} \)

\[
\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}
\]

- The tangent of the angle \( \theta \) = \( \frac{\text{the length of the opposite side}}{\text{the length of the adjacent}} \)

\[
\tan \theta = \frac{\text{opposite side}}{\text{adjacent}}
\]

Secant, cosecant, and cotangent: the inverse trigonometric functions.

- Secant is the inverse of cosine: \( \sec \theta = \frac{1}{\cos \theta} \)

- Cosecant is the inverse of sine: \( \csc \theta = \frac{1}{\sin \theta} \)

- Cotangent is the inverse of tangent: \( \cot \theta = \frac{1}{\tan \theta} \)

Six trigonometric functions:

<table>
<thead>
<tr>
<th>Trigonometric function</th>
<th>Read</th>
<th>Diagram</th>
<th>Memory aid</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin \theta )</td>
<td>Sine of ( A )</td>
<td><img src="image-url" alt="Diagram" /></td>
<td>Soh</td>
</tr>
<tr>
<td>( \cos \theta )</td>
<td>Cosine of ( A )</td>
<td></td>
<td>Cah</td>
</tr>
<tr>
<td>( \tan \theta )</td>
<td>Tangent of ( A )</td>
<td><img src="image-url" alt="Diagram" /></td>
<td>Toa</td>
</tr>
<tr>
<td>( \sec \theta )</td>
<td>Secant of ( A )</td>
<td><img src="image-url" alt="Diagram" /></td>
<td>Inverse of cosine</td>
</tr>
<tr>
<td>( \csc \theta )</td>
<td>Cosecant of ( A )</td>
<td><img src="image-url" alt="Diagram" /></td>
<td>Inverse of sine</td>
</tr>
<tr>
<td>( \cot \theta )</td>
<td>Cotangent of ( A )</td>
<td><img src="image-url" alt="Diagram" /></td>
<td>Inverse of tangent</td>
</tr>
</tbody>
</table>
**Example:** Find the sine, cosine, and tangent for each of the following.

The sine of one angle in the right triangle is equal to the cosine of the other angle in that same right triangle.

**Example:** Find the sine, cosine, and tangent for each of the following.

**Memory Aid:**

<table>
<thead>
<tr>
<th>Sine, cosine, and tangent</th>
<th>Trigonometric function</th>
<th>Memory aid</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td>sin $\theta = \frac{\text{OPP}}{\text{HYP}}$</td>
<td>Soh</td>
<td><img src="#" alt="Sine Diagram" /></td>
</tr>
<tr>
<td>Cosine</td>
<td>cos $\theta = \frac{\text{ADJ}}{\text{HYP}}$</td>
<td>Cah</td>
<td><img src="#" alt="Cosine Diagram" /></td>
</tr>
<tr>
<td>Tangent</td>
<td>tan $\theta = \frac{\text{OPP}}{\text{ADJ}}$</td>
<td>Toa</td>
<td><img src="#" alt="Tangent Diagram" /></td>
</tr>
</tbody>
</table>
Find the trigonometric functions of an angle:

**Example:** Find each of the following using a scientific calculator.

1) \( \sin 132^\circ = ? \)

Type in: \( \text{sin} \ 132 \)

Display: 0.7431…

\( \sin 132^\circ \approx 0.7431 \)

Or 132 \( \text{sin} \) with some calculators.

2) \( \cos 25^\circ = ? \)

Type in: \( \text{cos} \ 25 \)

Display: 0.9063 …

\( \cos 25^\circ \approx 0.9063 \)

Or 25 \( \text{cos} \) with some calculators.

3) \( \tan 48^\circ = ? \)

Type in: \( \text{tan} \ 48 \)

Display: 1.11061…

\( \tan 48^\circ \approx 1.1106 \)

Or 48 \( \text{tan} \) with some calculators.

Find an angle when given the trigonometric function (ratio):

**Example:** Find each of the following using a scientific calculator.

1) \( \sin A = 0.5446, \quad < A = ? \)

Type in: \( \text{2ndF} \ \text{sin}^{-1} \ 0.5446 \)

Display: 32.997333… \quad < A \approx 33^\circ \)

Or \( \text{INV} \) with some calculators.

2) \( \tan B = 0.57736, \quad < B = ? \)

Type in: \( \text{2ndF} \ \text{tan}^{-1} \ 0.57736 \)

Display: 30.000418… \quad < B \approx 30^\circ \)

Or \( \text{INV} \) with some calculators.
Solving Triangles

Angles in a triangle: the sum of the three internal angles in a triangle is always $180^\circ$.

$\angle A + \angle B + \angle C = 180^\circ$

Pythagorean theorem review: a relationship between the three sides of a right triangle.

$$c = \sqrt{a^2 + b^2}$$

There are six elements (or parts) in a triangle, that is, three sides and three internal angles.

Solving a triangle: to solve a triangle means to know all three sides and all three angles.

Example: 1) Solve for the variable.

$$\tan 32^\circ = \frac{7}{x}, \quad x = ?$$

$$x \cdot \tan 32^\circ = \frac{7}{x} \cdot x$$

$$x \cdot \tan 32^\circ = \frac{7}{\tan 32^\circ}$$

$$x = \frac{7}{\tan 32^\circ} \approx 11.2$$

2) Find side $c$ if $b = 10$ m and $\angle B = 36^\circ$.

$$\sin 36^\circ = \frac{10\text{m}}{c}$$

$$\sin 36^\circ \cdot c = \frac{10\text{m}}{\sin 36^\circ} \cdot c$$

$$\frac{\sin 36^\circ}{\sin 36^\circ} \cdot c = \frac{10\text{m}}{\sin 36^\circ}$$

$$c = \frac{10\text{m}}{\sin 36^\circ} \approx 17.01 \text{ m}$$

Example: Solve the triangle ($\angle A = ? \quad b = ? \quad c = ?$).

- $\angle A = 180^\circ - (\angle C + \angle B)$
  $$= 180^\circ - (90^\circ + 37.4^\circ)$$
  $$= 52.6^\circ$$

- $\tan B = \frac{b}{a}$

  $$b = a (\tan B) = 10 (\tan 37.4^\circ) \approx 7.65 \text{ mm}$$

- $c = \sqrt{a^2 + b^2} = \sqrt{10^2 + 7.65^2} \approx 12.59 \text{ mm}$

Pythagorean theorem.
Example: Find the missing part of each triangle.

1) \[ \cos 50^\circ = \frac{b}{3} \]
\[ 3 \cdot \cos 50^\circ = \frac{b}{3} \cdot 3 \]
\[ 3 \left( \cos 50^\circ \right) = b \]
\[ b = 3 \left( \cos 50^\circ \right) \approx 1.928 \text{ m} \]

2) \[ \sin A = \frac{2}{5.1} \]
\[ < A = \sin^{-1} \left( \frac{2}{5.1} \right) \approx 23.1^\circ \]

Example: Solve the right triangle.

1) \(< B: < B = 180^\circ - (< C + < A)\]
\[ = 180^\circ - (90^\circ + 35^\circ) = 55^\circ \]
\[ b: \tan 35^\circ = \frac{2}{b} \]
\[ b \cdot \tan 35^\circ = \frac{2 \cdot b}{b} \]
\[ b \tan 35^\circ = \frac{2}{\tan 35^\circ} \]
\[ b = \frac{2}{\tan 35^\circ} \approx 2.856 \text{ m} \]
\[ c: c = \sqrt{a^2 + b^2} = \sqrt{2^2 + 2.856^2} \approx 3.487 \text{ m} \]

2) \[ a: a = \sqrt{c^2 - b^2} = \sqrt{4^2 - 2^2} \approx 3.464 \text{ m} \]
\[ < A: \cos A = \frac{2}{4} = 0.5 \]
\[ < A = \cos^{-1} A = \cos^{-1} 0.5 = 60^\circ \]
\[ < B: < B = 180^\circ - (90^\circ + 60^\circ) = 30^\circ \]
Check: \[ < A + < B + < C = 180^\circ, \quad 60^\circ + 30^\circ + 90^\circ = 180^\circ \] Correct!
Angles of Depression and Elevation

**Angle of depression:** the angle between a horizontal line and the line of sight for an object below the horizontal. The word "depression" means "fall" or "drop".

**Angle of elevation:** the angle between a horizontal line and the line of sight for an object above the horizontal. The word "elevation" means "rise" or "move up".

**Example:** 1) Find the angle of elevation.

\[
\tan B = \frac{3}{2} = 1.5
\]

\[
< B = \tan^{-1} B = \tan^{-1} \frac{3}{2} \approx 56.3^\circ
\]

2) Find \(y\) if the angle of depression is \(36^\circ\).

\[
\sin 36^\circ = \frac{5}{y}
\]

\[
y = \frac{5}{\sin 36^\circ} \approx 8.507 \text{ mm}
\]

(Divide both sides by \(\sin 36^\circ\) and multiply both sides by \(y\).)

**Example:** From the top of a rock wall, the angle of depression to a swimmer is \(56^\circ\). If the wall is 20m high, how far from the base of the wall is the swimmer?

\[
90^\circ - 56^\circ = 34^\circ
\]

\[
\tan 34^\circ = \frac{x}{20}
\]

\[
x = 20 \tan 34^\circ \approx 13.49 \text{ m}
\]

(Multiply both sides by 20 and reverse the sides of the equation.)

**Example:** Mike has let 25 m of string out on his kite. He is flying it 11.5 m above his eye level. Find the angle of elevation of the kite.

\[
\sin \theta = \frac{11.5}{25} \approx 0.46
\]

\[
\theta = \cos^{-1} 0.46 \approx 62.6^\circ
\]
Applications of Trigonometry

Example: When Brandon stands 37 m from the base of a building and sights the top of the building, he is looking up at an angle of 43°. How high is the building?

\[
\tan 43^0 = \frac{x}{37}
\]

(37) tan 43° = \frac{x}{37} \cdot 37

Multiply both sides by 37.

\[x = (37) \tan 43^0 \approx 34.5 \text{ m}\]

The building is approximately 34.5 m high.

Example: Tom tries to swim straight across a river. He can swim at 1.6 m/sec, but the river is flowing at 1.2 m/sec. At what angle to his intended direction will Tom actually travel?

\[
\tan \theta = \frac{1.2}{1.6} = 0.75
\]

\[\theta = \tan^{-1} 0.75 \approx 36.87^0\]

Tom will travel about 36.87° off course.

Example: An equilateral triangle has a height of 12 mm. Find the length of each side.

\[
\sin 60^0 = \frac{12}{x}
\]

Each angle = 60° (an equilateral triangle.)

\[x = \frac{12}{\sin 60^0} \approx 13.86 \text{ mm}\]

(Multiply both sides by x and divide both sides by sin 60°.)

The length of each side is about 13.86 mm.
An angle can vary from 0 to 360 degrees (360°).

### Classifying angles:

<table>
<thead>
<tr>
<th>Angle</th>
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<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight angle</td>
<td>An angle of exactly 180°.</td>
<td><img src="image" alt="180° angle" /></td>
</tr>
<tr>
<td>Right angle</td>
<td>An angle of exactly 90°.</td>
<td><img src="image" alt="90° angle" /></td>
</tr>
<tr>
<td>Acute angle</td>
<td>An angle between 0 and 90°.</td>
<td><img src="image" alt="A &lt; 90°" /></td>
</tr>
<tr>
<td>Obtuse angle</td>
<td>An angle between 90 and 180°.</td>
<td><img src="image" alt="90° &lt; A &lt; 180°" /></td>
</tr>
<tr>
<td>Reflex angle</td>
<td>An angle between 180 and 360°.</td>
<td><img src="image" alt="180° &lt; A &lt; 360°" /></td>
</tr>
<tr>
<td>Complementary angles</td>
<td>Two angles whose sum is exactly 90°.</td>
<td><img src="image" alt="&lt; A + &lt; B = 90°" /></td>
</tr>
<tr>
<td>Supplementary angles</td>
<td>Two angles whose sum is exactly 180°.</td>
<td><img src="image" alt="&lt;A + &lt;B = 180°" /></td>
</tr>
<tr>
<td>Vertical angles</td>
<td>Two angles formed by the intersection of two straight lines.</td>
<td><img src="image" alt="A and &lt; B are vertical angles." /></td>
</tr>
</tbody>
</table>

### Classify triangles:

<table>
<thead>
<tr>
<th>Name of triangle</th>
<th>Definition</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral triangle</td>
<td>A triangle that has three equal sides and three equal angles. ( a = b = c ), (&lt; A = &lt; B = &lt; C = 60° )</td>
<td><img src="image" alt="Equilateral triangle" /></td>
</tr>
<tr>
<td>Isosceles triangle</td>
<td>A triangle that has two equal sides and two equal angles. ( a = b ), (&lt; A = &lt; B )</td>
<td><img src="image" alt="Isosceles triangle" /></td>
</tr>
<tr>
<td>Acute triangle</td>
<td>A triangle that has three acute angles (&lt; 90°).</td>
<td><img src="image" alt="Acute triangle" /></td>
</tr>
<tr>
<td>Right triangle</td>
<td>A triangle that has a right angle (= 90°).</td>
<td><img src="image" alt="Right triangle" /></td>
</tr>
<tr>
<td>Obtuse triangle</td>
<td>A triangle that has an obtuse angle (&gt; 90°).</td>
<td><img src="image" alt="Obtuse triangle" /></td>
</tr>
<tr>
<td>Scalene triangle</td>
<td>A triangle that has three unequal sides.</td>
<td><img src="image" alt="Scalene triangle" /></td>
</tr>
</tbody>
</table>

**Angles in a triangle:** the sum of the three angles in a triangle is always 180°.

\[ < X + < Y + < Z = 180° \]
How to use a protractor:
- Place the protractor so that the center hole is over the angle’s vertex.
- Line up the base line of the protractor with one of the sides of the angle.
- Read the angle over the second side of the angle.

Sides and angles:
- Sides are labeled with lower case letters (or two capital letters).
- Angles are labeled with uppercase letters.
- Side $a$ will be the side opposite angle $A$; side $b$ will be the side opposite angle $B$; and side $c$ will be the side opposite angle $C$.

Hypotenuses, adjacent, and opposite:
- The longest side of the triangle is the hypotenuses (the side opposite the 90° angle).
- “Opposite” and “adjacent” refer to sides that are opposite or adjacent to the two acute angles ($< A$ and $< B$) of the triangle.
- Adjacent side: the side next to the acute angle.
- Opposite side: the side opposite the acute angle.

Six trigonometric functions:

<table>
<thead>
<tr>
<th>Trigonometric function</th>
<th>Diagram</th>
<th>Memory aid</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin $\theta = \frac{\text{opposite}}{\text{hypotenuse}}$</td>
<td>[Diagram]</td>
<td>Soh</td>
</tr>
<tr>
<td>cos $\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$</td>
<td>[Diagram]</td>
<td>Cah</td>
</tr>
<tr>
<td>tan $\theta = \frac{\text{opposite}}{\text{adjacent side}}$</td>
<td>[Diagram]</td>
<td>Toa</td>
</tr>
<tr>
<td>csc $\theta = \frac{1}{\sin A}$</td>
<td>[Diagram]</td>
<td>Inverse of sine</td>
</tr>
<tr>
<td>sec $\theta = \frac{1}{\cos A}$</td>
<td>[Diagram]</td>
<td>Inverse of cosine</td>
</tr>
<tr>
<td>cot $\theta = \frac{1}{\tan A}$</td>
<td>[Diagram]</td>
<td>Inverse of tangent</td>
</tr>
</tbody>
</table>

Pythagorean theorem review: a relationship between the three sides of a right triangle.

$$c = \sqrt{a^2 + b^2}$$

Solving a triangle: to solve a triangle means to know all three sides and all three angles.

Angle of depression: the angle between a horizontal line and the line of sight for an object below the horizontal.

Angle of elevation: the angle between a horizontal line and the line of sight for an object above the horizontal.
Topic A

1. Label each of the following angles.

   a) 
   
   b) 
   
   c) 
   
   d) 

2. What is the complementary angle to 42 degrees?

3. What is the supplementary angle to 146 degrees?

4. What is the size of the angle $x$?

5. a) Two angles $A$ and $33^0$ that add together to measure $180^0$ are said to be _____?

   b) What is the size of angle $A$ and $B$?

6. What is the size of angle $C$ in the following figure?

7. What is the size of angle $C$, $D$ and the side $b$ in the following figure?

8. Match the following triangles to the letter with the best definition.

   a) Equilateral triangle i. has two equal sides
   b) Isosceles triangle ii. has three unequal sides
   c) Supplementary angles iii. Two angles whose sum is exactly $180^0$.
   d) Scalene triangle iv. has three equal sides

9. Find the missing measurement and then name the kind of triangle.

   a) 
   
   $x =$ ?

   $r =$ ?
b) $a = ?$

\[
\begin{array}{c}
\text{43 ft} \\
39^\circ \\
39^\circ
\end{array}
\]

\[
B = ?
\]

c) $C = ?$

\[
\begin{array}{c}
\text{21 m} \\
124^\circ \\
21 m
\end{array}
\]

\[
B = ?
\]

d) $x = ?$

\[
\begin{array}{c}
\text{32 cm} \\
54^\circ \\
54^\circ
\end{array}
\]

**Topic B**

10. Fill in the blanks in each of the following

a) Side $ZY$ (or $x$) is _______ angle $X$.
b) Side $XZ$ (or $y$) is _______ angle $X$.
c) Side $XY$ (or $z$) is the _______.
d) Side $ZY$ (or $x$) is _______ angle $Y$.
e) Side $XZ$ (or $y$) is _______ angle $Y$.
f) Side $XZ$ (or $y$) is opposite to angle ______.

11. Find the sine, cosine, and tangent of each acute angle.

12. Find the sine, cosine, and tangent of each acute angle.

**Topic C**

13. Use a calculator to find the trigonometric value of each angle.

a) $\sin 57^\circ = ?$
b) $\cos 36^\circ = ?$
c) $\tan 87^\circ = ?$
d) $\sin ( ) = 0.2165$
e) $\cos ( ) = 0.4567$
f) $\tan ( ) = 1.2356$
14. Solve for the variable.

\[ \tan 57^\circ = \frac{12}{x}, \quad x = ? \]

15. Find side c if \( b = 24 \) cm and \( \angle B = 41^\circ \).

16. Solve the triangle \( \angle A = ?, \quad b = ?, \quad c = ? \).

17. Find the missing part of each triangle.
   a) 
   \[ \triangle ABC \] with angles \( 46^\circ \), \( 9 \) cm, \( 7 \) cm, \( b \) ?
   b) 
   \[ \triangle ABC \] with sides \( 7.8 \) m, \( 49^\circ \), \( 3.6 \) m, \( 6.92 \) m, \( a \) ?, \( b \) ?, \( c \) ?

18. Solve the right triangle. \( \angle B = ?, \quad b = ?, \quad c = ? \)
   a) 
   \[ \triangle ABC \] with \( \angle A = 45^\circ \), \( b \), \( c \), \( 6 \) m, \( B \)
   b) \( \angle A = ?, \quad \angle B = ?, \quad \angle B = ? \)
   \[ \triangle ABC \] with sides \( 5 \) ft, \( 3 \) ft, \( a \), \( b \), \( c \)

19. a) Find the angle of elevation.
   b) Find \( y \) if the angle of depression is \( 32^\circ \).

20. From the top of a wall, the angle of depression to a boy is \( 43^\circ \). If the wall is \( 24 \) m high, how far from the base of the wall is the boy?

21. Todd has let \( 34 \) m of string out on his kite. He is flying it \( 22.4 \) m above his eye level. Find the angle of elevation of the kite. \( \frac{11}{36^\circ} \)

22. When Ann stands \( 28 \) m from the base of a building and sights the top of the building, she is looking up at an angle of \( 39^\circ \). How high is the building?

23. Damon tries to swim straight across a river. He can paddle at \( 1.3 \) m/sec, but the river is flowing at \( 1.5 \) m/sec. At what angle to his intended direction will Damon actually travel?

24. An equilateral triangle has a height of \( 41 \) cm. Find the length of each side.
Unit 11

Exponents, Roots and Scientific Notation

Topic A: Exponents
- Basic exponent properties review
- Degree of a polynomial

Topic B: Properties of exponents
- Properties of exponents
- Properties of exponents – examples
- Simplifying exponential expressions

Topic C: Scientific notation and square roots
- Scientific notation
- Square roots
- Simplifying square roots

Unit 11: Summary

Unit 11: Self-test
Topic A: Exponents

Basic Exponent Properties Review

Exponent review: \(a^n\) or \(\text{Base}^{\text{Exponent}}\)

<table>
<thead>
<tr>
<th>Exponential notation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a^n = a \cdot a \cdot a \ldots a)</td>
<td>(2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16) Read “2 to the 4th.”</td>
</tr>
</tbody>
</table>

Zero Exponent \(a^0 = 1\) (0\(^0\) is undefined)

One Exponent \(a^1 = a\)

\(1^n = 1\)

Example: Write the following exponential expressions in expanded form.

<table>
<thead>
<tr>
<th>Exponential expressions</th>
<th>Expanded form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) (4^3)</td>
<td>(4 \cdot 4 \cdot 4)</td>
</tr>
<tr>
<td>2) ((-u)^3)</td>
<td>((-u) (-u) (-u))</td>
</tr>
<tr>
<td>3) (-u^3)</td>
<td>(-u \cdot u \cdot u)</td>
</tr>
<tr>
<td>4) ((2x^3y^0)^2)</td>
<td>((2x^3)^0 (2x^3)^0)</td>
</tr>
<tr>
<td>5) ((\frac{-5}{7}w)^3)</td>
<td>((\frac{-5}{7}w) \cdot (\frac{-5}{7}w) \cdot (\frac{-5}{7}w))</td>
</tr>
</tbody>
</table>

Example: Write each of the following in exponential form.

<table>
<thead>
<tr>
<th>Expanded form</th>
<th>Exponential notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ((0.3)(0.3)(0.3))</td>
<td>((0.3)^3)</td>
</tr>
<tr>
<td>2) ((4t)(4t)(4t)(4t))</td>
<td>((4t)^4)</td>
</tr>
<tr>
<td>3) ((3x)(2y)(x)(2y))</td>
<td>(12x^2y^2)</td>
</tr>
</tbody>
</table>

Example: Evaluate.

1) \(2x^3 + y\), for \(x = 2\), \(y = 3\)

\[2x^3 + y = 2 \cdot 2^3 + 3\]
\[= 2 \cdot 8 + 3 = 19\]

2) \((2a)^4 - b\), for \(a = 1\), \(b = 4\)

\[(2a)^4 - b = (2 \cdot 1)^4 - 4\]
\[= 2^4 - 4 = 12\]
The degree of a term with one variable: the exponent of its variable.

Example: \(9x^3\) degree: 3
\(-7u^5\) degree: 5
\(2a\) degree: 1

The degree of a term with more variables: the sum of the exponents of its variables.

Example: \(-8x^2y^4z^3\) degree: 2 + 4 + 3 = 9

The degree of a polynomial with more variables: the highest degree of any individual term.

Example: \(9t^2u + 4t^3u^2v^5 - 6t + 5\) degree: 10

Examples of degree of a polynomial:

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>(5x^3 - x^2 + 21)</th>
<th>(2x^2y - 5z + 7x^4y^2z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term</td>
<td>(5x^3), (-x^2), 21</td>
<td>(2x^2y), (-5z), (7x^4y^2z)</td>
</tr>
<tr>
<td>Degree of the term</td>
<td>3, 2, 0</td>
<td>3, 1, 7</td>
</tr>
<tr>
<td>Degree of the polynomial</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Example: What is the degree of the following term / polynomial?

1) \(3xy^3\) degree: 4
2) \(2bc^3d^5 + 5e^2 - fg^2 + 2e^0\) degree: 9

Descending order: the exponent of a variable decreases for each succeeding term.

Example: \(9x^4 - 7x^3 + x^2 - x + 2\) \(a^1 = a\)
\(21uv^3 - uv^2 + 4v - 67\) The descending order of exponent \(v\).

Ascending order: the exponent of a variable increases for each succeeding term.

Example: \(13 - 8a + 34a^2 - 12a^3\) \(a^1 = a\)
\(-7 + \frac{3}{5}wz + 3.5w^2z^2 - 5z^3 + z^4\) The ascending order of power \(z\).
**Topic B: Properties of Exponents**

### Properties of Exponents:

<table>
<thead>
<tr>
<th>Name</th>
<th>Rule</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Product rule</strong></td>
<td>( a^m a^n = a^{m+n} )</td>
<td>( 2^3 2^2 = 2^5 = 32 )</td>
</tr>
<tr>
<td><strong>Quotient rule</strong></td>
<td>( \frac{a^m}{a^n} = a^{m-n} )</td>
<td>( \frac{y^4}{y^2} = y^{4-2} = y^2 )</td>
</tr>
<tr>
<td><strong>Power of a power</strong></td>
<td>( (a^m)^n = a^{mn} )</td>
<td>( (x^3)^2 = x^6 )</td>
</tr>
<tr>
<td><strong>Power of a product</strong></td>
<td>( (a \cdot b)^n = a^n b^n )</td>
<td>( (2 \cdot 3)^2 = 2^2 3^2 = 4 \cdot 9 = 36 )</td>
</tr>
<tr>
<td><strong>Power of a quotient</strong></td>
<td>( \frac{(a^m b^n)^p}{b^{np}} = a^{mp} b^{np} )</td>
<td>( \frac{(3 \cdot s^4)^2}{s^4} = 3^2 4^2 = 9 16 )</td>
</tr>
<tr>
<td><strong>Negative exponent</strong></td>
<td>( a^{-n} = \frac{1}{a^n} )</td>
<td>( 4^{-2} = \frac{1}{4^2} = \frac{1}{16} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{a^{-n}} = a^n )</td>
<td>( \frac{1}{4^{-2}} = 4^2 = 16 )</td>
</tr>
<tr>
<td><strong>Zero exponent</strong></td>
<td>( a^0 = 1 )</td>
<td>( 15^0 = 1 )</td>
</tr>
<tr>
<td><strong>One exponent</strong></td>
<td>( a^1 = a )</td>
<td>( 7^1 = 7 ) , ( 1^{13} = 1 )</td>
</tr>
</tbody>
</table>

### Properties of exponents explained:

- **Product rule** (multiplying the same base): when multiplying two powers with the same base, keep the base and add the exponents. \( a^m a^n = a^{m+n} \)

  Example: \( 2^3 2^2 = (2 \cdot 2 \cdot 2) (2 \cdot 2) = 2^5 = 32 \)

  Or \( 2^3 2^2 = 2^5 = 32 \) A short cut, \( a^m a^n = a^{m+n} \)

- **Quotient rule** (dividing the same base): when dividing two powers with the same base, keep the base and subtract the exponents. \( \frac{a^m}{a^n} = a^{m-n} \)

  Example: \( \frac{2^4}{2^2} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = 2^2 = 4 \)

  Or \( \frac{2^4}{2^2} = 2^2 = 4 \) A short cut, \( \frac{a^m}{a^n} = a^{m-n} \)

This law can also show that why \( a^0 = 1 \) (zero exponent \( a^0 \)): \( \frac{a^2}{a^2} = a^{2-2} = a^0 = 1 \)
- **Power rule:**
  - Power of a power: when raise a power to a power, just multiply the exponents.
    \[ (a^m)^n = a^{mn} \]
    
    **Example:**
    \[ (4^3)^2 = (4^3)^2 = (4 \cdot 4 \cdot 4)^2 = 64^2 = 4096 \]
    
    **Or**
    \[ (4^3)^2 = 4^3 \cdot 2 = 4^6 \cdot 4096 \]
    A short cut, \((a^m)^n = a^{mn}\)

  - Power of a product: when raise a power to different bases, distribute the exponent to each base.
    \[ (a \cdot b)^n = a^n b^n \]
    
    **Example:**
    \[ (2 \cdot 3)^2 = (2 \cdot 3)^2 = 6 \cdot 6 = 36 \]
    
    **Or**
    \[ (2 \cdot 3)^2 = 2^2 \cdot 3^2 = 4 \cdot 9 = 36 \]
    A short cut, \((a \cdot b)^n = a^n b^n\)

- Power of a product (different bases): when raise a power to a power with different bases, multiply each exponent inside the parentheses by the power outside the parentheses.
  \[ (a^m \cdot b^n)^p = a^{mp} b^{np} \]
  
  **Example:**
  \[ (2^2 \cdot 3^2)^2 = (2^2 \cdot 3^2)^2 = (2^2 \cdot 2^2) (3^2 \cdot 3^2) = 16 \cdot 81 = 1296 \]
  
  **Or**
  \[ (2^2 \cdot 3^2)^2 = 2^2 \cdot 2^2 \cdot 3^2 \cdot 3^2 = 2^4 \cdot 3^4 = 16 \cdot 81 = 1296 \]
  A short cut, \((a \cdot b)^n = a^n b^n\)

- **Power of a quotient** (different bases):
  - When raise a fraction to a power, distribute the exponent to the numerator and denominator of the fraction.
    \[ \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \]
    
    **Example:**
    \[ \left( \frac{2}{3} \right)^3 = \left( \frac{2}{3} \right)^3 \cdot \left( \frac{2}{3} \right)^3 \cdot \left( \frac{2}{3} \right)^3 = \frac{2^3}{3^3} = \frac{8}{27} \]
    
    **Or**
    \[ \left( \frac{2}{3} \right)^3 = \frac{2^3}{3^3} = \frac{8}{27} \]
    A short cut, \(\left( \frac{a}{b} \right)^n = \frac{a^n}{b^n}\)

  - When raise a fraction with powers to a power, multiply each exponent in the numerator and denominator by the power outside the parentheses.
    \[ \left( \frac{a^m}{b^n} \right)^p = \frac{a^{mp}}{b^{np}} \]
    
    **Example:**
    \[ \left( \frac{2^2}{3^3} \right)^3 = \left( \frac{2^2}{3^3} \right)^3 \cdot \left( \frac{2^2}{3^3} \right)^3 \cdot \left( \frac{2^2}{3^3} \right)^3 = \frac{2^6}{3^9} = \frac{64}{531,441} \]
    
    **Or**
    \[ \left( \frac{2^2}{3^3} \right)^3 = \frac{2^6}{3^9} = \frac{64}{531,441} \]
    A short cut, \(\left( \frac{a^m}{b^n} \right)^p = \frac{a^{mp}}{b^{np}}\)

- **Negative exponent:** a negative exponent is the reciprocal of the number with a positive exponent.
  \[ a^{-n} = \frac{1}{a^n} \]
  \[ \frac{1}{a^{-n}} = a^n \]
  \[ a^{-n} \] is the reciprocal of \(a^n\).

  **Example:**
  \[ 3^{-4} = \frac{1}{3^4} = \frac{1}{81} \]
  
  **Example:**
  \[ \frac{1}{3^{-4}} = 3^4 = 81 \]
  \[ \frac{1}{a^{-n}} = a^n \]
Properties of Exponents – Examples

Example: Simplify (do not leave negative exponents in the answer).

1) \((-4)^1 = -4\)  \(a^1 = a\)
2) \((-2345)^0 = 1\)  \(a^0 = 1\)
3) \((-0.3)^3 = -0.027\)  \(a^n = a \cdot a \cdot a \ldots\)
4) \(-5^2 = -(5^2) = -25\)
5) \(x^2 \cdot x^3 = x^{2+3} = x^5\)  \(a^m \cdot a^n = a^{m+n}\)
6) \(\frac{y^6}{y^4} = y^{6-4} = y^2\)  \(\frac{a^m}{a^n} = a^{m-n}\)
7) \((x^4)^{-3} = x^{4 \cdot (-3)} = x^{-12} = \frac{1}{x^{12}}\)  \((a^m)^n = a^{m \cdot n}, \ \frac{1}{a^{-n}} = a^n\)
8) \(7b^{-1} = 7 \cdot \frac{1}{b} = \frac{7}{b}\)  \(a^{-n} = \frac{1}{a^n} \quad a^1 = a\)
9) \([(-4) \cdot (0.7)]^2 = (-4)^2 \cdot 0.7^2 = (16)(0.49) = 7.84\)  \((a \cdot b)^n = a^n b^n\)
10) \((2t^3 \cdot w^2)^4 = 2^4 t^{3 \cdot 4} \cdot w^{2 \cdot 4} = 16 t^{12} w^8\)  \((a^m \cdot b^n)^p = a^{mp} b^{np}\)
11) \(\frac{1}{3} - 2 = 3^2 = 9\)  \(\frac{1}{a^{-n}} = a^n\)
12) \(\frac{u^4}{z^2} = \frac{u^2}{z^2} = \frac{2}{x^2}\)  \(\frac{a^m}{b^n} = \frac{a^n}{b^m}, \ a^{-n} = \frac{1}{a^n}, \ \frac{1}{a^n} = a^n\)
13) \(\frac{(x^4)^2}{y^{-3} \cdot (2)} = \frac{x^8}{y^{-6}} = x^8 y^6\)  \((\frac{a^m}{b^n})^p = \frac{a^{mp}}{b^{np}}, \ \frac{1}{a^{-n}} = a^n\)
14) \((2^{-3})^3 = \frac{1}{2^3} \cdot 2^3 = 1\)  \(\frac{1}{a^{-n}} = a^n\)
15) \(\frac{7x^4 y^{-5}}{9y^2} = \frac{7x^4 \cdot y^{-5-3}}{1} = \frac{7x^4 \cdot y^{-8}}{y^3} = \frac{7x^4}{y^3}\)  \(a^0 = 1, \ a^m = a^{m-n}, \ a^{-n} = \frac{1}{a^n}\)
16) \(\left(\frac{e^{-3} f^2}{g^{-2}}\right)^2 = \frac{e^{-3} \cdot f^{2 \cdot (-2)}}{g^{(-2) \cdot (-2)}} = \frac{e^{-6} f^{-4}}{g^4} = \frac{e^{-6}}{g^4 f^4}\)  \((\frac{a^m}{b^n})^p = \frac{a^{mp}}{b^{np}}, \ \frac{1}{a^{-n}} = a^n\)

Using a calculator:  
4^2 = ?  \[4^2 = 16\]  (The display reads 16)  
3^4 = ?  \[3^4 = 81\]  (The display reads 81)  
(Or \[3^4\] or \[4^2\] on some calculators.)
Simplifying Exponential Expressions

Steps for simplifying exponential expressions:

- Remove parentheses using “power rule” if necessary. \((a^m \cdot b^n)^p = a^{mp} b^{np}\)
- Regroup coefficients and variables.
- Use “product rule” and “quotient rule”.
- Simplify.
- Use “negative exponent” rule to make all exponents positive if necessary.

Example: Simplify.

1) \((3x^3y^2)^2(2x^{-3}y^{-1})^3(-248z^{-19})^0\)

Remove brackets. \((a^m b^n)^p = a^{mp} b^{np}\), \(a^0 = 1\)

Regroup coefficients and variables.

Simplify. \(a^m a^n = a^{m+n}\)

Make exponent positive. \(a^{-n} = \frac{1}{a^n}\), \(a^1 = a\)

\[
= 3^2 x^{3 \cdot 2} y^{2 \cdot 2} \cdot 2^3 x^{-3 \cdot 3} \cdot y^{-1 \cdot 3} \cdot 1
= (3^2 \cdot 2^3)(x^6 x^{-9})(y^4 y^{-3})
= 72 x^{-3} y^1
= \frac{72y}{x^3}
\]

2) \(\left(\frac{2x^4}{3x^3y^2}\right)^2 = \left(\frac{2x^4}{3x^3y^2}\right)^2\)

Remove brackets. \((a \cdot b)^n = a^n b^n\)

Regroup coefficients and variables.

Simplify. \(\frac{a^m}{a^n} = a^{m-n}\)

\[
= \frac{2^2 x^{4 \cdot 2} y^{5 \cdot 2}}{3^2 x^{3 \cdot 2} y^{2 \cdot 2}}
= \frac{4 \cdot x^8 \cdot y^{10}}{9 \cdot x^6 \cdot y^4}
= \frac{\frac{4}{9} x^2 y^6}{x^6 y^4}
\]

Example: Evaluate for \(a = 2\), \(b = 1\), \(c = -1\).

1) \((-29a^{-5}b^4c^{-7})^0 = 1\)

\(a^0 = 1\)

2) \(\left(\frac{a}{b}\right)^{-4} = \left(\frac{2}{1}\right)^{-4}\)

Substitute 2 for \(a\) and 1 for \(b\).

\[
= \frac{2^{-4}}{1^{-4}} = \frac{1^4}{2^4} = \frac{1}{16}
\]

3) \((a + b - c)^a = [2 + 1 - (-1)]^2 = 4^2 = 16\)

Substitute 2 for \(a\), 1 for \(b\), and -1 for \(c\).
### Scientific Notation and Square Roots

**Scientific notation** is a special way of concisely expressing very large and small numbers.

**Example:**
- $300,000,000 = 3 \times 10^8 \text{ m/sec}$ The speed of light.
- $0.0000000000000000016 = 1.6 \times 10^{-19} \text{ C}$ An electron.

**Scientific notation**: a product of a number between 1 and 10 and power of 10.

<table>
<thead>
<tr>
<th>Scientific notation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N \times 10^{±n}$</td>
<td>$67504.3 = 6.75043 \times 10^4$</td>
</tr>
</tbody>
</table>

**Scientific vs. non-scientific notation**:

<table>
<thead>
<tr>
<th>Scientific notation</th>
<th>Not scientific notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7.6 \times 10^3$</td>
<td>$76 \times 10^2$</td>
</tr>
<tr>
<td>$8.2 \times 10^{13}$</td>
<td>$0.82 \times 10^{14}$</td>
</tr>
<tr>
<td>$5.37 \times 10^7$</td>
<td>$53.7 \times 10^6$</td>
</tr>
</tbody>
</table>

**Writing a number in scientific notation**:

- **Step**
  - Move the decimal point *after* the *first nonzero digit*.
  - Determine $n$ (the power of 10) by counting the number of places you moved the decimal.
  - If the decimal point is moved to the *right*: $\times 10^n$
  - If the decimal point is moved to the *left*: $\times 10^n$

**Example**: Write in scientific notation.

1) $2340000 = 2.34 \times 10^6$
2) $0.000000439 = 4.39 \times 10^{-7}$

**Example**: Write in standard (or ordinary) form.

1) $64275 \times 10^4 = 64275$  
2) $2.9 \times 10^{-3} = 0.0029$

**Example**: Simplify and write in scientific notation.

1) $(4.9 \times 10^{-3})(3.82 \times 10^9) = (4.9 \times 3.82)(10^{-3+9}) = (18.718 \times 10^5)$  
   *Multiply coefficients of $10^n$, $a^m a^n = a^{m+n}$*  
   *18 > 10, this is not in scientific notation.*  
   *N = 1, this is in scientific notation.*

2) $\frac{5 \times 10^5(2.3 \times 10^{-2})}{4.5 \times 10^7} = \frac{5 \times 2.3}{4.5} \times \frac{(10^5 \times 10^{-2})}{10^7}$  
   *Regroup coefficients of $10^n$*  
   *$a^m a^n = a^{m+n}$, $a^m/a^n = a^{m-n}$*
Square Roots

Square root (\(\sqrt{\quad}\)): a number with the symbol \(\sqrt{\quad}\) that is the opposite of the square of a number, such as \(\sqrt{9} = 3\) and \(3^2 = 9\), respectively.

\[
\begin{array}{c|c}
\text{Square \((3^2)\)} & \text{Square root \((\sqrt{9})\)} \\
2 & 4 \\
\end{array}
\]

Perfect square: a number that is the exact square of a whole number.

<table>
<thead>
<tr>
<th>Perfect square</th>
<th>Square root</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1 \times 1 = 1^2 = 1)</td>
<td>(\sqrt{1} = 1)</td>
</tr>
<tr>
<td>(2 \times 2 = 2^2 = 4)</td>
<td>(\sqrt{4} = 2)</td>
</tr>
<tr>
<td>(3 \times 3 = 3^2 = 9)</td>
<td>(\sqrt{9} = 3)</td>
</tr>
<tr>
<td>(4 \times 4 = 4^2 = 16)</td>
<td>(\sqrt{16} = 4)</td>
</tr>
<tr>
<td>(5 \times 5 = 5^2 = 25)</td>
<td>(\sqrt{25} = 5)</td>
</tr>
<tr>
<td>(6 \times 6 = 6^2 = 36)</td>
<td>(\sqrt{36} = 6)</td>
</tr>
<tr>
<td>(7 \times 7 = 7^2 = 49)</td>
<td>(\sqrt{49} = 7)</td>
</tr>
<tr>
<td>(8 \times 8 = 8^2 = 64)</td>
<td>(\sqrt{64} = 8)</td>
</tr>
<tr>
<td>(9 \times 9 = 9^2 = 81)</td>
<td>(\sqrt{81} = 9)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Examples:

<table>
<thead>
<tr>
<th>Square root</th>
<th>Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{100} = 10)</td>
<td>(10^2 = 100)</td>
</tr>
<tr>
<td>(\sqrt{49} = 7)</td>
<td>(7^2 = 49)</td>
</tr>
<tr>
<td>(\sqrt{121} = 11)</td>
<td>(11^2 = 121)</td>
</tr>
<tr>
<td>(\sqrt{169} = 13)</td>
<td>(13^2 = 169)</td>
</tr>
<tr>
<td>(\frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5})</td>
<td>(4^2 = 16) (5^2 = 25)</td>
</tr>
</tbody>
</table>

Using a calculator: \(\sqrt{81} = ?\)

\[
\boxed{2nd \ 81 \ \sqrt{\quad} \ =} \quad \text{(The display reads 9)}
\]

Or \(\sqrt{\quad} \ 81 \ =\) for some calculators.

Example: Find the square roots.

1) \(\sqrt{144} = \sqrt{12^2} = 12\)

\[
\boxed{2nd \ 144 \ \sqrt{\quad} \ =}
\]

2) \(\frac{\sqrt{64}}{\sqrt{225}} = \frac{\sqrt{8^2}}{\sqrt{15^2}} = \frac{8}{15}\)

\[
\boxed{2nd \ 225 \ \sqrt{\quad} \ =}
\]
Simplifying Square Roots

Order of operations:

<table>
<thead>
<tr>
<th>Order of operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. the brackets or parentheses and absolute values (innermost first)</td>
</tr>
<tr>
<td>2. exponent or square root (from left-to-right)</td>
</tr>
<tr>
<td>3. multiplication or division (from left-to-right)</td>
</tr>
<tr>
<td>4. addition or subtraction (from left-to-right)</td>
</tr>
</tbody>
</table>

Memory aid - BEDMAS

<table>
<thead>
<tr>
<th>B</th>
<th>E (R)</th>
<th>D M</th>
<th>A S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brackets</td>
<td>Exponents or Square Root</td>
<td>Divide or Multiply</td>
<td>Add or Subtract</td>
</tr>
</tbody>
</table>

Example: Calculate.

1) $6 - 2\sqrt{81} = 6 - 2 \cdot 9$
   $= 6 - 18 = -12$

2) $3.2^2 - 3\sqrt{2} + 3^2 = 10.24 - 3 \sqrt{11}$
   $\approx 10.24 - 3 (3.32)$
   $= 10.24 - 9.96 = 0.28$

3) $\frac{\sqrt{64}}{\sqrt{250-249}} = \frac{8}{\sqrt{1}} = 8$

Simplifying square roots:

- Factor the number inside the square root sign.
  (Find the perfect square(s) that will divide the number).
- Rewrite the square root as a multiplication problem.
- Reduce the perfect squares ("pulling out" the integer(s)).

Example: Simplify.

1) $\sqrt{180} = \sqrt{45 \cdot 4} = \sqrt{9 \cdot 5 \cdot 4} = \sqrt{3^2 \cdot 5 \cdot 2^2} = 3 \cdot \sqrt{5} \cdot 2 = 6\sqrt{5}$

2) $\sqrt{92} = \sqrt{\frac{4 \cdot 23}{8}} = \frac{2\sqrt{23}}{8} = \frac{\sqrt{23}}{4}$
Unit 11: Summary

Exponents, Roots & Scientific Notation

The degree of a term with one variable: the exponent of its variable.
The degree of a term with more variables: the sum of the exponents of its variables.
The degree of a polynomial with more variables: the highest degree of any individual term.

Descending order: the exponent of a variable decreases for each succeeding term.
Ascending order: the exponent of a variable increases for each succeeding term.

Properties of exponents:

<table>
<thead>
<tr>
<th>Name</th>
<th>Rule</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product rule</td>
<td>(a^m \cdot a^n = a^{m+n})</td>
<td>(2^3 \cdot 2^2 = 2^5 = 32)</td>
</tr>
<tr>
<td>Quotient rule (the same base)</td>
<td>(\frac{a^m}{a^n} = a^{m-n}) ((a \neq 0))</td>
<td>(\frac{y^4}{y^2} = y^{4-2} = y^2)</td>
</tr>
<tr>
<td>Power of a power</td>
<td>((a^m)^n = a^{mn})</td>
<td>((x^3)^2 = x^{3 \cdot 2} = x^6)</td>
</tr>
<tr>
<td>Power of a product (different bases)</td>
<td>((a \cdot b)^n = a^n b^n)</td>
<td>((2 \cdot 3)^2 = 2^2 \cdot 3^2 = 4 \cdot 9 = 36)</td>
</tr>
<tr>
<td>Power of a quotient (different bases)</td>
<td>((\frac{a}{b})^n = \frac{a^n}{b^n}) ((b \neq 0))</td>
<td>((\frac{2}{3})^2 = \frac{2^2}{3^2} = \frac{4}{9})</td>
</tr>
<tr>
<td>((\frac{a}{b})^n = \frac{a^m}{b^n}) ((b \neq 0))</td>
<td>(\left(\frac{q^2}{p^3}\right)^3 = \frac{q^{2 \cdot 3}}{p^{3 \cdot 3}} = \frac{q^6}{p^9})</td>
<td></td>
</tr>
<tr>
<td>Negative exponent (a^{-n})</td>
<td>(a^{-n} = \frac{1}{a^n}) ((a \neq 0))</td>
<td>(4^{-2} = \frac{1}{4^2} = \frac{1}{16})</td>
</tr>
<tr>
<td>(\frac{1}{a^{-n}} = a^n) ((a \neq 0))</td>
<td>(\frac{1}{4^{-2}} = 4^2 = 16)</td>
<td></td>
</tr>
<tr>
<td>Zero exponent (a^0)</td>
<td>(a^0 = 1)</td>
<td>(15^0 = 1)</td>
</tr>
<tr>
<td>One exponent (a^1)</td>
<td>(a^1 = a) ((\text{But } 1^1 = 1))</td>
<td>(7^1 = 7), (1^{13} = 1)</td>
</tr>
</tbody>
</table>

Steps for simplifying exponential expressions:

- Remove parentheses using “power rule” if necessary.
  \((a^m \cdot b^n)^p = a^{mp} b^{np}\)
- Regroup coefficients and variables.
- Use “product rule” and “quotient rule”.
- Simplify.
- Use “negative exponent” rule to make all exponents positive if necessary.
Scientific notation: a product of a number between 1 and 10 and power of 10.

<table>
<thead>
<tr>
<th>Scientific notation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N \times 10^n$</td>
<td>$1 \leq N &lt; 10$</td>
</tr>
<tr>
<td>$n$ - integer</td>
<td>$n = 3$</td>
</tr>
<tr>
<td>$n$ - integer</td>
<td>$n = 7$</td>
</tr>
<tr>
<td>Standard form</td>
<td>$67504.3 = 6.75043 \times 10^4$</td>
</tr>
<tr>
<td>Scientific notation</td>
<td>$0.0079 = 7.9 \times 10^{-3}$</td>
</tr>
<tr>
<td>3 places to the right</td>
<td></td>
</tr>
<tr>
<td>Scientific notation</td>
<td>$37213000. = 3.7213 \times 10^7$</td>
</tr>
<tr>
<td>7 places to the left</td>
<td></td>
</tr>
</tbody>
</table>

Writing a number in scientific notation:

- Move the decimal point after the first nonzero digit. 0.0079 37213000.
- Determine $n$ (the power of 10) by counting the number of places you moved the decimal.
- If the decimal point is moved to the right: $\times 10^n$ 0.0079 = $7.9 \times 10^{-3}$ 3 places to the right
- If the decimal point is moved to the left: $\times 10^n$ 37213000. = $3.7213 \times 10^7$ 7 places to the left

Square root ($\sqrt{}$): a number with the symbol $\sqrt{}$ that is the opposite of the square of a number.

Perfect square: a number that is the exact square of a whole number.

Order of operations

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Memory aid - BEDMAS

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<th>E (R)</th>
<th>D</th>
<th>M</th>
<th>A</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brackets</td>
<td>Exponents or Square Root</td>
<td>Divide or Multiply</td>
<td>Add or Subtract</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Simplifying square roots

- Factor the number inside the square root sign. \( \sqrt{75} \)
  (Find the perfect square(s) that will divide the number). 25 3
- Rewrite the square root as a multiplication problem. \( \sqrt{75} = \sqrt{25 \times 3} \)
- Reduce the perfect squares ("pulling out" the integer(s)). \( \sqrt{75} = \sqrt{5^2 \times 3} = 5 \sqrt{3} \)
Unit 11: Self-Test

Exponents, Roots & Scientific Notation

Topic A
1. Write the following exponential expressions in expanded form.
   a) $7^4$
   b) $(-t)^3$
   c) $(5a^4b^0)^2$
   d) $\left(\frac{-7}{11}x\right)^3$
2. Write each of the following in the exponential form.
   a) $(0.5) (0.5) (0.5) (0.5)$
   b) $(6w) (6w) (6w)$
   c) $(7u) (3v) (u) (2v)$
3. Evaluate.
   a) $4x^2 + 5y$, for $x = 1$, $y = 4$
   b) $(2a)^3 - 3b$, for $a = 5$, $b = 6$
4. What is the degree of the following term/polynomial?
   a) $15ab^4$
   b) $6xy^2z^4 + 5y^6 - xz + 2z^0$
5. Arranging polynomials in descending order:
   a) $x^2 + 2 - 7x^3 - x + 9x^4$
   b) $4v - 67 + 21uv^3 - uv^2$
6. Arranging polynomials in ascending order.
   a) $26x^2 - 17x^3 - 5x + 43$
   b) $4.3t^2w^2 + \frac{4}{7}tw + w^4 - 8w^3 - 9$

Topic B
7. Simplify (do not leave negative exponents in the answer).
   a) $(-92)^1$
   b) $(-38076)^0$
   c) $(-0.4)^3$
   d) $-8^2$
   e) $y^4y^3$
   f) $\frac{x^9}{x^6}$
g) \((t^4)^5\)

h) \(13a^{-1}\)

i) \([-4 \cdot (0.2)^3\]

j) \((3a^2 \cdot b^3)^4\)

k) \(\frac{1}{4^{-3}}\)

l) \((\frac{w}{u})^{-3}\)

m) \((\frac{a^3}{b^{-4}})^2\)

n) \((3^{-4})(3^4)\)

o) \(\frac{5x^5y^{-6}}{11^9 \cdot x^3y^4}\)

p) \((\frac{u^{-2}y^3}{w^{-4}})^{-3}\)

q) \((2x^2y^3)^3(3x^{-1}y^{-2})^2(-2345w^{-34})^0\)

r) \((\frac{3x^3y^4}{4x^2y^3})^3\)

8. Evaluate for \(x = 3, y = 2, z = -2\).
   a) \((-145x^{-6}y^5z^{-8})^0\)
   b) \((\frac{x}{y})^{-3}\)
   c) \((x - y + 2z)^y\)

Topic C

9. Write in scientific notation.
   a) \(45,600,000\)
   b) \(0.00000523\)

10. Write in standard (or ordinary) form.
    a) \(3.578 \times 10^3\)
    b) \(4.3 \times 10^{-5}\)

11. Simplify and write in scientific notation.
    a) \((5.42 \times 10^{-2})(4.38 \times 10^7)\)
    b) \(\frac{(5\times10^5)(2.4\times10^{-3})}{3.2\times10^8}\)

12. Simplify.
    a) \(\sqrt{196}\)
    b) \(\frac{\sqrt{121}}{\sqrt{225}}\)
    c) \(\sqrt{320}\)
    d) \(\frac{\sqrt{117}}{\sqrt{81}}\)
Unit 12

Solving Word Problems

Topic A: Value mixture problems
Solving value mixture problems

Topic B: Concentration mixture problems
Solving mixture problems

Topic C: Motion and business problems
- Distance, speed and time problems
- Business problems

Topic D: Mixed problems
Solving mixed problems

Unit 12: Summary

Unit 12: Self-test
**Topic A: Value Mixture Problems**

**Steps for solving word problems:**

<table>
<thead>
<tr>
<th>Steps for Solving Word Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Organize the facts given from the problem (make a table).</td>
</tr>
<tr>
<td>• Identify and label the unknown quantity (let ( x = \text{unknown} )).</td>
</tr>
<tr>
<td>• Draw a diagram if it will make the problem clearer.</td>
</tr>
<tr>
<td>• Convert words into a mathematical equation.</td>
</tr>
<tr>
<td>• Solve the equation and find the solution(s).</td>
</tr>
<tr>
<td>• Check and state the answer.</td>
</tr>
</tbody>
</table>

**Table for value mixture problems:**

<table>
<thead>
<tr>
<th>Item</th>
<th>Value of the item</th>
<th>Number of items</th>
<th>Total value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item A</td>
<td>value of A</td>
<td># of A</td>
<td>(value of A) ( \times ) (# of A) = amount of A</td>
</tr>
<tr>
<td>Item B</td>
<td>value of B</td>
<td># of B</td>
<td>(value of B) ( \times ) (# of B) = amount of B</td>
</tr>
<tr>
<td>Item C</td>
<td>value of C</td>
<td># of C</td>
<td>(value of C) ( \times ) (# of C) = amount of C</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Total or mixture</td>
<td>...</td>
<td>...</td>
<td>total value</td>
</tr>
</tbody>
</table>

Let \( x = \text{unknown} \)

Value of item A + Value of item B + Value of item C + … = Total value of the mixture

**Example:** Susan has $5.95 in nickels, dimes and quarters. If she has two less than three times quarters of dimes, and three more nickels than quarters. How many of each coin does she have?

- Let \( x = \text{number of quarters} \)
- Organize the facts:

<table>
<thead>
<tr>
<th>Coin</th>
<th>Value of the coin</th>
<th>Number of coins</th>
<th>Total value (in cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter</td>
<td>25¢</td>
<td>( x )</td>
<td>( 25 \times x )</td>
</tr>
<tr>
<td>Dime</td>
<td>10¢</td>
<td>( 3x - 2 )</td>
<td>( 10 \times (3x - 2) )</td>
</tr>
<tr>
<td>Nickel</td>
<td>5¢</td>
<td>( x + 3 )</td>
<td>( 5 \times (x + 3) )</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>( $5.95 = 595¢ )</td>
</tr>
</tbody>
</table>

Convert $ to ¢.

- Equation: \( 25x + 10(3x - 2) + 5(x + 3) = 595 \)  
  value of quarters + value of dimes + value of nickels = 595¢

- Solve for \( x \): \( 25x + 30x - 20 + 5x + 15 = 595 \)
  
  \[ 60x - 5 = 595 \]
  
  \[ 60x = 600 \]
  
  \[ x = \frac{10}{6} \]

Remove parentheses. Combine like terms. Solve for \( x \).
Check:

<table>
<thead>
<tr>
<th>Number of quarters</th>
<th>( x = 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of dimes</td>
<td>( 3x - 2 = 3(10) - 2 = 28 )</td>
</tr>
<tr>
<td>Number of nickels</td>
<td>( x + 3 = 10 + 3 = 13 )</td>
</tr>
</tbody>
</table>

\[ 25x + 10(3x - 2) + 5(x + 3) = 595 \]  
\[ 25 \cdot 10 + 10(3 \cdot 10 - 2) + 5(10 + 3) = 595 \]  
\[ 250 + 280 + 65 = 595 \]  
\[ \sqrt{595} = 595 \]  
Correct!

State the answer:

<table>
<thead>
<tr>
<th>Number of quarters</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of dimes</td>
<td>28</td>
</tr>
<tr>
<td>Number of nickels</td>
<td>13</td>
</tr>
</tbody>
</table>

Example: Damon purchased $1.00, $1.19, and $1.20 Canadian stamps with a total value of $23.72. If the number of $1.19 stamps is 7 more than the number of $1.00 stamps, and the number of $1.20 stamps is 8 more than three times of $1.00 stamps. How many of each did Damon receive?

- Let \( x \) = number of $1.00 stamps
- Organize the facts:

<table>
<thead>
<tr>
<th>Stamps</th>
<th>Value of the stamps</th>
<th>Number of stamps</th>
<th>Total value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.00</td>
<td>$1.00</td>
<td>( x )</td>
<td>( 1.00 \times x )</td>
</tr>
<tr>
<td>$1.19</td>
<td>$1.19</td>
<td>( 7 + x )</td>
<td>( 1.19 \times (7 + x) )</td>
</tr>
<tr>
<td>$1.20</td>
<td>$1.20</td>
<td>( 8 + 3x )</td>
<td>( 1.20 \times (8 + 3x) )</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>$23.72</td>
</tr>
</tbody>
</table>

Equation:  
\[ 1.00x + 1.19(7 + x) + 1.20(8 + 3x) = 23.72 \]

Solve for \( x \):  
\[ 5.79x + 17.93 = 23.72 \]  
\[ 579x + 1793 = 2372 \]  
\[ 579x = 579 \]  
\[ x = 1 \]

State the answer:

<table>
<thead>
<tr>
<th>Number of $1.00</th>
<th>( x = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of $1.19</td>
<td>( 7 + x = 7 + 1 = 8 )</td>
</tr>
<tr>
<td>Number of $1.20</td>
<td>( 8 + 3x = 8 + 3 \cdot 1 = 11 )</td>
</tr>
</tbody>
</table>
Topic B:  Concentration Mixture Problems

Solving Mixture Problems

Table of concentration mixture:

<table>
<thead>
<tr>
<th>Item</th>
<th>Concentration</th>
<th>Volume</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item A</td>
<td>concentration of A</td>
<td>volume of A</td>
<td>(concentration of A) × (volume of A) = amount of A</td>
</tr>
<tr>
<td>Item B</td>
<td>concentration of B</td>
<td>volume of B</td>
<td>(concentration of B) × (volume of B) = amount of B</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Mixture</td>
<td>concentration of mixture</td>
<td>volume of mixture</td>
<td>(concentration of mixture) × (volume of mixture) = amount of mixture</td>
</tr>
</tbody>
</table>

Let $x = \text{unknown}$

Amount of item A + Amount of item B + ... = Amount of the mixture

Example: A shrimp meal is 35% protein and a fish meal is 25% protein. Susan wants a 750 grams mixture that is 30% protein. How many grams of protein each meal should she have?

- Let $x = \text{the protein volume of the shrimp meal}$
- The protein volume of fish meal = $750 - x$
  
  The protein volume of mixture – The protein volume of shrimp meal = The protein volume of fish meal
  
  (If there is a total mixture protein volume of 750 g, then $750 - x$ must be the protein volume of fish meal.)

Organize the facts:

<table>
<thead>
<tr>
<th>Meal</th>
<th>Concentration</th>
<th>Protein volume</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shrimp meal</td>
<td>35% = 0.35</td>
<td>$x$</td>
<td>$0.35x$</td>
</tr>
<tr>
<td>Fish meal</td>
<td>25% = 0.25</td>
<td>$750 - x$</td>
<td>$0.25(750 - x)$</td>
</tr>
<tr>
<td>Mixture</td>
<td>30% = 0.30</td>
<td>750</td>
<td>$0.3(750)$</td>
</tr>
</tbody>
</table>

(concentration of shrimp meal) × (volume of shrimp meal)
(concentration of fish meal) × (volume of fish meal)
(concentration of mixture) × (volume of mixture)

Equation: $0.35x + 0.25(750 - x) = (0.3) (750)$

Remove parentheses.

Amount of shrimp meal + Amount of fish meal = Amount of mixture

Solve for $x$: $0.35x + 187.5 - 0.25x = 225$

$0.1x = 37.5$

Divide both sides by 0.1.

State the answer: - Shrimp meal: $x = 375 \text{ g}$
- Fish meal: $750 - x = 750 - 375 = 375 \text{ g}$
**Example:** How much 8% sugar solution must be added to 15 liters of 27% solution to make a 20% solution?

- Let \( x \) = volume of 8% solution
- Volume of 20% = \( x + 15 \)

Volume of 20% = Volume of 8% + Volume of 27%

**Organize the facts:**

<table>
<thead>
<tr>
<th>Solution</th>
<th>Concentration</th>
<th>Volume</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>8%</td>
<td>0.08</td>
<td>( x )</td>
<td>0.08 ( x )</td>
</tr>
<tr>
<td>27%</td>
<td>0.27</td>
<td>15</td>
<td>(0.27)(15)</td>
</tr>
<tr>
<td>20%</td>
<td>0.2</td>
<td>( x + 15 )</td>
<td>0.2 ( x + 15 )</td>
</tr>
</tbody>
</table>

**Equation:** \( 0.08x + (0.27)(15) = 0.2(x + 15) \)

Amount of 8% + Amount of 27% = Amount of 20%

**Solve for \( x \):**

\[ 0.08x + 4.05 = 0.2x + 3 \]

- 0.12 \( x \) = -1.05

\[ x = 8.75 \]

**State the answer:** 8.75 liters of 8% sugar solution must be added to 15 liters of 27% solution.
Topic C: Motion and Business Problems

Distance, Speed and Time Problems

Formulas of motion:
- Distance = Speed \cdot Time \quad d = r \cdot t
- Speed = \frac{Distance}{Time} \quad r = \frac{d}{t}
- Time = \frac{Distance}{Speed} \quad t = \frac{d}{r}

Example: Adam walks for 4.4 hours at a rate of 2 km per hour. **How far** does he walk?
Equation: \quad d = r \cdot t
\quad t = 4.4 \text{ h}, \quad r = 2 \text{ km/h}, \quad d = ?
\quad = (2 \text{ km/h}) (4.4 \text{ h}) = 8.8 \text{ km}

Table of motions:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Speed (r)</th>
<th>Time (t)</th>
<th>Distance (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition A</td>
<td>r</td>
<td>t</td>
<td>(d = r \cdot t)</td>
</tr>
<tr>
<td>Condition B</td>
<td>r</td>
<td>t</td>
<td>(d = r \cdot t)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>60 km</td>
</tr>
</tbody>
</table>

Example: Two cyclists are 60 km apart and are travelling towards each other. Their speeds differ by 1.5 km per hour. **What is the speed of each cyclist if they meet after 2 hours**?

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Condition} & \text{Speed (r)} & \text{Time (t)} & \text{Distance (d = r \cdot t)} \\
\hline
\text{Bike A} & r & 2 & 2r \\
\text{Bike B} & r - 1.5 & 2 & 2(r - 1.5) \\
\hline
\text{Total} & & & 60 \text{ km} \\
\hline
\end{array}
\]

- Equation: \quad 2r + 2(r - 1.5) = 60 \quad \text{Distance of A + Distance of B = 60km}
  \quad 2r + 2r - 3 = 60 \quad \text{Remove parentheses.}
  \quad 4r = 63 \quad \text{Combine like terms.}
  \quad r = 15.75 \text{ km/h} \quad \text{Divide both sides by 4.}
- Bike A: \quad r = 15.75 \text{ km/h}
- Bike B: \quad r - 1.5 = 15.75 - 1.5 = 14.25 \text{ km/h}

Example: Mike **boats** at a speed of 28 km per hour in still water. The river flows at a speed of 5 km per hour. **How long** will it take Mike to boat 3 km downstream? 3 km upstream?

<table>
<thead>
<tr>
<th>Condition</th>
<th>Speed (r)</th>
<th>Distance (d)</th>
<th>Time ((t = \frac{d}{r}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream</td>
<td>(r = 28 + 5 = 33 \text{ km/h})</td>
<td>(d = 3 \text{ km})</td>
<td>(t = \frac{d}{r} = \frac{3 \text{ km}}{33 \text{ km/h}} \approx 0.091 \text{ h})</td>
</tr>
<tr>
<td>Upstream</td>
<td>(r = 28 - 5 = 23 \text{ km/h})</td>
<td>(d = 3 \text{ km})</td>
<td>(t = \frac{d}{r} = \frac{3 \text{ km}}{23 \text{ km/h}} \approx 0.13 \text{ h})</td>
</tr>
</tbody>
</table>

Downstream (fast): speed of boat + speed of river
Upstream (slower): speed of boat - speed of river
Business Problems

Business math formulas:

<table>
<thead>
<tr>
<th>Business problems</th>
<th>Formulas</th>
</tr>
</thead>
</table>
| Percent increase  | \[ x = \frac{N-O}{O} \]  
| Percent decrease  | \[ x = \frac{O-N}{O} \]  
| Sales tax         | \[ \text{Sales tax} = \text{Sales} \times \text{Tax rate} \]  
| Commission        | \[ \text{Commission} = \text{Sales} \times \text{Commission rate} \]  
| Discount          | \[ \text{Discount} = \text{Original price} \times \text{Discount rate} \]  
|                   | \[ \text{Sale price} = \text{Original price} - \text{Discount} \]  
| Markup            | \[ \text{Markup} = \text{Selling price} \times \text{Markup rate} \]  
|                   | \[ \text{Original price} = \text{Selling price} - \text{Markup} \]  
| Simple interest   | \[ I = \text{Principal} \times \text{Interest rate} \times \text{Time} \]  
|                   | \[ \text{Balance} = \text{Principal} + \text{Interest} \]  
| Compound interest | \[ \text{Balance} = \text{Principal} \times (1 + \text{Interest rate}) \times \text{Time} \]  
|                   | \[ \text{Balance} = \text{Principal} + \text{Interest} \]  

Example: A product increased production from 230 last month to 250 this month. Find the percent increase.

- New value (N): 250 This month.
- Original value (O): 230 Last month.
- Percent increase: \[ x = \frac{N-O}{O} = \frac{250-230}{230} \approx 0.087 = 8.7\% \] About 8.7% increase.

Example: A product was reduced from $59 to $39. What was the percent reduction?

Percent decrease: \[ x = \frac{O-N}{O} = \frac{59-39}{59} \approx 0.339 = 33.9\% \] 33.9% decrease.

Example: Find the sales tax for a $999 laptop with a tax rate of 7%.

Sales tax = Sales \times Tax rate  
\[ = (999) (0.07) = 69.93 \]

Example: Find the commission for a $950,000 house with a commission rate of 5%.

Commission = Sales \times Commission rate  
\[ = (950,000) (0.05) = 47,500 \]
Example: A men’s coat was originally priced at $159, and is on sale at a 25% discount. Find the discount and sale price.

- **Discount** = Original price $\times$ Discount rate
  
  $\text{Discount} = ($159) (0.25)$
  
  $= \$39.75$

- **Sale price** = Original price − Discount
  
  $\text{Sale price} = \$159 − \$39.75$
  
  $= \$119.25$

Example: A condo was sold at $399,000, with a markup rate of 8%. What was the markup and original price?

- **Markup** = Selling price $\times$ Markup rate
  
  $\text{Markup} = ($399,000) (0.08)$
  
  $= \$31,920$

- **Original price** = Selling price $-$ Markup
  
  $\text{Original price} = \$399,000 − \$31,920$
  
  $= \$367,080$

Example: Jo borrowed $150,000 mortgage from a bank. Find the interest at 3% per year for 3.5 years, and also find the total amount that Jo paid the bank.

- **Interest** = Principle $\times$ Interest rate $\times$ Time
  
  $\text{Interest} = P \times r \times t $(150,000) (0.03) (3.5)
  
  $= \$15,750$

- **Balance** = Principle + Interest
  
  $\text{Balance} = \$150,000 + \$15,750$
  
  $= \$165,750$

Example: David deposited $3,000 in an account at 4.5% interest compounded per year for 5 years. How much was in the account at the end of 5 years?

\[
\text{Balance} = \text{Principal} \times (1 + \text{Interest rate})^t
\]

\[
= \text{P} \times (1 + 0.045)^5
\]

$\approx \$3738.55$
**Topic D: Mixed Problems**

### Solving Mixed Problems

**Example:** After a *ten percent reduction*, a toy is on sale for *twenty-nine dollars*. What was the *original price*?

- Let \( x \) = original price
- Equation: \( x - 10\% \ x = 29 \)  
  \[ 1 \cdot x - 0.1 \cdot x = 29 \]  
  \[ 0.9x = 29 \]
- Answer: \( x = \frac{29}{0.9} \approx 32.22 \)  
  The original price was $32.22.

**Example:** William receives a *1.5% raises* bring his salary to *$39,000*. What was his salary *before the raise*?

- Let \( x \) = Tom’s salary before the raise  
  Raise = (1.5\%)(Previous salary) = 1.5\% \ x
- Equation: \( x + 1.5\% \ x = 39,000 \)  
  Previous salary + Raise = Current salary  
  \[ 1 \cdot x + 0.015 \cdot x = 39,000 \]  
  \[ 1.015 \cdot x = 39,000 \]
- Answer: \( x = \frac{39000}{1.015} \approx 38423.65 \)  
  Tom’s salary before the raise was $38423.65.

**Example:** Bob deposits a certain amount of money in a *chequing account* that earns *2.5%* in annual interest, and deposits *$2000 less than* that in a *saving account* that pays *1.5%* in annual interest. If the total interest from both accounts at the end of the year is *$95*, how much is deposited in each account?

- Let \( x \) = money deposited in the saving account

<table>
<thead>
<tr>
<th>Account</th>
<th>Deposit</th>
<th>Interest rate</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chequing account</td>
<td>( x )</td>
<td>2.5%</td>
<td>0.025( x )</td>
</tr>
<tr>
<td>Saving account</td>
<td>( x - 2000 )</td>
<td>1.5%</td>
<td>0.015(( x - 2000 ))</td>
</tr>
</tbody>
</table>

- Equation: \( 0.025x + 0.015(\( x - 2000 \)) = 95 \)  
  \[ 0.025x + 0.015x - 30 = 95 \]  
  Combine like terms.  
  \[ 0.04x = 125 \]  
- Answer: Chequing account: \( x = \frac{125}{0.04} = 3125 \)  
  $3125 in the chequing account.  
  Saving account: \( x - 2000 = 3125 - 2000 = 1125 \)  
  $1125 in the saving account.
Example: A string that is 103 meters long is cut into four pieces. The second is four times as long as the first. The third piece is five meters longer than the first. The fourth piece is twice as long as the third. How long is each piece of string?

- Let \( x \) = the length of the first piece.

<table>
<thead>
<tr>
<th>1st piece</th>
<th>( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd piece</td>
<td>( 4x )</td>
</tr>
<tr>
<td>3rd piece</td>
<td>( x + 5 )</td>
</tr>
<tr>
<td>4th piece</td>
<td>( 2(x + 5) )</td>
</tr>
</tbody>
</table>

- Equation: \( x + 4x + (x + 5) + 2(x + 5) = 103 \) \( 1st + 2nd + 3rd + 4th = 103 \)
  \( x + 4x + x + 5 + 2x + 10 = 103 \)
  \( 8x + 15 = 103 \)
  \( 8x = 88 \)
  \( x = \boxed{11 \text{ m}} \)

- Answer:

<table>
<thead>
<tr>
<th>1st piece</th>
<th>( x = \boxed{11 \text{ m}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd piece</td>
<td>( 4x = 4(11) = \boxed{44 \text{ m}} )</td>
</tr>
<tr>
<td>3rd piece</td>
<td>( x + 5 = 11 + 5 = \boxed{16 \text{ m}} )</td>
</tr>
<tr>
<td>4th piece</td>
<td>( 2(x + 5) = 2(11 + 5) = \boxed{32 \text{ m}} )</td>
</tr>
</tbody>
</table>

Example: A fruit punch that contains 25% fruit juice. How much water would you have to add to 1 liter of punch to get a new drink that contains 10% fruit juice?

- Let \( x \) = water to add to 1 L of punch to get a 10% fruit juice.

<table>
<thead>
<tr>
<th>Concentration</th>
<th>Volume</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fruit punch</td>
<td>25 %</td>
<td>1 (L)</td>
</tr>
<tr>
<td>New drink</td>
<td>10 %</td>
<td>( x + 1 )</td>
</tr>
</tbody>
</table>

- Equation: \( 0.25(1) = 0.1(x + 1) \)
  \( 0.25 = 0.1x + 0.1 \)
  \( 25 = 10x + 10 \)
  \( 15 = 10x \)
  \( x = \boxed{1.5 \text{ L}} \)

- Answer: It needs to add 1.5 L of water to get a new drink that contains 10% fruit juice.
Unit 12: Summary

Solving Word Problems

Steps for solving word problems:

- Organize the facts given from the problem (make a table).
- Identify and label the unknown quantity (let \( x = \text{unknown} \)).
- Draw a diagram if it will make the problem clearer.
- Convert words into a mathematical equation.
- Solve the equation and find the solution(s).
- Check and state the answer.

Table for value mixture problems:

<table>
<thead>
<tr>
<th>Item</th>
<th>Value of the item</th>
<th>Number of items</th>
<th>Total value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item A</td>
<td>value of A</td>
<td># of A</td>
<td>(value of A) ( \times ) (# of A) = amount of A</td>
</tr>
<tr>
<td>Item B</td>
<td>value of B</td>
<td># of B</td>
<td>(value of B) ( \times ) (# of B) = amount of B</td>
</tr>
<tr>
<td>Item C</td>
<td>value of C</td>
<td># of C</td>
<td>(value of C) ( \times ) (# of C) = amount of C</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Total or mixture</td>
<td></td>
<td></td>
<td>total value</td>
</tr>
</tbody>
</table>

Value of item A + Value of item B + Value of item C + … = Total value of the mixture

Formulas of motion:

Distance = Speed \cdot Time \quad d = rt \quad t = \frac{d}{r} \quad r = \frac{d}{t}

Table of motions:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Speed (r)</th>
<th>Time (t)</th>
<th>Distance (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition A</td>
<td>r</td>
<td>t</td>
<td>( d = rt )</td>
</tr>
<tr>
<td>Condition B</td>
<td>r</td>
<td>t</td>
<td>( d = rt )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Downstream (fast): speed of boat + speed of river
- Upstream (slower): speed of boat - speed of river

Table of concentration mixture:

<table>
<thead>
<tr>
<th>Item</th>
<th>Concentration</th>
<th>Volume</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item A</td>
<td>concentration of A</td>
<td>volume of A</td>
<td>(concentration of A) ( \times ) (volume of A) = amount of A</td>
</tr>
<tr>
<td>Item B</td>
<td>concentration of B</td>
<td>volume of B</td>
<td>(concentration of B) ( \times ) (volume of B) = amount of B</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Mixture</td>
<td>concentration of mixture</td>
<td>volume of mixture</td>
<td>(concentration of mixture) ( \times ) (volume of mixture) = amount of mixture</td>
</tr>
</tbody>
</table>

Amount of item A + Amount of item B + … = Amount of the mixture
# Business math formulas:

<table>
<thead>
<tr>
<th>Business problems</th>
<th>Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent increase</td>
<td>Percent increase = ( \frac{\text{New value} - \text{Original value}}{\text{Original value}} ), ( x = \frac{\text{N} - \text{O}}{\text{O}} )</td>
</tr>
<tr>
<td>Percent decrease</td>
<td>Percent decrease = ( \frac{\text{Original value} - \text{New value}}{\text{Original value}} ), ( x = \frac{\text{O} - \text{N}}{\text{O}} )</td>
</tr>
<tr>
<td>Sales tax</td>
<td>Sales tax = ( \text{Sales} \times \text{Tax rate} )</td>
</tr>
<tr>
<td>Commission</td>
<td>Commission = ( \text{Sales} \times \text{Commission rate} )</td>
</tr>
<tr>
<td>Discount</td>
<td>Discount = ( \text{Original price} \times \text{Discount rate} )</td>
</tr>
<tr>
<td></td>
<td>Sale price = ( \text{Original price} - \text{Discount} )</td>
</tr>
<tr>
<td>Markup</td>
<td>Markup = ( \text{Selling price} \times \text{Markup rate} )</td>
</tr>
<tr>
<td></td>
<td>Original price = ( \text{Selling price} - \text{Markup} )</td>
</tr>
<tr>
<td>Simple interest</td>
<td>Interest = ( \text{Principle} \cdot \text{Interest rate} \cdot \text{Time} ), ( I = \text{P} \cdot \text{r} \cdot \text{t} )</td>
</tr>
<tr>
<td></td>
<td>Balance = ( \text{Principle} + \text{Interest} )</td>
</tr>
<tr>
<td>Compound interest</td>
<td>Balance = ( \text{Principle} (100% + \text{Interest rate})^t )</td>
</tr>
<tr>
<td></td>
<td>Balance = ( P (100% + r)^t )</td>
</tr>
</tbody>
</table>
Unit 12: Self-Test

Solving Word Problems

Topic A

1. Robert has $2.50 in nickels, dimes and quarters. If he has two more than five times quarters of dimes, and two less nickels than quarters. How many of each coin does he have?

2. William purchased $1.00, $1.19, and $1.20 Canadian stamps with a total value of $27.13. If the number of $1.19 stamps is 5 more than the number of $1.00 stamps, and the number of $1.20 stamps is 6 more than four times of $1.00 stamps. How many of each did Damon receive?

Topic B

3. A lamb meal is 36% protein and a pork meal is 25% protein. Peter wants an 860 grams mixture that is 28% protein. How many grams of protein each meal should he have?

4. How much 5% salt solution must be added to 18 liters of 32% solution to make a 25% solution?

Topic C

5. Two cyclists are 72 km apart and are travelling towards each other. Their speeds differ by 2 km per hour. What is the speed of each cyclist if they meet after 3 hours?

6. Linda boats at a speed of 17 km per hour in still water. The river flows at a speed of 3 km per hour. How long will it take Linda to boat 4 km downstream? 4 km upstream?

7. A product increased production from 400 last month to 420 this month. Find the percent increase.

8. A product was reduced from $80 to $62. What was the percent reduction?

9. Find the sales tax for a $679 laptop with a tax rate of 9%.

10. Find the commission for a $699,000 townhouse with a commission rate of 4%.
11. A women’s dress was originally priced at $199, and is on sale at a 15% discount. Find the discount and sale price.

12. A condo was sold at $469,000, with a markup rate of 5%. What was the markup and original price?

13. Smith borrowed $100,000 mortgage from a bank. Find the interest at 4% per year for 5 years, and also find the total amount that Smith paid the bank.

14. Susan deposited $2,500 in an account at 3.2% interest compounded per year for 2 years. How much was in the account at the end of 2 years?

**Topic D**

15. After a five percent reduction, a toy is on sale for thirty-nine dollars. What was the original price?

16. Ruth receives a 2.5% raises bring her salary to $34,000. What was her salary before the raise?

17. Amy deposits a certain amount of money in a chequing account that earns 1.5% in annual interest, and deposits $1500 less than that in a saving account that pays 1.2% in annual interest. If the total interest from both accounts at the end of the year is $76.50, how much is deposited in each account?

18. A string that is 52 meters long is cut into four pieces. The second is three times as long as the first. The third piece is seven meters longer than the first. The fourth piece is three times as long as the third. How long is each piece of string?

19. A fruit punch is 45% fruit juice. How much water would you have to add to 1.5 liter of punch to get a new drink that is 25% fruit juice?
Unit 13
More about Polynomials

Topic A: Adding and subtracting polynomials

- Polynomials review
- Adding and subtracting polynomials

Topic B: Multiplication of polynomials

- Multiplying polynomials
- Special binomial products

Topic C: Polynomial division

- Dividing polynomials
- Long division of polynomials
- Missing terms in long division

Unit 13 Summary

Unit 13 Self-test
### Topic A: Adding and Subtracting Polynomials

#### Polynomials Review

**Review of basic algebraic terms:**

<table>
<thead>
<tr>
<th>Algebraic term</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
</table>
| Algebraic expression | A mathematical phrase that contains numbers, variables, and arithmetic operations (+, −, ×, ÷, etc.). | 5x + 2
|                      |                                                                           | 3a − (4b + 6)                                                         |
|                      |                                                                           | \( \frac{2x}{3} + 4y − z^2 + 11 \)                                   |
| Constant             | A number.                                                                 | x + 2 constant: 2                                                     |
| Variable             | A letter that can be assigned different values.                           | 3 − x variable: x                                                     |
| Coefficient          | The number that is in front of a variable.                               | -6 x coefficient: -6                                                 |
|                      |                                                                           | xc^3 coefficient: 1                                                  |
| Term                 | A term can be a constant, a variable, or the product of a number and variable(s). (Terms are separated by a plus or minus sign.) | 3x − \( \frac{2}{5} \) + 13y^2 + 73xy                               |
|                      |                                                                           | Terms: 3x, \( -\frac{2}{5} \), 13y^2, 73xy                           |
| Like terms           | The terms that have the same variables and exponents.                    | 2x − y^2 − \( \frac{2}{5} \) + 5x − 7 + 13y^2                        |
|                      |                                                                           | Like terms: 2x and 5x, \( -\frac{2}{5} \) and 13y^2, \( -\frac{7}{2} \) and -7 |
| Factor               | A number or variable that multiplies with another.                      | 24 = 2 · 3 · 4 factors: 2, 3, 4                                      |
|                      |                                                                           | 5xy = 5 · x · y factors: 5, x, y                                     |
|                      | A number or expression can have many factors.                            |                                                                       |

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Example</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monomial (one term)</td>
<td>7a</td>
<td>7</td>
</tr>
<tr>
<td>Binomial (two terms)</td>
<td>3x − 5</td>
<td>3</td>
</tr>
<tr>
<td>Trinomial (three terms)</td>
<td>-4x^2 + xy + 7</td>
<td>-4, 1</td>
</tr>
<tr>
<td>Polynomial (one or more terms)</td>
<td>2pq + 4p^3 + p +11</td>
<td>2, 4, 1</td>
</tr>
</tbody>
</table>

**The degree of a term with more variables:** the sum of the exponents of its variables.

Example: \(-3x^3y^5z^2\) degree: 3 + 5 + 2 = 10

**The degree of a polynomial with more variables:** the highest degree of any individual term.

Example: \(4ab^3 + 3a^2b^2c^3 − 5a + 1\) degree: 7

4 7 1

**Additive (or negative) inverse or opposite:** the opposite of a term (two terms whose sum is 0).

Example: 1) The additive inverse of \(5\) is \(-5\) 5 + (-5) = 0

2) The additive inverse of \(-\frac{3}{4}y\) is \(\frac{3}{4}y\) \(-\frac{3}{4}y + \frac{3}{4}y = 0\)

3) The additive inverse of \(4ab^3 − 3a^2 + b^3\) is \(-4ab^3 + 3a^2 − b^3\)
Add or subtract polynomials:

**Example:** Add \(4x^3 - 5x^2 - x + 3\) and \(3x^3 + 3x^2 - 5x + 2\).

**Steps**

1. Regroup like terms:
   \[(4x^3 - 5x^2 - x + 3) + (3x^3 + 3x^2 - 5x + 2)\]
2. Combine like terms:
   \[= (4x^3 + 3x^3) + (-5x^2 + 3x^2) + (-x - 5x) + (3 + 2)\]
   \[= 7x^3 - 2x^2 - 6x + 5\]

**Solution**

**Example:** Subtract \(6x^2 + 7x - 5\) and \(3x^2 - 4x + 16\).

**Steps**

1. Remove parenthesis:
   \[(6x^2 + 7x - 5) - (3x^2 - 4x + 16)\]
2. Regroup like terms:
   \[= (6x^2 - 3x^2) + (7x + 4x) + (-5 - 16)\]
3. Combine like terms:
   \[= 3x^2 + 11x - 21\]

**Solution**

Add or subtract polynomials using the column method:

**Example:** Add \(4x^3 - 3x^2 + 6x - 5\) and \(3x^3 + 2x + 3\).

**Steps**

1. Line up like terms in columns:
   \[4x^3 - 3x^2 + 6x - 5\]
2. Add:
   \[\begin{array}{c}
   3x^3 \\
   + 2x \\
   \hline
   7x^3 - 3x^2 + 8x - 2
   \end{array}\]

**Solution**

**Example:** Subtract \((7x^2 - 3x + 4)\) and \((3x^2 - 1)\).

**Steps**

1. Line up like terms in columns:
   \[7x^2 - 3x + 4\]
2. Change signs in the minuend and add:
   \[\begin{array}{c}
   -3x^2 \\
   + 1 \\
   \hline
   4x^2 - 3x + 5
   \end{array}\]

**Solution**

Or \((7x^2 - 3x + 4) - (3x^2 - 1) = 7x^2 - 3x + 4 - 3x^2 + 1\)
   \[= 4x^2 - 3x + 5\]
Topic B: Multiplication of Polynomials

### Multiplying Monomials

**Example:** \((-4a^2b^3)(5a^3b^5) = (-4 \cdot 5)(a^2 \cdot a^3)(b^3 \cdot b^5)\)

Multiply the coefficients and add the exponents.

\[= -20a^5b^8\]

### Multiplying Monomial and Polynomial

**Example:** \(5x^2(4x^3 - 3x) = (5x^2)(4x^3) - (5x^2)(3x)\)

Distribute:

\[= 20x^5 - 15x^3\]

**Example:** \(3xy^3(4xy^2 + x^3y - y)\)

Distribute:

\[= (3xy^3)(4xy^2) + (3xy^3)(x^3y) + (3xy^3)(-y)\]

Multiply the coefficients and add the exponents.

\[= 12x^5y^5 + 3x^4y^4 - 3xy^4\]

### Multiplying Binomials (2 terms × 2 terms)

**Example:** Find the following product.

\[(4a - 5)(2a - 3) = 4a \cdot 2a + 4a (-3) - 5 \cdot 2a - 5 (-3)\]

F O I L

\[= 8a^2 - 12a - 10a + 15\]

\[= 8a^2 - 22a + 15\]

### Multiplying Binomial and Polynomial

**Example:** Multiply: \(2x - 3x^2\) and \(x^2 + x - 4\)

**Steps**

**Solution**

\[(2x - 3x^2)(x^2 + x - 4)\]

- Use the distributive property:

\[= 2x \cdot x^2 + 2x \cdot x + 2x (-4) - 3x^2 \cdot x^2 - 3x^2 \cdot x - 3x^2 (-4)\]

- Multiply coefficients and add exponents:

\[= 2x^3 + 2x^2 - 8x - 3x^4 - 3x^3 + 12x^2\]

- Combine like terms and write in descending order:

\[= -3x^4 - x^3 + 14x^2 - 8x\]

### Multiplying Polynomials Mentally (no need to write out each step)

**Example:** Multiply.

a) \(2x^3 (3x^2 - 2) = 6x^5 - 4x^3\)

\[a (b + c) = ab + ac\]

b) \((a - 3)(2a - 1) = 2a^2 - 7a + 3\)

FOIL
Special binomial products – squaring binominals

<table>
<thead>
<tr>
<th>Special products</th>
<th>Formula</th>
<th>Initial expansion</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference of squares</td>
<td>((a + b) \ (a - b) = a^2 - b^2)</td>
<td>((a + b) \ (a - b) = a^2 - ab + ba - b^2)</td>
<td>((x + 3) \ (x - 3) = x^2 - 3^2 = x^2 - 9) or ((x - 3) \ (x + 3) = x^2 - 3^2 = x^2 - 9) ((a = x, b = 3))</td>
</tr>
<tr>
<td>Square of sum</td>
<td>((a + b)^2 = a^2 + 2ab + b^2)</td>
<td>((a + b)^2 = (a + b) \ (a + b))</td>
<td>((y + 2)^2 = y^2 + 2 \cdot y \cdot 2 + 2^2) (= y^2 + 4y + 4)</td>
</tr>
<tr>
<td>Square of difference</td>
<td>((a - b)^2 = a^2 - 2ab + b^2)</td>
<td>((a - b)^2 = (a - b) \ (a - b))</td>
<td>((z - 5)^2 = z^2 - 2 \cdot z \cdot 5 + 5^2) (= z^2 - 10z + 25)</td>
</tr>
</tbody>
</table>

Special binomial products: special forms of binomial products that are worth memorizing.

Memory aid: \((a \pm b)^2 = (a^2 \pm 2ab + b^2)\) Notice the reversed plus or minus sign in the second term.

Example: Find the following products.

1) \((5x + 3) \ (5x - 3) = 5x^2 - 3^2\) \(= 25x^2 - 9\) \((a + b) \ (a - b) = a^2 - b^2\) \(a = 5x\ , \ b = 3\)

2) \((2t - 1)^2 = (2t)^2 - 2(2t) + 1^2\) \(= 4t^2 - 4t + 1\) \((a - b)^2 = a^2 - 2ab + b^2\) \(a = 2t\ , \ b = 1\)

3) \((3w + \frac{1}{3})^2 = (3w)^2 + 2(3w) \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2\) \(= 9w^2 + 2w + \frac{1}{9}\) \((a + b)^2 = a^2 + 2ab + b^2\) \(a = 3w\ , \ b = \frac{1}{3}\)

4) \((5u - \frac{1}{2}v)^2 = (5u)^2 - 2(5u) \left(\frac{1}{2}v\right) + \left(\frac{1}{2}v\right)^2\) \(= 25u^2 - 5uv + \frac{1}{4}v^2\) \((a - b)^2 = a^2 - 2ab + b^2\) \(a = 5u\ , \ b = \frac{1}{2}v\)

5) \((\frac{1}{3}t - \frac{1}{2}) \ (\frac{1}{3}t + \frac{1}{2}) = (\frac{1}{3}t)^2 - (\frac{1}{2})^2\) \(= \frac{1}{9}t^2 - \frac{1}{4}\) \((a + b) \ (a - b) = a^2 - b^2\) \(a = \frac{1}{3}t\ , \ b = \frac{1}{2}\)
Dividing Polynomials

### Dividing a monomial by a monomial

**Example:** \( \frac{14a^5}{a^2} = 14a^{5-2} \)

\[ = 14a^3 \]

Apply \( \frac{a^m}{a^n} = a^{m-n} \)

**Example:** \( \frac{-28u^6v^2}{7u^4v^5} \)

**Steps**

- Divide the coefficients:
  \[ -28 \cdot \frac{1}{7} = -4 \]

- Divide like variables (apply \( \frac{a^m}{a^n} = a^{m-n} \)):
  \[ \frac{u^6}{u^4} = u^{6-4} = u^2 \]
  \[ \frac{v^2}{v^5} = v^{2-5} = v^{-3} \]

**Solution**

\[ -4 \cdot \frac{u^2}{v^3} \]

- \( a^{-m} = \frac{1}{a^m} \)

### Dividing a polynomial by a monomial

**Example:** \( \frac{15a^2 + 5a - 4}{5a} \)

**Steps**

- Split the polynomial into three parts:

**Solution**

\[ \frac{15a^2 + 5a - 4}{5a} = \frac{15a^2}{5a} + \frac{5a}{5a} - \frac{4}{5a} \]

\[ = 3a + 1 - \frac{4}{5a} \]

**Example:** \( \frac{4x^2 + 8x + 2x + 4}{x + 2} \)

**Steps**

- Group:

**Solution**

\[ \frac{4x^2 + 8x + 2x + 4}{x + 2} = \frac{(4x^2 + 8x) + (2x + 4)}{x + 2} \]

\[ = \frac{4x(x + 2) + 2(x + 2)}{x + 2} \]

\[ = 4x + 2 + \frac{2(x + 2)}{x + 2} \]

\[ = 4x + 2 + 2 \]

\[ = 4x + 2 = 2(x + 1) \]
**Long Division of Polynomials**

**Long division for numbers:**

Example:

\[
\begin{array}{c|c}
\text{Quotient} & 7 \\
\text{Divisor) Dividend} & 3 \overline{)22} \\
\hline
- & - \\
\text{Remainder} & 21 \\
\end{array}
\]

**Polynomial long division:** a method used for dividing a polynomial by another polynomial of the same or lower degree (it is very similar to long division for numbers).

Example: \( \frac{4x^2 + 8x + 1}{2x} \)

**Steps**

- Write in divisor \( \overline{\text{Dividend}} \) form:
  \[ 2x \overline{\text{) } 4x^2 + 8x + 1} \]
  \[ \text{Solution: } 2 \overline{\text{) } 481} \]

- Divide the first term:
  \[ 2x \overline{\text{) } 4x^2 + 8x + 1} \]
  \[ - 4x^2 \] 
  \[ \text{Remainder: } 2 \overline{\text{) } 481} \]
  \[ - 4 \] 
  \[ 2 \cdot 2 = 4 \]

- Divide the second term:
  \[ 2x \overline{\text{) } 4x^2 + 8x + 1} \]
  \[ - 8x \] 
  \[ \text{Remainder: } 2 \overline{\text{) } 481} \]
  \[ - 8 \] 
  \[ 2 \cdot 4 = 8 \]

- Quotient = \( 2x + 4 \), remainder = 1

- Tip: continue until the degree of the remainder is less than the degree of the divisor.
  (i.e. \( l = 1 \cdot x^0 \) and \( 2x = 2x^1 \), \( 0 < 1 \))

- Check: \( \text{Dividend = Quotient} \cdot \text{Divisor} + \text{Remainder} \)
  \[ 4x^2 + 8x + 1 = (2x + 4) \cdot (2x) + 1 \]
  \[ \sqrt{4x^2 + 8x + 1 = 4x^2 + 8x + 1} \] Correct!
If there is a missing consecutive power term in a polynomial (i.e. if there are \(x^3\) and \(x\) but not \(x^2\)), add in the missing term with a coefficient of 0.

**Example:** \[ \frac{7 - 4x^2 + x^3}{1 + x} \]

**Steps**

- Rewrite both polynomials in descending order:
  
  **Descending order:** \(Ax^3 + Bx^2 + Cx + D\), \(Ax + B\)

- Write in **divisor \(\) Dividend** form and insert a 0 coefficient for the missing power term.

- Divide as usual:

  - Quotient = \(x^2 - 5x + 5\), remainder = \(2\)

- Check: Dividend = Quotient \(\) Divisor + Remainder

\[
7 - 4x^2 + x^3 = (x^2 - 5x + 5)(x + 1) + 2
\]

\[
7 - 4x^2 + x^3 = (x^3 + x^2 - 5x^2 - 5x + 5x + 5) + 2
\]

\[
7 - 4x^2 + x = x^3 - 4x^2 + 7
\]
Basic algebraic terms:

<table>
<thead>
<tr>
<th>Algebraic term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic expression</td>
<td>A mathematical phrase that contains numbers, variables, and arithmetic operations.</td>
</tr>
<tr>
<td>Constant</td>
<td>A number.</td>
</tr>
<tr>
<td>Variable</td>
<td>A letter that can be assigned different values.</td>
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<tr>
<td>Coefficient</td>
<td>The number that is in front of a variable.</td>
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<td>A term can be a constant, a variable, or the product of a number and variable(s). (Terms are separated by a plus or minus sign.)</td>
</tr>
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<td>Like terms</td>
<td>The terms that have the same variables and exponents.</td>
</tr>
<tr>
<td>Factor</td>
<td>A number or variable that multiplies with another.</td>
</tr>
</tbody>
</table>

The degree of a term with more variables: the sum of the exponents of its variables.

The degree of a polynomial with more variables: the highest degree of any individual term.

Additive (or negative) inverse or opposite: the opposite of a term.

Add or subtract polynomials:

- Regroup like terms.
- Combine like terms.

Add polynomials using the column method:

- Line up like terms in columns.
- Add.

Subtract polynomials using the column method:

- Line up like terms in columns.
- Change signs in minuend and add.

Multiplying binomial and polynomial:

- Use the distributive property. $a (b + c) = ab + ac$
- Multiply coefficients and add exponents. $\frac{a^m}{a^n} = a^{m-n}$
- Combine like terms and write in descending order.
### Special binomial products – squaring binominals

<table>
<thead>
<tr>
<th>Special products</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference of squares</td>
<td>((a + b)(a - b) = a^2 - b^2)</td>
</tr>
<tr>
<td>Square of sum</td>
<td>((a + b)^2 = a^2 + 2ab + b^2)</td>
</tr>
<tr>
<td>Square of difference</td>
<td>((a - b)^2 = a^2 - 2ab + b^2)</td>
</tr>
</tbody>
</table>

Memory aid: \((a \pm b)^2 = (a^2 \pm 2ab + b^2)\)

### Dividing a monomial by a monomial

- Divide coefficients.
- Divide like variables (apply \(\frac{a^m}{a^n} = a^{m-n}\)).

### Dividing a polynomial by a monomial

- Split the polynomial into parts.
- Divide a monomial by a monomial.

**Polynomial long division:** A method used for dividing a polynomial by another polynomial of the same or lower degree (it is very similar to long division for numbers).

#### Example: \(\frac{8-3x+x^3}{2+x}\)

**Steps**

- **Rewrite both polynomials in descending order:**
  
  Descending order: \(Ax^3 + Bx^2 + Cx + D, \ Ax + B\)

- **Write in divisor \(\) Dividend form and insert a 0 coefficient for the missing power term.**

- **Divide as usual:**

  \[
  \begin{array}{c|c|c}
  & x^3 - 3x + 8 & \\
  \hline
  x + 2 & x^3 + 0x^2 - 3x + 8 & \\
  \hline
  \end{array}
  \]

  \[
  \begin{array}{c|c|c}
  & x^2 - 2x + 1 & \\
  \hline
  x + 2 & x^3 + 0x^2 - 3x + 8 & (x^2)(x) = x^3 \\
  \hline
  - & x^3 + 2x^2 & (x^2)(2) = 2x^2 \\
  \hline
  \end{array}
  \]

  \[
  \begin{array}{c|c|c}
  & -2x^2 - 3x & \\
  \hline
  x + 2 & -2x^2 - 4x & (-2x)(x) = -2x^2 \\
  \hline
  - & -2x^2 - 4x & (-2x)(2) = -4x \\
  \hline
  \end{array}
  \]

  \[
  \begin{array}{c|c|c}
  & x + 8 & \\
  \hline
  x + 2 & x + 2 & (1)(x) = x \\
  \hline
  - & x + 2 & (1)(2) = 2 \\
  \hline
  \end{array}
  \]

  Quotient = \(x^2 - 2x + 1\), remainder = 6

- **Tip:** continue until the degree of the remainder is less than the degree of the divisor.

- **Check:** Dividend = Quotient \(\) Divisor + Remainder

\[
\frac{\text{Quotient}}{\text{Divisor}} \frac{\text{Dividend}}{\text{Remainder}}
\]
Unit 13: Self-Test

More about Polynomials

Topic A

1. Determine the degree of the following.
   a) \(-8x^4y^3z^5\)
   b) \(21x^5y + 32x^2y^3z + 6x^3y^4z^2\)
   c) \(3.5a^4b + 6.1a^4b^3c - 7.3a + 5.4\)

2. Determine the additive inverse.
   a) \(8y\)
   b) \(-\frac{5}{8}x\)
   c) \(9xy^2 - 4x^2 + y^3\)

3. Add \(5x^4 - 3x^3 - x + 7\) and \(4x^4 + 2x^3 - 7x + 3\).

4. Subtract \(8x^2 + 5x - 4\) and \(4x^2 - 2x + 14\).

5. Add or subtract polynomials using the column method:
   a) Add \(7a^3 - 4a^2 + 3a - 6\) and \(4a^3 + 6a + 8\).
   b) Subtract \((9x^2 - 4x + 8)\) and \((4x^2 - 3)\).

Topic B

   a) \((-6x^3y^2)(4x^4y^3)\)
   b) \(4a^2(3a^4 - 6a)\)
   c) \(7xy^2(2xy^4 + x^3y - 3y)\)
d) \((3x - 4)(4x - 5)\)
e) \((3a - 2a^2)(a^2 + a - 5)\)

7. Find the following product.

a) \(4t^4(2t^3 - 5)\)
b) \((x - 5)(3x - 2)\)
c) \((6a + 5)(6a - 5)\)
d) \((3w - 1)^2\)
e) \((5u + \frac{1}{2})^2\)
f) \((6x - \frac{1}{3}y)^2\)
g) \((\frac{1}{5}z - \frac{1}{4})(\frac{1}{5}z + \frac{1}{4})\)

Topic C

8. Divide the following.

a) \(\frac{56x^6}{x^3}\)
b) \(\frac{-81a^5b^3}{9a^3b^6}\)
c) \(\frac{28y^2 + 7y - 3}{7y}\)
d) \(\frac{6a^2 + 18a + 3a + 9}{a + 3}\)

9. Use long division to divide the following.

a) \(\frac{9x^2 + 6x + 2}{3x}\)
b) \(\frac{30 - 3x^2 + 2x^3}{2 + x}\)
Unit 14

Factoring Polynomials

Topic A: Factoring
- Highest / greatest common factor
- Factoring polynomials by grouping
- Factoring difference of squares

Topic B: Factoring trinomials
- Factoring $x^2 + bx + c$
- Factoring $ax^2 + bx + c$
- More on factoring $ax^2 + bx + c$
- Factoring trinomials: AC method
- Factoring special products

Topic C: Application of factoring
- Quadratic equations
- Solving quadratic equations
- Application of quadratic equations

Unit 14 Summary

Unit 14 Self-test
Topic A: Factoring

**Highest / Greatest Common Factor**

**Factoring whole numbers**: write the number as a product (multiply) of its prime factors.

**Prime factor**: it is a prime number that has only two factors, 1 and itself.

**Example**: Factor 42.

\[ 42 = 2 \cdot 3 \cdot 7 \]

2, 3 and 7 are prime factors.

**Common factor**: a number or an expression that is a factor of each term of a group of terms.

**Greatest / highest common factor (GCF or HCF)**: the product of the common factors.

**Examples**:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Factors</th>
<th>Common factor</th>
<th>GCF or GCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>2 \cdot 3 \cdot 5</td>
<td>2, 3</td>
<td>6</td>
</tr>
<tr>
<td>42</td>
<td>2 \cdot 3 \cdot 7</td>
<td>2, 3</td>
<td>6</td>
</tr>
<tr>
<td>2xy³ + 6xy²</td>
<td>2 \cdot x \cdot y \cdot y²</td>
<td>2, x, y²</td>
<td>2xy²</td>
</tr>
</tbody>
</table>

**Factoring a polynomial**: express a polynomial as a product of other polynomials. (Factoring is the reverse of multiplication.)

**Examples**:

**Expression**

\[ 3xy (2x - 4xy + 3) = 6x²y - 12x²y² + 9xy = 3xy (2x) - 3xy (4xy) + 3xy \cdot 3 = 3xy (2x - 4xy + 3) \]

**Factors**

\[ 2 \cdot 3 \cdot 5 \cdot \frac{1}{3} \cdot w = 2 \cdot x \cdot y \cdot y² \cdot \frac{1}{3} \cdot w = 2 \cdot x \cdot y² = 2xy² \]

**GCF or HCF**

3xy

**Tips**:

- Factor each term and pull out the GCF.
- If the first term is negative, factor out a negative GCF to make the first term positive.
Factoring Polynomials by Grouping

Steps for factoring polynomials by grouping:

<table>
<thead>
<tr>
<th>Steps</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regroup terms with the GCF.</td>
<td>Factor $16x^2 - 4x + 28x - 7$.</td>
</tr>
<tr>
<td>Factor out the GCF from each group.</td>
<td>$16x^2 - 4x + 28x - 4 = (16x^2 - 4x) + (28x - 7)$</td>
</tr>
<tr>
<td>Factor out the GCF again from last step.</td>
<td>$= 4x (4x - 1) + 7(4x - 1)$</td>
</tr>
<tr>
<td></td>
<td>$= (4x - 1) (4x + 7)$</td>
</tr>
</tbody>
</table>

Factoring completely: continue factoring until no further factors can be found.

Example: Factor the following completely.

1) $35xy^2 - 7x^2y + 5y - x = (35xy^2 - 7x^2y) + (5y - x)$
   $= 7xy (5y - x) + (5y - x) \cdot 1$
   $= (5y - x) (7xy + 1)$

2) $3xy + yz - 5yz + 6xy = (3xy + 6xy) + (yz - 5yz)$
   $= 3xy (1 + 2) + yz (1 - 5)$
   $= 3xy (3) + yz (-4)$
   $= 9xy - 4yz$

3) $t^3 - t^2w - tw^2 + w^3 = (t^3 - t^2w) - (tw^2 - w^3)$
   $= t^2 (t - w) - w^2 (t - w)$
   $= (t - w) (t^2 - w^2)$
   $= (t - w) (t + w) (t - w)$
   $= (t - w)^2 (t + w)$

Tip: Identify patterns of common factors such as $5y - x$, $t - w$ ...
### Factoring Difference of Squares

**Factoring difference of squares:**

<table>
<thead>
<tr>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^2 - b^2 = (a + b) (a-b) ) or ( a^2 - b^2 = (a - b) (a+b) )</td>
<td>( x^2 - 49 = x^2 - 7^2 = (x + 7) (x - 7) ) ( y^2 - 81 = y^2 - 9^2 = (y - 9) (y + 9) )</td>
</tr>
</tbody>
</table>

**Note:**
- \( a^2 + b^2 \) cannot be factored.
- Always factor out the greatest common factor (GCF) first.
- Determine the perfect square or the square root of each term.

Recall that **factoring is the reverse of multiplication.**

\[
a^2 - b^2 = (a + b) (a-b) \]

#### Example:

Factor the following completely.

1) \( 2x^2 - 18 = 2 (x^2 - 9) \)
   - Factor out 2.
   - \( 9 = 3^2 \) or \( \sqrt{9} = 3 \)
   - \( a^2 - b^2 = (a + b) (a-b) \): \( a = x, \ b = 3 \)
   - \( 2x^2 - 18 = 2 (x^2 - 3^2) = 2 (x + 3) (x - 3) \)

2) \( 1 - 64u^2 = 1^2 - 8^2 u^2 \)
   - \( 1 = 1^2, \ 64 = 8^2 \) or \( \sqrt{64} = 8 \)
   - \( a^2 b^n = (a b)^n \)
   - \( a^2 - b^2 = (a + b) (a-b) \): \( a = 1, \ b = 8u \)
   - \( 1 - 64u^2 = 1^2 - (8u)^2 = (1+ 8u) (1 - 8u) \)

3) \( 100t^2 - 256 = 10^2t^2 - 16^2 \)
   - \( 256 = 16^2 \) or \( \sqrt{256} = 16 \)
   - \( a^2 b^n = (a b)^n \)
   - \( a^2 - b^2 = (a + b) (a-b) \): \( a = 10t, \ b = 16 \)
   - \( 100t^2 - 256 = (10t)^2 - 16^2 = (10t + 16) (10t - 16) \)

4) \( 9x^2 - 16y^2 = 3^2x^2 - 4^2y^2 = (3x)^2 - (4y)^2 \)
   - \( a^2 b^n = (a b)^n \)
   - \( a^2 - b^2 = (a + b) (a-b) \): \( a = 3x, \ b = 4y \)
   - \( 9x^2 - 16y^2 = (3x + 4y) (3x - 4y) \)

5) \( 36x^8 - 0.04 = 6^2 (x^4)^2 - 0.2^2 \)
   - \( 0.04 = 0.2^2 \) or \( \sqrt{0.04} = 0.2 \) , \( x^4 = (x^4)^1 \)
   - \( a^2 b^n = (a b)^n \)
   - \( a^2 - b^2 = (a + b) (a-b) \): \( a = 6x^4, \ b = 0.2 \)
   - \( 36x^8 - 0.04 = (6x^4)^2 - 0.2^2 = (6x^4 + 0.2) (6x^4 - 0.2) \)
**Topic B: Factoring Trinomials**

**Factoring \( x^2 + bx + c \): cross-multiplication method**

- Setting up two sets of parenthesis.
- Factor the first term \( x^2 \): \( x = x \cdot x \)
- Factor the last term \( c \) (by trial and error): \( c = c_1 \cdot c_2 \)
- Cross multiply and then add up to the middle term.
- Complete the parenthesis with \( x + c_1 \) and \( x + c_2 \).
- Check using FOIL.

**Factoring \( x^2 + bx + c \) using the cross-multiplication method**

<table>
<thead>
<tr>
<th>In general</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 + bx + c = ( ) ( ) )</td>
<td>( x^2 - 8x + 15 = ( ) ( ) )</td>
</tr>
<tr>
<td>( x )</td>
<td>( x )</td>
</tr>
<tr>
<td>( x \cdot x \equiv x^2 )</td>
<td>( x \cdot x \equiv x^2 )</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>(-5)</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>(-3)</td>
</tr>
<tr>
<td>( c_1 \cdot c_2 = c )</td>
<td>(( -3)(-5) = 15 )</td>
</tr>
<tr>
<td>((c_1)(x) + (c_2)(x) \equiv b x )</td>
<td>((-5) x + (-3) x \equiv -8x ) yes!</td>
</tr>
<tr>
<td>( x^2 + bx + c = (x + c_1)(x + c_2) )</td>
<td>( x^2 - 8x + 15 = (x - 5)(x - 3) ) Check: (-5 + (-3) = -8 ) yes!</td>
</tr>
</tbody>
</table>

**Summary:** Factoring \( x^2 + bx + c \)

- \( x^2 + (c_1 + c_2) x + c_1c_2 = (x + c_1)(x + c_2) \)
- Check: \( c_1 + c_2 x = b x \)

**Example:** Factor the following:

1) \( a^2 - 11a + 30 = ( ) ( ) \)

- \( a = -5 \)
- \( a \cdot a = a^2 \) \((-5)\cdot(-6) = 30 \)
- \((-5)a + (-6) a = -11a \) yes!
- Check: \(-5 + (-6) = -11 \) yes!
- Answer: \( a^2 - 11a + 30 = (a - 5)(a - 6) \)

2) \( 3x^2 + 24x - 27 = 3(x^2 + 8x - 9) \)

- \( x = 1 \)
- \( x \cdot x = x^2 \) \((-1)(9) = -9 \)
- \((-1)x + 9x = 8x \) yes!
- Check: \(-1 + 9 = 8 \) yes!
- Answer: \( 3(x^2 + 8x - 9) = 3(x - 1)(x + 9) \)

**Note:** Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form \((ax^2 + bx + c)\) first.

**Trial and error process**

<table>
<thead>
<tr>
<th>( a^2 - 11a + 30 )</th>
<th>( a^2 - 11a + 30 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = -5 )</td>
<td>( a = 3 )</td>
</tr>
</tbody>
</table>
| \( a \cdot a = a^2 \) | \(-5\cdot6 = 30 \)
| \((-5)a + (-6) a = -11a \) yes! | \( 3a + 10a = -11a \) no |
| Answer: \( a^2 - 11a + 30 = (a - 5)(a - 6) \) | \( 6a + 5a = -11a \) no |

<table>
<thead>
<tr>
<th>( x^2 + 8x - 9 )</th>
<th>( x^2 + 8x - 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 3 )</td>
<td>( x = -3 )</td>
</tr>
</tbody>
</table>
| \( x \cdot x = x^2 \) | \((-1)(9) = -9 \)
| \((-1)x + 9x = 8x \) yes! | \( 3x + (-3)x = 8x \) no |
| Answer: \( x^2 + 8x - 9 = (x - 1)(x + 9) \) | \((-3)x + 3x = 8x \) no |
Factoring \( ax^2 + bx + c \)

## Procedure for factoring \( ax^2 + bx + c \) using the cross-multiplication method:

### Steps

- Setting up two sets of parenthesis.
- Factor the first term \( ax^2 \): \( ax^2 = a_1x \cdot a_2x \)
- Factor the last term \( c \) (by trial and error): \( c = c_1 \cdot c_2 \)
- Cross multiply and then add up to the middle term.
- Complete the parenthesis with \((a_1x + c_1)\) and \((a_2x + c_2)\).
- Check using FOIL.

### In general

\[
ax^2 + bx + c = (a_1x + c_1)(a_2x + c_2)
\]

### Example

\[
3x^2 - 2x - 8 = (x - 2)(3x + 4)
\]

### Tip:

- Write the factors with their appropriate signs (+ or −) to get the right middle term.

---

### Factoring \( ax^2 + bx + c \) using the cross-multiplication method

<table>
<thead>
<tr>
<th>In general</th>
<th>Example</th>
</tr>
</thead>
</table>
| \[
ax^2 + bx + c = (a_1x + c_1)(a_2x + c_2)
\] | \[
4x^2 + 7x + 3 = (4x + 3)(x + 1)
\] |

### Tip:

- Cross multiply and then add up to the middle term.

### Summary: Factoring \( ax^2 + bx + c \)

\[
\frac{a_1}{a_2} = \frac{(c_1a_2 + c_2a_1)}{x + c_1} = \frac{(a_1x + c_1)(a_2x + c_2)}{c_1 \cdot c_2}
\]

### Note:

- Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form \((ax^2 + bx + c)\) first.
More on Factoring \( ax^2 + bx + c \)

**Example:** Factor \( 6y^2 - 17y - 14 \).

\[
6y^2 - 17y - 14 = (\phantom{-} \phantom{-}2 \phantom{-})(\phantom{-}\phantom{-}7)
\]

\[
\begin{array}{l}
3y \cdot 2y = 6y^2 \\
2(-7) = -14
\end{array}
\]

\[
(2) (2y) + (-7) (3y) = -17y \quad \text{yes!}
\]

\[
6y^2 - 17y - 14 = (3y + 2)(2y - 7)
\]

**Trial and error process**

1) \[
\begin{array}{l}
6y^2 - 17y - 14 \\
y \quad -7 \\
6y \quad 2
\end{array}
\]

\[
(-7)(6y) + 2y \not= -17y \quad \text{no}
\]

2) \[
\begin{array}{l}
6y^2 - 17y - 14 \\
3y \quad -7 \\
2y \quad -2
\end{array}
\]

\[
7(2y) + (-2)(3y) \not= -17y \quad \text{no}
\]

3) \[
\begin{array}{l}
6y^2 - 17y - 14 \\
6y \quad -7 \\
2y \quad -2
\end{array}
\]

\[
2y + (-7)(6y) \not= -17y \quad \text{no}
\]

Check: \[
(3y + 2)(2y - 7) = 6y^2 - 21y + 4y - 14
\]

\[
\sqrt{(2y + 2)(2y - 7)} = 6y^2 - 17y - 14 \quad \text{Correct!}
\]

**Example:** Factor the following completely.

1) \[
28x - 24 + 20x^2 = 20x^2 + 28x - 24
\]

\[
= 4(5x^2 + 7x - 6) = 4(\phantom{-} \phantom{-}x \phantom{-})(\phantom{-}\phantom{-})
\]

\[
\begin{array}{l}
x \quad -2 \\
5x \quad -3
\end{array}
\]

\[
4(5x^2 + 7x - 6) = 4(x + 2)(5x - 3)
\]

Note: Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form \((ax^2 + bx + c)\) first.

2) \[
8a^2 - 6ab - 5b^2 = (\phantom{-} \phantom{-}b \phantom{-})(\phantom{-}\phantom{-})
\]

\[
\begin{array}{l}
2a \quad -b \\
4a \quad -5b
\end{array}
\]

\[
8a^2 - 6ab - 5b^2 = (2a + b)(4a - 5b)
\]

3) \[
2r^4 + 14r^2 + 20 = 2(r^4 + 7r^2 + 10) = 2(\phantom{-} \phantom{-}r^2 \phantom{-})(\phantom{-}\phantom{-})
\]

\[
\begin{array}{l}
r^2 \quad -2 \\
r^2 \quad -5
\end{array}
\]

\[
2r^4 + 14r^2 + 20 = 2(r^2 + 2)(r^2 + 5)
\]

**Unit 14** Factoring Polynomials
Factoring Trinomials: AC Method

AC method for factoring trinomials: \( ax^2 + bx + c \)

<table>
<thead>
<tr>
<th>Factoring ( ax^2 + bx + c = 0 ) by Grouping</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Steps</strong></td>
<td><strong>Solve</strong> 14x + 6 = -8x^2</td>
</tr>
<tr>
<td>• Convert to standard form (descending order) if necessary.</td>
<td>8x^2 + 14x + 6 = 0</td>
</tr>
<tr>
<td>• Factor out the greatest common factor (GCF).</td>
<td>2 (4x^2 + 7x + 3) = 0</td>
</tr>
<tr>
<td>• Multiply ( a ) and ( c ) in ( ax^2 + bx + c ).</td>
<td>ac = 4 \cdot 3 = 12</td>
</tr>
<tr>
<td>• Factor the product ( ac ) that sum to the middle coefficient ( b ).</td>
<td>4 \cdot 3 = 12, 4 + 3 = 7</td>
</tr>
<tr>
<td>• Rewrite the middle term as the sum using the factors found in last step.</td>
<td>2 (4x^2 + 7x + 3) = 0</td>
</tr>
<tr>
<td>• Factor by grouping.</td>
<td>2 (4x^2 + 4x + 3x + 3) = 0</td>
</tr>
<tr>
<td></td>
<td>2 [4x (x + 1) + 3 (x + 1)] = 0</td>
</tr>
<tr>
<td></td>
<td>2 (x + 1) (4x + 3) = 0</td>
</tr>
</tbody>
</table>

**Example:** Factor \( 6x^2 - 16 = 4x \) using \( ac \) method.

**Steps**

- Write in standard form:
- Factor out the greatest common factor:
- Multiply \( a \) and \( c \) in \( ax^2 + bx + c \):
- Factor the product \( ac \) that sum to the middle coefficient \( b \).

(There are different pairs to get the product of \( ac \) of -24. Try to find two numbers that multiply to \( ac \) and add to obtain \( b = -2 \).)

<table>
<thead>
<tr>
<th>Some factors of ( ac )</th>
<th>(-24)</th>
<th>Sum of factors</th>
<th>( (b = -2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3 &amp; 8</td>
<td>-3 + 8 = 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-4 &amp; 6</td>
<td>-4 + 6 = 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 &amp; -3</td>
<td>8 + (-3) = 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 &amp; -6</td>
<td>4 + (-6) = -2</td>
<td>Correct!</td>
<td></td>
</tr>
</tbody>
</table>

The right choices are 4 an -6, since they both add up to \( b = -2 \). 4 (-6) = -24, 4 + (-6) = -2

- Rewrite the middle term as \( 4x - 6x \).
- Factor by grouping.

\[
2(3x^2 - 2x - 8) = 0 \\
2 (3x^2 + 4x - 6x - 8) = 0 \\
2 [x (3x + 4) - 2(3x + 4)] = 0 \\
2(3x + 4) (x - 2) = 0
\]
Recall that **factoring is the reverse of multiplication.**

Special products:

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
</table>
| Square of sum (perfect square trinomial) | $a^2 + 2ab + b^2 = (a + b)^2$ | $x^2 + 10x + 25 = (x + 5)^2$  
  $x = 5$  
  Check: $(x + 5)^2 = x^2 + 2 \cdot x \cdot 5 + 5^2 = x^2 + 10x + 25$ $\sqrt{}$  
  $a = x$, $b = 5$ |
| Square of difference (perfect square trinomial) | $a^2 - 2ab + b^2 = (a - b)^2$ | $9y^2 - 24y + 16 = (3y - 4)^2$  
  $3y = 4$  
  Check: $(3y - 4)^2 = (3y)^2 - 2(3y) \cdot 4 + 4^2 = 9y^2 - 24y + 16$ $\sqrt{}$  
  $a = 3y$, $b = 4$ |

**Note:** The quickest way to factor an expression is to recognize it as a special product.

**Memory aid:** $(a^2 \pm ab + b^2) = (a \pm b)^2$  
Notice the reversed plus or minus sign in the second term.

**To use perfect square trinomial formulas:** use cross-multiplication method to factor a perfect square. Then use the square formula to check.

**Example:** Factor the following completely.

1) $28z + 49 + 4z^2 = 4z^2 + 28z + 49$  
$\frac{2z}{2z} \frac{7}{7} = (2z + 7)$  
$\frac{2z}{2z} \frac{7}{7} = (2z + 7)^2$  
Check: $(2z + 7)^2 = (2z)^2 + 2 \cdot 2z \cdot 7 + 7^2 = 4z^2 + 28z + 49$ $\sqrt{}$  
$a^2 + 2ab + b^2 = (a + b)^2$ : $a = 2z$, $b = 7$

2) $50p^2 - 40p + 8 = 2(25p^2 - 20p + 4)$  
$\frac{5p}{5p} \frac{-2}{-2} = 2(5p - 2)^2$  
Check: $2(5p - 2)^2 = 2[(5p)^2 - 2(5p) \cdot 2] = 2(25p^2 - 20p + 4)$ $\sqrt{}$  
$a^2 - 2ab + b^2 = (a - b)^2$ : $a = 5p$, $b = 2$

3) $16n^{10} - 48n^5 + 36 = 4(4n^{10} - 12n^5 + 9)$  
$\frac{2n^5}{2n^5} \frac{-3}{-3} = 4(2n^5 - 3)^2$  
Check: $(2n^5 - 3)^2 = (2n^5)^2 - 2(2n^5) \cdot 3 + 3^2 = 4n^{10} - 12n^5 + 9$ $\sqrt{}$  
$a^2 - 2ab + b^2 = (a - b)^2$ : $a = 2n^5$, $b = 3$
Topic C: Application of Factoring

Quadratic Equations

**Quadratic equation**: an equation that has a squared term, such as $7x^2 + 3x - 5 = 0$.

<table>
<thead>
<tr>
<th>Quadratic equations in standard form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ax^2 + bx + c = 0$</td>
</tr>
<tr>
<td>$a \neq 0$</td>
</tr>
</tbody>
</table>

**Incomplete quadratic equation**

<table>
<thead>
<tr>
<th>Incomplete quadratic equation</th>
<th>Example</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ax^2 + bx = 0$ ($c = 0$)</td>
<td>$4x^2 - 3x = 0$</td>
<td>4</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>$ax^2 + c = 0$ ($b = 0$)</td>
<td>$8x^2 + 5 = 0$</td>
<td>8</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

**Zero-product property**:

If $A \cdot B = 0$, then either $A = 0$ or $B = 0$ (or both).

Note: “or” means possibility of both.

**Solving incomplete quadratic equations**

<table>
<thead>
<tr>
<th>Incomplete quadratic equation</th>
<th>Steps</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use the zero-product property to solve $ax^2 + bx = 0$</td>
<td>- Express in $ax^2 + bx = 0$&lt;br&gt; - Factor: $x(ax + b) = 0$&lt;br&gt; - Apply the zero-product property: $x = 0$ or $ax + b = 0$&lt;br&gt; - Solve for $x$: $x = 0$, $x = -\frac{b}{a}$</td>
<td>Solve $11x^2 - 6x = 0$&lt;br&gt; $11x^2 + 6x = 0$&lt;br&gt; $x(11x + 6) = 0$&lt;br&gt; $x = 0$, $11x + 6 = 0$&lt;br&gt; $x = 0$, $x = -\frac{6}{11}$</td>
</tr>
<tr>
<td>Use the square root method to solve $ax^2 - c = 0$ (or $ax^2 = c$)</td>
<td>- Express in $ax^2 = c$&lt;br&gt; - Divide both sides by $a$: $x^2 = \frac{c}{a}$&lt;br&gt; - Take the square root of both sides: $x = \pm \sqrt{\frac{c}{a}}$</td>
<td>Solve $64x^2 - 9 = 0$&lt;br&gt; $64x^2 = 9$&lt;br&gt; $x^2 = \frac{9}{64}$&lt;br&gt; $x = \pm \frac{3}{8}$</td>
</tr>
</tbody>
</table>
Solving Quadratic Equations

**Solve a quadratic equation:** a quadratic equation \( ax^2 + bx + c = 0 \) can be written as:

\[
(x + a)(x + b) = 0
\]

Set each term equal to zero:

\[
x + a = 0 \quad x + b = 0
\]

Solutions:

\[
x = -a \quad x = -b
\]

Solve for \( x \).

**Example:** Solve for \( x \).

\((x + 6)(x - 11) = 0\)

\[
x + 6 = 0 \quad x - 11 = 0
\]

\[
x = -6 \quad x = 11
\]

Solve for \( x \).

**Example:** Solve the quadratic equation \( x^2 - x - 20 = 0 \).

1) \( x^2 - x - 20 = 0 \)

\[
x = -4 \quad x = 5
\]

**Example:** Solve for \( x \).

\((x + 6)(x - 11) = 0\)

\[
x + 6 = 0 \quad x - 11 = 0
\]

\[
x = -6 \quad x = 11
\]

**Example:** Solve the quadratic equation \( x^2 - x - 20 = 0 \).

2) \( 6x^2 - 13x = 15 \)

\[
6x^2 - 13x - 15 = 0
\]

\[
6x - 5 \quad -3
\]

\((6x + 5)(x - 3) = 0\)

\[
6x + 5 = 0 \quad x - 3 = 0
\]

\[
x = -\frac{5}{6} \quad x = 3
\]

**Example:** Solve the quadratic equation \( x^2 - x - 20 = 0 \).

3) \( x^2 - \frac{2}{3}x = \frac{1}{3}x \)

\[
x = \frac{1}{3} \quad x = \frac{2}{3}
\]

\((x + \frac{1}{3})(x - \frac{2}{3}) = 0\)

\[
x + \frac{1}{3} = 0 \quad y - \frac{2}{3} = 0
\]

\[
x = -\frac{1}{3} \quad x = \frac{2}{3}
\]
Application of Quadratic Equations

Review number problems - examples

<table>
<thead>
<tr>
<th>English phrase</th>
<th>Algebraic expression/equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 more than the difference of the square of a number and 11 is 32.</td>
<td>((x^2 - 11) + 6 = 32)</td>
</tr>
<tr>
<td>The quotient of 5 and the product of 9 and a number is 7 less than the number.</td>
<td>(\frac{5}{9x} = x - 7)</td>
</tr>
<tr>
<td>The product of 9 and the square of a number decreased by 13 is 21.</td>
<td>(9x^2 - 13 = 21)</td>
</tr>
<tr>
<td>15 more than the quotient of 4x by 7 is 5 times the square of a number.</td>
<td>(15 + \frac{4x}{7} = 5x^2)</td>
</tr>
</tbody>
</table>

Review consecutive integers

<table>
<thead>
<tr>
<th>English phrase</th>
<th>Algebraic expression</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three consecutive odd integers</td>
<td>(x, x + 2, x + 4)</td>
<td>If (x = 1, x + 2 = 3, x + 4 = 5)</td>
</tr>
<tr>
<td>Three consecutive even integers</td>
<td>(x, x + 2, x + 4)</td>
<td>If (x = 2, x + 2 = 4, x + 4 = 6)</td>
</tr>
<tr>
<td>The product of two consecutive odd integers is 35.</td>
<td>(x(x + 2) = 35)</td>
<td></td>
</tr>
<tr>
<td>Three consecutive even integers whose sum is 12.</td>
<td>(x + (x + 2) + (x + 4) = 12)</td>
<td></td>
</tr>
</tbody>
</table>

Examples:

1) The product of a number and 4 more than the square of the number is 21. Find the number(s).
   - Let \(x\) = the number
   - Equation \(x^2 + 4x = 21\)
   - Solve for \(x\): \(x + 4x - 21 = 0\)
     
     \[
     \begin{align*}
     x + 7 & = 0 \\
     x & = -7
     \end{align*}
     \]
     \[
     \begin{align*}
     x - 3 & = 0 \\
     x & = 3
     \end{align*}
     \]

2) The product of two consecutive even integers is 48. Find the integers.
   - Let \(x\) = the first even integer
   - Equation \(x(x + 2) = 48\)
   - Solve for \(x\): \(x^2 + 2x - 48 = 0\)
     
     \[
     \begin{align*}
     x + 6 & = 0 \\
     x & = -6
     \end{align*}
     \]
     \[
     \begin{align*}
     x + 8 & = 0 \\
     x & = -8
     \end{align*}
     \]

If \(x = 6, x + 2 = 8\) If \(x = -8, x + 2 = -6\)
Dimension (length and width) problems:

**Example:** Robert is going to replace the old carpet in his bedroom, which is a rectangle and has a *length 3 meters greater than its width*. If the *area* of his bedroom is 54 square meters \((m^2)\), what will be the dimensions of the carpet?

**Steps**
- Organize the facts.
- Draw a diagram.
- Equation:
- Solve the equation.
- Zero-product property:
- Solutions:
- Answer (the size of the carpet):

**Solution**

<table>
<thead>
<tr>
<th>Facts</th>
<th>Area ( A = 54m^2 )</th>
<th>Length = Width + 3m</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unknowns</strong></td>
<td>Width = ( x ) , Length = ( x + 3 )</td>
<td></td>
</tr>
</tbody>
</table>

**Steps**
- Organize the facts.

**Solution**

\[
x (x + 3) = 54
\]

\[
x^2 + 3x = 54
\]

*Distribute.*

\[
x^2 + 3x - 54 = 0 \quad ax^2 + bx + c = 0
\]

*Factor:*

\[
(x + 9)(x - 6) = 0
\]

*Zero-product property:

\[
x + 9 = 0 \quad x - 6 = 0
\]

\[
x = -9 \quad x = 6
\]

Since the width of a rectangle cannot be negative, eliminate \( x = -9 \).

**Answer:**

**Width** = \( x = 6 \) m

**Length** = \( x + 3 = 6 + 3 = 9 \) m

**Triangle problem:**

**Example:** A triangle is 1 meter wider than it is tall. The area is 36 \(m^2\). Find the base and the height.

- Organize the facts.

**Solution**

<table>
<thead>
<tr>
<th>Facts</th>
<th>Area ( A = 36m^2 )</th>
<th>Base = Height + 1m</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unknowns</strong></td>
<td>Height = ( x ) , Base = ( x + 1 )</td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{1}{2}(x + 1)x = 36 \quad \text{Area: } A = \frac{1}{2} bh
\]

\[
x^2 + x = 72 \quad \text{Multiply both sides by 2.}
\]

\[
x^2 + x - 72 = 0 \quad ax^2 + bx + c = 0
\]

*Factor:*

\[
(x + 9)(x - 8) = 0
\]

*Zero-product property:

\[
x + 9 = 0 \quad x - 8 = 0
\]

\[
x = -9 \quad x = 8
\]

**Height** = \( x = 8 \) m

(Since the height of a triangle cannot be negative, eliminate \( x = -9 \).)

**Base** = \( x + 1 = 8 + 1 = 9 \) m
Factoring whole numbers: write the number as a product of its prime factors.

Common factor: a number or an expression that is a factor of each term of a group of terms.

Greatest / highest common factor (GCF or HCF): the product of the common factors.

Factoring a polynomial: express a polynomial as a product of other polynomials. It is the reverse of multiplication.

Steps for factoring polynomials by grouping:
- Regroup terms with the GCF.
- Factor out the GCF from each group.
- Factor out the GCF again from last step.

Special products:

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference of squares</td>
<td>$a^2 - b^2 = (a + b)(a - b)$</td>
</tr>
<tr>
<td>Square of sum</td>
<td>$a^2 + 2ab + b^2 = (a + b)^2$</td>
</tr>
<tr>
<td>Square of difference</td>
<td>$a^2 - 2ab + b^2 = (a - b)^2$</td>
</tr>
</tbody>
</table>

Memory aid: $(a^2 \pm ab + b^2) = (a \pm b)^2$

Cross-multiplication method:

**Factoring $x^2 + bx + c$ using the cross-multiplication method**

<table>
<thead>
<tr>
<th>In general</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + bx + c = ( ) ( )$</td>
<td>$x^2 - 8x + 15 = ( ) ( )$</td>
</tr>
</tbody>
</table>

$x \cdot x = x^2$

$c_1 \cdot c_2 = c$

$(c_1)(x) + (c_2)(x) = bx$

$x^2 + bx + c = (x + c_1)(x + c_2)$

Examples:

- $(5)x + (-3)x = -8x$ yes!
- $x^2 - 8x + 15 = (x - 5)(x - 3)$

Tips:
- Cross multiply and then add up to the middle term.
- Write the factors with their appropriate signs (+ or −) to get the right middle term.

**Factoring $ax^2 + bx + c$ using the cross-multiplication method**

<table>
<thead>
<tr>
<th>In general</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ax^2 + bx + c = ( ) ( )$</td>
<td>$4x^2 + 7x + 3 = ( ) ( )$</td>
</tr>
</tbody>
</table>

$a \cdot x \cdot a \cdot x = ax^2$

$c = c_1 \cdot c_2$

$(c_1)(a_2x) + (c_2)(a_1x) = bx$

$a_1x + a_2x + a_3x = ax^2$

$a = a_1x + c_1$ $a_2x + c_2$

Tips:
- Cross multiply and then add up to the middle term.
- Always factor out the greatest common factor (GCF) and rewrite in descending order or standard form $(ax^2 + bx + c)$ first.
Factoring polynomials:

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two terms</td>
<td>- If it is a perfect square:</td>
</tr>
<tr>
<td>(binomial)</td>
<td>$a^2 - b^2 = (a + b)(a - b)$</td>
</tr>
<tr>
<td></td>
<td>- If not, use the distributive property:</td>
</tr>
<tr>
<td></td>
<td>$ac + bc = c(a + b)$</td>
</tr>
<tr>
<td>Three terms</td>
<td>$ax^2 + bx + c$ : use the cross-multiplication or AC methods.</td>
</tr>
<tr>
<td>(trinomial)</td>
<td></td>
</tr>
<tr>
<td>Four terms</td>
<td>Factor by grouping</td>
</tr>
</tbody>
</table>

**Quadratic equation:** an equation that has a squared term.

**Quadratic equations in standard form**

| $ax^2 + bx + c = 0$  | $a 
eq 0$ |

Incomplete quadratic equation

<table>
<thead>
<tr>
<th>Incomplete quadratic equation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$ax^2 + bx = 0$</td>
<td>$(c = 0)$</td>
</tr>
<tr>
<td>$ax^2 + c = 0$</td>
<td>$(b = 0)$</td>
</tr>
</tbody>
</table>

**Zero-product property:**

<table>
<thead>
<tr>
<th>Zero-product property</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>If $A \cdot B = 0$, then</td>
<td>either $A = 0$ or</td>
</tr>
<tr>
<td>$A$ and $B$ are algebraic</td>
<td>$B = 0$ (or both)</td>
</tr>
<tr>
<td>expressions.</td>
<td></td>
</tr>
</tbody>
</table>

**Solving incomplete quadratic equations**

| Incomplete quadratic         | Steps                                                                 |
| equation                     |                                                                     |
| Use the zero-product         | - Express in $ax^2 + bx = 0$                                        |
| property to solve            | - Factor: $x(ax + b) = 0$                                           |
| $ax^2 + bx = 0$              | - Apply the zero-product property:                                  |
|                              | $x = 0, ax + b = 0$                                                 |
|                              | - Solve for $x$: $x = 0, x = -\frac{b}{a}$                        |
| Use the square root method   | - Express in $ax^2 = c$                                             |
| to solve $ax^2 - c = 0$      | - Divide both sides by $a$: $x^2 = \frac{c}{a}$                     |
|                              | - Take the square root of both sides: $x = \pm \frac{\sqrt{c}}{a}$ |
Unit 14:  Self-Test

Factoring Polynomials

Topic A

1. Factor 60.

2. Find the greatest common factor (GCF) for the following.
   a) \(5x^2 - 20x\)
   b) \(3a^3b + 15a^4b - 21ab\)
   c) \(17y^2(y + 4) - (2y + 8)\)
   d) \(\frac{1}{4}x^3 - \frac{3}{4}xy^2 + \frac{5}{4}x\)
   e) \(-4y^3 - 8y^2 + 20y\)

3. Factor the following completely.
   a) \(25x^2 - 5x + 20x - 4\)
   b) \(48ab^2 - 8a^2b + 6b - a\)
   c) \(4uv + vw - 7vw + 21uv\)
   d) \(x^3 - x^2y - xy^2 + y^3\)
   e) \(5y^2 - 20\)
   f) \(1 - 49w^2\)
   g) \(81u^2 - 121\)
   h) \(25a^2 - 36b^2\)
   i) \(4y^6 - 0.09\)

Topic B

4. Factor the following:
   a) \(x^2 + 9x + 20\)
   b) \(x^2 - 10x + 24\)
   c) \(x^2 - 3x - 18\)
   d) \(2x^2 + 10x - 28\)
   e) \(4x^2 - 7x - 15\)
f) \(5y^2 + 9y - 18\)
g) \(24ab^2 - 4a^2b + 6b - a\)
h) \(6uv + vs - 7vs + 11uv\)

5. Factor the following using the ac method.
   a) \(6x^2 - 60 = 9x\)
   b) \(6x^2 + 4x - 16\)

6. Factor the following completely.
   (Use the cross-multiplication method to factor a perfect square. Then use the square formula to check.)
   a) \(9x^2 + 30x + 25\)
   b) \(27 + 12y^2 - 36y\)
   c) \(18r^8 - 24r^4 + 8\)

**Topic C**

7. Solve for \(x\).
   a) \(23x^2 = -7x\)
   b) \(81x^2 - 49 = 0\)
   c) \((x + 9)(x - 17) = 0\)

8. Solve the following quadratic equations.
   a) \(x^2 - x - 42 = 0\)
   b) \(7x^2 - 31x = 20\)
   c) \(x^2 + \frac{3}{16} = x\)

9. The product of a number and 5 more than the square of the number is 36. Find the number(s).

10. The product of two consecutive even integers is 24. Find the integers.

11. Lisa is going to replace old carpet in her living room, which is a rectangle and has a length 2 meters greater than its width. If the area of her living room is 63 square meters \((m^2)\), what will be the dimensions of the carpet?

12. A triangle is 2 meters wider than it is tall. The area is 24\(m^2\). Find the base and the height.
Unit 15

Graphing Linear Equations

Topic A: Cartesian graphing

- The Coordinate plane
- Graphing linear equations

Topic B: The slope of a straight line

- Slope
- Vertical and horizontal lines

Topic C: Graphing a linear equation

- Slope-intercept equation of a line
- Graphing using the slope and the y-intercept
- Graphing linear equations
  - Intercept method

Topic D: Writing equations of lines

- Finding an equation of a line

Unit 15 Summary

Unit 15 Self-test
Topic A: Cartesian Graphing

The coordinate plane (or Cartesian / rectangular coordinate system): a powerful tool to mark a point and solution of linear equations on a graph.

- Coordinate axes:
  - x axis - the horizontal line.
  - y axis - the vertical line.

- The origin: the intersection of the x and y axes (both lines are 0 at the origin).

Ordered pair \((x, y)\): a pair of numbers (each point on the plane corresponds to an ordered pair).

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>((x, y))</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st quadrant I</td>
<td>((+x, +y))</td>
<td>(+2, +3)</td>
</tr>
<tr>
<td>2nd quadrant II</td>
<td>((-x, +y))</td>
<td>(-2, +3)</td>
</tr>
<tr>
<td>3rd quadrant III</td>
<td>((-x, -y))</td>
<td>(-2, -3)</td>
</tr>
<tr>
<td>4th quadrant IV</td>
<td>((+x, -y))</td>
<td>(+2, -3)</td>
</tr>
</tbody>
</table>

Example: Plot the points and name the quadrants.

\((1, 3)\), \((-3, 2)\), \((-2, -2)\), \((2, -1)\)

\((1, 3): \text{I}, \quad (-3, 2): \text{II}, \quad (-2, -2): \text{III}, \quad (2, -1): \text{IV}\)

x - intercept \((x, 0)\): the point at which the graph crosses the x - axis.

Example: \((x, y) = (3, 0)\)

y - intercept \((0, y)\): the point at which the graph crosses the y - axis.

Example: \((x, y) = (0, 2)\)
Graphing Linear Equations

A linear (first-degree) equation: an equation whose graph is a straight line.

A linear equation in two variables: a linear equation that contains two variables, such as \(2x + y = 3\).

The standard form of linear equation in two variables: \(Ax + By = C\)

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Ax + By = C)</td>
<td>(5x - 7y = 4)</td>
</tr>
</tbody>
</table>

Solutions of equations: solutions for a linear equation in two variables are an ordered pair. They are the particular values of the variables in the equation that makes the equation true.

Example: Find the ordered pair solution of the given equation.

\[2x - 3y = 7, \quad \text{when } x = 2.\]

Replace \(x\) with 2.

\[2(2) - 3y = 7 \quad 4 - 3y = 7 \quad \text{Subtract 4 from both sides.}\]

\[-3y = 3 \quad y = -1 \quad \text{Divide -3 both sides.}\]

Check: \[2 \cdot 2 - 3(-1) = 7, \quad 7 = 7. \quad \text{The ordered pair solution is } (2, -1).\]

The graph of an equation is the diagram obtained by plotting the set of points where the equation is true (or satisfies the equation).

Procedure to graph a linear equation

Steps

- Choose two values of \(x\), calculate the corresponding \(y\), and make a table.
- Plot these two points on the coordinate plane.
- Connect the points with a straight line. (Any two points determine a straight line.)
- Check with the third point.

Is third point \((2, 1)\) on the line? Yes. Correct!

Example: Graph \[y = \frac{1}{2}x - 3\] and determine another point.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3</td>
<td>(0, -3)</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
<td>(2, -2)</td>
</tr>
</tbody>
</table>

Another solution: \((8, 1)\)
Topic B: The Slope of a Straight Line

Recall: the graph of a linear equation is a straight line.

Slope \((m)\) (grade or pitch): the slope of a straight line is the rate of change. It is a measure of the “steepness” or “incline” of the line and indicates whether the line rises or falls.

A line with a positive slope rises from left to right and a line with a negative slope falls.

The slope formula:

\[
\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

The slope of the straight line that passes through two points \((x_1, y_1)\) and \((x_2, y_2)\):

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m = \frac{y_1 - y_2}{x_1 - x_2} \quad x_1 \neq x_2
\]

Example: Determine the slope containing points (3, -2) and (4, 1).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{4 - 3} = \frac{3}{1} = 3
\]

or

\[
m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-2 - 1}{3 - 4} = \frac{-3}{-1} = 3
\]

Example: Determine the slope of \(6x - y - 5 = 0\).

\[
\begin{align*}
x & \quad \quad y = 6x - 5 \\
0 & \quad 6 \cdot 0 - 5 = -5 \quad (x_1, y_1) = (0, -5) \\
1 & \quad 6 \cdot 1 - 5 = 1 \quad (x_2, y_2) = (1, 1)
\end{align*}
\]

Choose calculate

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-5)}{1 - 0} = \frac{6}{1} = 6 \
6x - 5 = y \quad \text{(add } y \text{ both sides.)}
\]

Other points on the line will obtain the same slope \(m\).
Vertical and Horizontal Lines

**Horizontal line**: a line that is parallel to the x-axis.

- It has a zero slope \((m = 0)\).
- With a y-intercept \(y = b\) or \((0, b)\). Where \(b\) is any constant.

**Example**: \(y = -4\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4</td>
<td>(1, -4)</td>
</tr>
<tr>
<td>2</td>
<td>-4</td>
<td>(2, -4)</td>
</tr>
</tbody>
</table>

Given \(y_2 - y_1\):
\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-4)}{2 - 1} = \frac{0}{1} = 0
\]

The horizontal line \(y = -4\) has a zero slope.

**Vertical line**: a line that is parallel to the y-axis.

- It has an infinite slope \((m = \infty)\).
- With a x-intercept \(x = a\) or \((a, 0)\).

**Example**: \(x = -3\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>3</td>
<td>(-3, 3)</td>
</tr>
<tr>
<td>-3</td>
<td>-1</td>
<td>(-3, -1)</td>
</tr>
</tbody>
</table>

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{-3 - (-3)} = \frac{-4}{0} = \infty
\]

The vertical line \(x = -3\) has an infinite slope.

**Summary of horizontal and vertical lines**:

<table>
<thead>
<tr>
<th>Line</th>
<th>Equation</th>
<th>Slope ((m))</th>
<th>Example</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal line</td>
<td>(y = b)</td>
<td>(m = 0)</td>
<td>(y = 2)</td>
<td></td>
</tr>
<tr>
<td>Vertical line</td>
<td>(x = a)</td>
<td>(m = \infty)</td>
<td>(x = 1)</td>
<td></td>
</tr>
</tbody>
</table>

**Example**: 1) Graph \(y = -0.5\)  
2) Graph \(x = 4\)  
3) Graph \(x = 0\)
**Topic C: Graphing a Linear Equation**

**Slope-Intercept Equation of a Line**

Slope - intercept form of a linear equation

- **Slope - intercept equation of a line**
  
  $$y = mx + b$$
  
  - $m$: the slope of the line
  - $b$: y-intercept

**Recall: y - intercept**: the point at which the line crosses the y axis. $b = (0, y)$

**Example**: Identify the slope and y-intercept of the following equations.

1) $y = -3x - 5$
   
   - The slope: $m = -3$
   - y-intercept: $b = -5$ or $(0, -5)$

2) $3y - 2x = 1$
   
   $3y = 2x + 1$
   
   - Add $2x$ on both sides.
   - $y = \frac{2}{3}x + \frac{1}{3}$
   
   - Divide both sides by 3.
   
   - The slope: $m = \frac{2}{3}$
   
   - y-intercept: $b = \frac{1}{3}$ or $(0, \frac{1}{3})$

3) $4x + \frac{1}{3}y = 5$
   
   $4x \cdot 3 + \frac{1}{3}y \cdot 3 = 5 \cdot 3$
   
   - Multiply 3 by each term.
   
   $12x + y = 15$
   
   - Subtract $12x$ from both sides.
   
   $y = -12x + 15$
   
   - The slope: $m = -12$
   
   - y-intercept: $b = 15$ or $(0, 15)$
**Graphing Using the Slope and the y - Intercept**

**Slope-intercept equation:** \( y = mx + b \)

- \( m \) = slope
- \( b \) = \( y \)-intercept

**The slope and a point can determine a straight line.**

**Example:** Graph the equation using the slope and the \( y \)-intercept. \( y = \frac{-2}{3}x + 5 \)

- Plot the \( y \)-intercept (0, 5).
- Determine the rise and run: \( m = \frac{-2}{3} \)
  - The change in \( y \): the rise (move 2 units down, \( y \) is negative).
  - The change in \( x \): the run (move 3 units to the right, \( x \) is positive).
- Plot another point by moving 2 units down and 3 units to the right (3, 3).
- Connect the two points with a straight line.

**Example:** Graph the equation using the slope and the \( y \)-intercept. - \( 9x + 12 = -3y \)

- Convert to the slope - intercept form. \( 3y = 9x - 12 \)
  - Divide each term by (-1).
  - \( y = 3x - 4 \)
  - Divide both sides by 3.
- \( y \)-intercept: \( (0, -4) \)
- Slope: \( m = 3 = \frac{3}{1} \)

**Tip:** \( m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} \)

- \(+y\): move up
- \(-y\): move down
- \(+x\): move to the right
- \(-x\): move to the left
Graphing Linear Equations
- Intercept Method

**Recall:** The $x$-intercept is the point at which the line crosses the $x$-axis. $(a, 0)$
The $y$-intercept is the point at which the line crosses the $y$-axis. $(0, b)$

**Find the intercepts:**

**Example:** Determine the intercepts of the line $5x - y = 6$.

- The $x$-intercept: let $y = 0$, and solve for $x$. $5x - 0 = 6$, $5x = 6$
  
  $x = \frac{6}{5} = 1.2$
  Divide both sides by 5.
  
  $(1.2, 0)$

- The $y$-intercept: let $x = 0$, and solve for $y$. $5 \cdot 0 - y = 6$, $-y = 6$

  $y = -6$
  Divide both sides by -1.

  $(0, -6)$

**Procedure to graph a linear equation using the intercept method**

**Steps**

- Choose $x = 0$ and calculate the corresponding $y$.
- Choose $y = 0$ and calculate the corresponding $x$.
- Plot these two points on the coordinate plane.
- Connect the points with a straight line.
- Check with the third point.

  Is third point $(1, -1)$ on the line? Yes. Correct!

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 5x - 6$</th>
<th>$(x, y)$</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-6</td>
<td>(0, -6)</td>
<td>$y$-intercept</td>
</tr>
<tr>
<td>1.2</td>
<td>0</td>
<td>(1.2, 0)</td>
<td>$x$-intercept</td>
</tr>
</tbody>
</table>

\[
(5 \cdot 1 - y = 6, \quad -y = 6 - 5, \quad y = -1)
\]
## Topic D: Writing Equations of Lines

### Finding an Equation of a Line

**Equation of a straight line:**

<table>
<thead>
<tr>
<th>Straight-line equation</th>
<th>Equation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point-slope form</td>
<td>( y - y_1 = m (x - x_1) )</td>
<td>( y - 3 = -4 (x + 2) )  ( m = -4 ), ( y_1 = 3 ), ( x_1 = -2 )</td>
</tr>
<tr>
<td>Slope-intercept form</td>
<td>( y = mx + b )</td>
<td>( y = 3x - \frac{4}{5} ) ( m = 3 ), ( b = -\frac{4}{5} )</td>
</tr>
</tbody>
</table>

#### Finding an equation of a line from the graph:

**Example:** Write the slope intercept equation of the given line. \( y = mx + b \)

![Graph of a line](image)

- Choose two points on the given line, such as \((0, 5)\) and \((1, 2)\).
- The slope: \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 3}{1 - 0} = -3 \) \( (x_1, y_1) = (0, 5), \ (x_2, y_2) = (1, 2) \)
- \( y \)-intercept: \( b = 5 \) The line crosses the y-axis at \((0, 5)\).
- Equation of the line: \( y = -3x + 5 \)

#### Finding an equation of a line when the slope and a point are given:

**Example:** Write an equation for a line passing the point \((5, 3)\) with slope \( m = -4 \).

- Start with: \( y = mx + b \) Replace \((x, y)\) by \((5, 3)\) & \( m \) by \(-4\).
- Solve for \( b \): \( 3 = -4 \cdot 5 + b \) Add 20 on both sides.
- \( y \)-intercept: \( b = 23 \)
- Equation of the line: \( y = -4x + 23 \) \( y = mx + b: \ m = -4, \ b = 23 \)

#### Finding an equation of a line when two points are given:

**Example:** Write an equation for a line that passes through the points \((2, 1)\) and \((3, -5)\).

- The slope: \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 1}{3 - 2} = -6 \) \( (x_1, y_1) = (2, 1), \ (x_2, y_2) = (3, -5) \)
- Substitute values into point-slope equation: \( y - y_1 = m (x - x_1) \)
- Point-slope equation: \( y - 1 = -6 (x - 2) \) Replace \((x_1, y_1)\) with \((2, 1)\) & \( m \) with \(-6\).
- Slope-intercept form: \( y - 1 = -6x + 12 \) Remove parentheses.
- \( y = -6x + 13 \) Add 1 on both sides. \( y = mx + b \)
Unit 15: Summary

Graphing Linear Equations

The coordinate plane: a powerful tool to mark a point and solution of linear equation on a graph.

- Coordinate axes: x axis and y axis.
- The origin: the intersection of the x and y axes (both lines are 0 at the origin).

Ordered pair \((x, y)\): a pair of numbers (each point on the plane corresponds to an ordered pair).

Coordinate: the numbers in an ordered pair (the \(x\)-distance and the \(y\)-distance from a given origin).

Four quadrants:

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>The 1st quadrant I</td>
<td>(+x, +y)</td>
</tr>
<tr>
<td>The 2nd quadrant II</td>
<td>(-x, +y)</td>
</tr>
<tr>
<td>The 3rd quadrant III</td>
<td>(-x, -y)</td>
</tr>
<tr>
<td>The 4th quadrant IV</td>
<td>(+x, -y)</td>
</tr>
</tbody>
</table>

\(x\) – intercept \((x, 0)\): the point at which the graph crosses the \(x\)-axis.

\(y\) – intercept \((0, y)\): the point at which the graph crosses the \(y\)-axis.

A linear (first-degree) equation: an equation whose graph is a straight line.

A linear equation in two variables: a linear equation that contains two variables, such as \(5x + 2y = 7\).

The standard form of linear equation in two variables: \(Ax + By = C\)

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Ax + By = C)</td>
<td>(4x - 9y = 11)</td>
</tr>
</tbody>
</table>

Solutions of equations: solutions for a linear equation in two variables are an ordered pair. They are the particular values of the variables in the equation that makes the equation true.

Procedure to graph a linear equation:

- Choose two values of \(x\), calculate the corresponding \(y\), and make a table.
- Plot these two points on the coordinate plane.
- Connect the points with a straight line.
- Check with the third point – is third point on the line?
**Slope** \((m)\) (grade or pitch): the slope of a straight line is the rate of change. It is a measure of the “steepness” or “incline” of the line and indicates whether the line rises or falls.

- A line with a positive slope rises from left to right and a line with a negative slope falls.

**The slope formula:**

<table>
<thead>
<tr>
<th>The slope formula</th>
<th>(\text{Slope} = \frac{\text{the change in } y}{\text{the change in } x} = \frac{\text{rise}}{\text{run}})</th>
<th>The slope of the straight line that passes through two points ((x_1, y_1)) and ((x_2, y_2)): (m = \frac{y_2 - y_1}{x_2 - x_1}) or (m = \frac{y_1 - y_2}{x_1 - x_2}) (x_1 \neq x_2)</th>
</tr>
</thead>
</table>

**Horizontal and vertical lines:**

<table>
<thead>
<tr>
<th>Line</th>
<th>Equation</th>
<th>Slope ((m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal line</td>
<td>(y = b)</td>
<td>(m = 0)</td>
</tr>
<tr>
<td>Vertical line</td>
<td>(x = a)</td>
<td>(m = \infty)</td>
</tr>
</tbody>
</table>

**The slope and a point can determine a straight line.**

**Procedure to graph a linear equation using the intercept method:**

- Choose \(x = 0\) and calculate the corresponding \(y\).
- Choose \(y = 0\) and calculate the corresponding \(x\).
- Plot these two points on the coordinate plane.
- Connect the points with a straight line.
- Check with the third point - is third point on the line?

**Equation of a straight line:**

<table>
<thead>
<tr>
<th>Straight-line equation</th>
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</thead>
<tbody>
<tr>
<td>Point-slope form</td>
<td>(y - y_1 = m(x - x_1)) ({m = \text{slope}} \{b = y - \text{intercept}})</td>
</tr>
<tr>
<td>Slope-intercept form</td>
<td>(y = mx + b)</td>
</tr>
</tbody>
</table>

**Finding an equation of a line from the graph:**

- Choose two points on the given line.
- Calculate the slope: \(m = \frac{y_2 - y_1}{x_2 - x_1}\)
- Determine the \(y\)-intercept on the line: \(b\) or \((0, y)\) \(\text{The line crosses the } y\text{-axis.}\)
- Equation of the line: \(y = mx + b\) \(\text{Replace } m \text{ and } b \text{ with values.}\)

**Finding an equation of a line when the slope and a point are given:**

- Start with: \(y = mx + b\) \(\text{Replace } (x, y) \& m \text{ with given values.}\)
- Solve for \(b\).
- Equation of the line: \(y = mx + b\) \(\text{Replace } m \text{ and } b \text{ with values.}\)

**Finding an equation of a line when two points are given:**

- Calculate the slope: \(m = \frac{y_2 - y_1}{x_2 - x_1}\)
- Point-slope equation: \(y - y_1 = m(x - x_1)\) \(\text{Replace } (x_1, y_1) \& m \text{ with values.}\)
- Slope-intercept equation: \(y = mx + b\) \(\text{Solve for } y.\)
Unit 15: Self-Test

**Graphing Linear Equations**

**Topic A**

1. Plot the points and name the quadrants.
   - (2, -1)  
   - (-4, 3)  
   - (-1, -3)  
   - (3, 2)

2. Graph the following.
   a) \( y = 3x \)
   b) \( 7x - y = 3 \)
   c) \( x + 3y = 6 \)

3. Graph \( y = \frac{1}{3}x - 4 \) and determine another point.

4. Find the ordered pair solution of the given equation.
   a) \( 3x - 5y = 11 \), when \( x = 2 \).
   b) \( x - 0.6y = -3 \), when \( x = -6 \).
   c) \( \frac{3}{4}x - 4y = 5 \), when \( x = -4 \).

**Topic B**

5. Determine the slope containing points (4, -1) and (3, 5).

6. Determine the slope of \( 8x - y - 3 = 0 \).

7. Graph the following.
   a) \( y = -0.9 \)
   b) \( x = 3 \)
   c) \( y = 0 \)

**Topic C**

8. Identify the slope and y-intercept of the following equations.
   a) \( y = -7x - 11 \)
   b) \( 5y - 3x = 2 \)
9. Graph the equation using the slope and the y-intercept.
   a) \( y = -\frac{3}{4}x + 5 \)
   b) \(-6x + 9 = -3y\)

10. Determine the intercepts of the line \( 3x - y = 9 \).

11. Graph the equation using the intercept method.
   a) \( 4x - y = 8 \)
   b) \( y = -\frac{1}{2}x + 3 \)

**Topic D**

12. Write the slope intercept equation of the given line.
   a) ![Graph](image1)
   b) ![Graph](image2)

13. Write the equation for the following lines:
   a) The line with a slope of -4 passing the point (-2, 5).
   b) The line with a slope of \( \frac{3}{5} \) passing the point (5, -7).

14. Write an equation of the line that passes through each pair of points.
   a) (3, 2) and (4, -7).
   b) (-3, 0) and (0, 6).
   c) (0, 5) and (5, 3).
Unit R

1. \(2^2 \times 3^2\)
2. a) Ten million, twenty-four thousand, five hundred twenty-six  
   b) Forty-seven and two hundred sixty-eight thousandths
3. a) 6.439  
    b) 8.025  
    c) 2.7
4. a) \(\frac{30}{7}\)  
    b) \(1 \frac{4}{5}\)
5. \(\frac{1}{4}\)
6. 400
7. a) 0.45  
    b) 43.6%  
    c) \(\frac{1}{4}\)  
    d) 20%  
    e) \(\frac{2}{5}\)  
    f) \(\frac{3}{10}\)
8. a) 192  
    b) 105
9. a) \(\frac{5}{6}\)  
    b) \(\frac{1}{2}\)  
    c) \(1 \frac{5}{8}\)  
    d) \(\frac{2}{3}\)  
    e) \(6 \frac{5}{7}\)  
    f) \(2 \frac{1}{12}\)  
    g) \(1 \frac{1}{4}\)  
    h) 2  
    i) 12
j) \(\frac{1}{6}\)

k) \(\frac{2}{3}\)

Unit 1

1. 5

2. 9

3. a) 7
   b) No mode.

4. a) 7
   b) 8

5. a) 3
   b) 1
   c) 4

6. Let your instructor check your line graph.

7. Let your instructor check your circle graph.

8. a) 551
   b) 311.64
   c) 2839
   d) \(7\frac{31}{42}\)
   e) 248

9. a) 6,000,000
   b) 570
   c) 8,600
   d) 48,000

10. a) 80,800
    b) 9,600
    c) 3,000,000
    d) 20

Unit 2

1. a) Constant: -3  Coefficient: 2  Variable: \(x\)
    b) Constant: 13  Coefficient: \(-4 \& \frac{5}{7}\)  Variable: \(t\)

2. a) 5\(x\), 3, \(-y\)
    b) 2\(r\), 16\(r^2\), \(-\frac{3}{14}r\), 1

3. a) \(-\frac{5}{9}x\) and 5\(x\), 2\(y^2\) and 13\(y^2\), 7 and -1
    b) 0.6\(t\) and -7\(t\), 9\(uv\) and 1.67\(uv\)
4. a) 76
b) 58
5. a) 10y
   b) \( \frac{t}{6} \)
   c) \( 15 - (x + \frac{3}{y}) = 6 \)
   d) \( 6x - 7 = 15 \)
6. a) \( 375 + y \)
   b) 175 - y
   c) 45 - w
   d) \( \frac{x}{4}, \frac{x}{48} \)
7. a) \( x \)
   b) 4
8. a) \( 9 \cdot 9 \cdot 9 \)
   b) \((-y) \cdot (-y) \cdot (-y) \cdot (-y) \)
   c) \(0.5a^2b \cdot 0.5a^3b \)
   d) \( \frac{2}{7}x \)
9. a) \((0.06)^4 \)
   b) \((12y)^3 \)
   c) \((\frac{-2}{y}x)^2 \)
10. 1440
11. a) \( y^8 \)
    b) \( 5^3 \)
12. a) 8
    b) 9
13. a) 133
    b) 63
    c) 8

**Unit 3**

1. 21 cm
2. 14.1 cm
3. a) 5.6 in
   b) 11 ft
   c) 35.2 cm
   d) \( \frac{18}{19} \) yd
4. 7.85 in
5. a) 33 cm  
b) 26.85 cm  
c) 17.85 in  
d) 22.6 yd  
6. 17.8 in  
7. 18 m  
8. a. 50 m  
b. $750  
9. 36 m  
10. a) 17.25 cm²  
b) 16.57 in²  
c) 23.85 m²  
11. 47.47 m²  
12. 281.2 m²  
13. a) 50.65 cm³  
b) 45.144 mm³  
c) 3591.1 cm³  
d) 89.8 cm³  
e) 217.68 cm³  
14. 14815.8 m³  
15. 301.6 m³  
16. No  
17. 7263.4 cm³  
18. 98.8 cm²  
19. 32.74 in²  
20. LA ≈ 93.12 yd², SA ≈ 135.59 yd²  
21. LA ≈ 73.39 cm², SA ≈ 105.56 cm²  
22. 10.18 m²  
23. 1.72 m²  
24. 273.3 m²  

Unit 4

1. a) 0.439 m  
b) 223.6 g  
c) 0.0000483 kL  
d) 25 hg  
2. a) 7.23 kg  
b) 520 mm
c) 0.34 L
d) 52000 cL

3.
   a) 4000 mm
   b) 63006 g
   c) 5290 mL
   d) 28.87 km

4.
   a) 0.74 m²
   b) 90,000 m²
   c) 5,000,000 cm³
   d) 0.567 cm³

5.
   a) 4
   b) 38 g
   c) 5000 cm³
   d) 2.7 cL
   e) 76
   f) 18,000 cm³
   g) 257 L
   h) 0.039375 kL

6.
   a) 108 in
   b) 94 pt
   c) 7040 yd
   d) 4.638 lb

7.
   a) 2.438 m
   b) 7.6 kg
   c) 93 tsp
   d) 9 mi
   e) 724.2 km

Unit 5

1.  \( \frac{1}{3} \),  7.3  (Answers may vary.)

2.  a) 8
   b) -3,  0,  8
   c) -3,  0,  8,  4.7,  \( \frac{3}{5} \),  2.\( \sqrt{5} \)
   d) 5.4259…,  π,  \( \sqrt{5} \)

3.  a) Identity property of addition
    b) Commutative property of addition
c) Associative property of addition

d) Inverse property of addition

e) Distributive property

f) Associative property of multiplication

g) Commutative property of multiplication

h) Inverse property of multiplication

i) Distributive property

j) Multiplicative property of zero

k) Commutative property of addition

l) Associative property of multiplication

4. a) \((12 + 88) + 45 = 145\)
b) \((9 \cdot 8) 1000 = 72,000\)
c) \((3 + 2997) + 56 = 3056\)

5. a) \(4y^2 + 1.2y\)
b) \(10 - 15y^2\)
c) \(\frac{2}{9} - \frac{1}{6}x\)

6. a) \(6 < 8\)
b) \(0 > -6\)
c) \(-4 < -2\)
d) \(-\frac{3}{7} < \frac{1}{7}\)
e) \(-0.6 > -0.8\)
f) \(1\frac{1}{2} > \frac{3}{8}\)

7. a) \(-17 < -9 < -4 < 0 < 8 < 23\)
b) \(-8 < -3.24 < 0.05 < \frac{2}{5} < \frac{3}{5}\)
c) \(-\frac{1}{3} < -\frac{1}{7} < \frac{2}{5} < 1\frac{3}{4}\)

8. a) \(67\)
b) \(21\)
c) \(0.45\)
d) \(-49\)
e) \(\frac{1}{8}\)

9. a) \(116\)
b) \(25\)

10. a) \(37\)
b) \(-15\)
c) \(-2\frac{3}{5}\)
d) 5  
e) -13  
f) 7.5  
g) -13  
h) 2  
i) $-\frac{5}{7}$  
j) $\frac{17}{28}$  
k) 5  
l) -0.6  
m) 27  
n) -8  
o) 0  
p) Undefined  

11. a) 45  
b) $-\frac{5}{8}$  
c) 1  

12. a) 4  
b) 40  
c) -41  
d) Undefined  

**Unit 6**  
1. a) $5x^3, -8x^2, 2x$  
b) $-\frac{2}{3}y^4, 9a^2, a, -1$  

2. a) 2, -7, 9  
     Degree: 5  
b) -8, $-\frac{2}{3}, 11, 4, -23$  
     Degree: 6  

3. a) Binomial  
b) Monomial  
c) Trinomial  

4. a) $15x^3 - 23x^2 + 8x + 3$  
b) $\frac{2}{3}y^4 - 3y^3 - 45y^2 + 4y$  

5. a) $-x + 19y$  
b) $6a^2 - 31b$  
c) $4uv^2 + 10u^2v$  
d) $23r - 9r$  
e) $9m^2 + 64n$
6. a) $10a^2 + 13$
   b) $-19x + 39y$
   c) $7z^2 - 16z + 31$
   d) $-20y^2 + 47y - 33$
   e) $17ab - 28xy$
7. a) $a^9$
   b) $\frac{1}{x^{11}}$
   c) $\frac{1}{r^6}$
   d) $-42a^7 b^{11}$
   e) $\frac{1}{4}x^4 y^7 z^9$
   f) $\frac{1}{6}y^5$
   g) $\frac{-9}{m}$
8. a) $-12x^7 + 28x^4$
   b) $27a^4b^3 + 18a^5b^3 - 9a^3b$
   c) $7a + 1 - \frac{4}{5a}$
   d) $40y^2 - 11y - 63$
   e) $21r^2 + 28rt^2 - 6rt - 8t^3$
   f) $10a^3 b^3 + 21a^2 b^2 + 9ab$
   g) $x^2 - x + \frac{2}{9}$

Unit 7

1. a) Yes
   b) No
   c) Yes
2. a) $x = 19$
   b) $y = \frac{1}{4}$
   c) $m = 23$
   d) $t = 8$
   e) $x = \frac{1}{8}$
   f) $y = -52$
   g) $x = 28$
   h) $y = -\frac{7}{9}$
   i) $x = 7$
   j) $t = -2$
k) \( y = 0.8 \)

l) \( y = \frac{8}{9} \)

3. 
   a) \( t = \frac{3}{14} \)
   b) \( m = 9 \)
   c) \( x = 2 \)
   d) \( y = \frac{1}{2} \)
   e) \( x \approx 0.069 \)
   f) \( t = -0.05 \)
   g) \( x = -\frac{4}{5} \)

4. 
   a) Contradiction equation
   b) Identity equation
   c) Conditional equation
   d) Contradiction equation
   e) Conditional equation
   f) Identity equation

5. 
   a) \((x - 7) + 9\)
   b) \(\frac{7}{9x}\)
   c) \(11x - 8\)

6. 
   a) \(4xy - 13 = x + y + 6\)
   b) \(x^2 + y^2 = xy - 26\)
   c) \(5 + \frac{5x}{23} = 11x\)
   d) \((x + 1) - x = 1\)
   e) \(x + (x + 2) + (x + 4) = 15\)
   f) \(x(x + 2) = 48\)
   g) \(x + (x + 2) + (x + 4) = 21\)

7. 
   a) \(7x = 42\), \(x = 6\)
   b) \(4x - 3 = \frac{x}{4} - 9\), \(x = -1.6\)
   c) \((5x - 3) + x + (4 + 5x - 3) = 20\), \(2, 7, 11\)
   d) \(x + (x + 2) + (x + 4) = 27\), \(7, 9, 11\)
   e) \(x + 7x + (30 + 7x) = 180\), \(10^0, 70^0, 100^0\)
   f) \(128 = 2(l - 8) + 2l\), \(36m, 28m\)
   g) \(x = 199.99 + 20\% x\), \(x = $249.99\)
   h) \(x = 379.99 - (10\%) (379.99)\), \(x = $341.99\)

**Unit 8**

1. 121.43
2. 195 km
3. 2 h
4. 186.13
5. \( A = 385 \text{ cm}^2 \), \( P = 92 \text{ cm} \)
6. 696 ft²
7. \( C = 15.08 \text{ ft} \), \( A = 18.1 \text{ ft}^2 \)
8. $337.50
9. 18°C
10. a) \( r = \frac{d}{t} \)
    b) \( t = \frac{l}{Fr} \)
    c) \( l = \frac{p-2w}{2} \)
    d) \( F = \frac{v}{s}C + 32 \), 75.2
    e) \( m = \frac{p-C}{c} \)
    f) \( z = \frac{x-35y^2}{y} \)
    g) \( b = \frac{2A}{h^2} \)
    h) \( z = y - xt \)
    i) \( h = \frac{35w}{wh^2} \)
    j) \( w = \frac{y-x}{2x+3}, 0.091 \)
11. 20.86 cm
12. 0.946 m
13. 14.91 ft
14. 283.65 km
15. 68.35 ft

**Unit 9**

1. a) \( \frac{1}{3} \)
    b) \( \frac{3}{11} \)
    c) \( \frac{7 \text{ people}}{30 \text{ tickets}} \)
    d) \( \frac{11}{31} \)
    e) \( \frac{8 \text{ km}}{37 \text{ min}} \)
2. 0.14%
3. 1.25%
4. 76.5 km/h
5. 2 L
6. 8-lb.
7. a) \(\frac{5}{110} = \frac{15}{330}\)
   b) \(\frac{24}{1970} = \frac{12}{985}\)
8. $3.69
9. 16 ft
10. $18,000
11. 117
12. 300
13. 40%
14. 20.1%
15. a) 3 cm
    b) 11.2 m
    c) 5.25 cm

Unit 10

1. a) Acute angles
    b) Obtuse angles
    c) Obtuse angle
    d) Reflex angle
2. 48°
3. 34°
4. 44°
5. a) Supplementary
    b) \(< A = 147°, < B = 33°\)
6. \(< C = 40°\)
7. \(< C = 72°, < D = 108°, b = 5\) cm
8. a) vi  b) i  c) iii  d) ii
9. a) \(< \theta = 60°, x = 23\) cm  It is an equilateral triangle (an acute triangle).
    b) \(< B = 102°, a = 43\) ft  It is an isosceles triangle (an obtuse triangle).
    c) \(< B < C = 28°\),  It is an isosceles triangle (an obtuse triangle).
    d) \(< Z = 72\) opposite°, \(x = 32\) cm  It is an isosceles triangle (an acute triangle).
10. a) opposite
    b) adjacent
    c) hypotenuse
d) adjacent

e) opposite

f) $Y$

11. \[\sin X = \frac{5 \text{ cm}}{7.81 \text{ cm}} \approx 0.6402, \quad \sin Z = \frac{6 \text{ cm}}{7.81 \text{ cm}} \approx 0.7682\]
\[\cos X = \frac{6 \text{ cm}}{7.81 \text{ cm}} \approx 0.7682, \quad \cos Z = \frac{5 \text{ cm}}{7.81 \text{ cm}} \approx 0.6402\]
\[\tan X = \frac{5 \text{ cm}}{6 \text{ cm}} \approx 0.8333, \quad \tan Z = \frac{6 \text{ cm}}{5 \text{ cm}} = 1.2\]

12. \[\sin O = \frac{6.32 \text{ ft}}{7.62 \text{ ft}} \approx 0.8294, \quad \sin Q = \frac{6.32 \text{ ft}}{7.62 \text{ ft}} \approx 0.8294\]
\[\cos O = \frac{6.32 \text{ ft}}{7.62 \text{ ft}} \approx 0.5577, \quad \cos Q = \frac{6.32 \text{ ft}}{7.62 \text{ ft}} \approx 0.5577\]
\[\tan O = \frac{6.32 \text{ ft}}{6.32 \text{ ft}} \approx 1.4871, \quad \tan Q = \frac{6.32 \text{ ft}}{4.25 \text{ ft}} \approx 1.4871\]

13. a) 0.8387

b) 0.8090
c) 19.0811
d) 12.5°
e) 62.83°
f) 51.02°

14. $x \approx 7.793$

15. $c = 36.58 \text{ cm}$

16. $< A = 51^0, \quad b \approx 4.86 \text{ m}, \quad c \approx 7.72 \text{ m}$

17. a) $b \approx 6.25 \text{ cm}$

b) $< A = 41^0$

18. a) $< B = 45^0, \quad b = 6 \text{ m}, \quad c \approx 8.458 \text{ m}$

b) $a = 4 \text{ ft}, \quad < A \approx 53.13^0, \quad < B = 36.87^0$

19. a) $< B \approx 32^0$

b) $y \approx 16.04 \text{ m}$

20. $x \approx 25.74 \text{ m}$

21. $< \theta \approx 41.21^0$

22. $x \approx 22.67 \text{ m}$

23. $< \theta \approx 49.09^0$

24. $x \approx 47.34 \text{ cm}$

Unit 11

1. a) $7 \cdot 7 \cdot 7 \cdot 7$

b) $(-t) (-t) (-t)$

c) $(5a^2b^0) (5a^2b^0)$

d) $\left(\frac{7}{11}x\right) \left(\frac{7}{11}x\right) \left(\frac{7}{11}x\right)$
2. a) $(0.5)^4$
   b) $(6w)^3$
   c) $42u^2v^3$

3. a) 24
   b) 982

4. a) 5
   b) 7

5. a) $9x^4 - 7x^3 + x^2 - x + 2$
   b) $21uv^3 - uv^2 + 4v - 67$

6. a) $43 - 5x + 26x^2 - 17x^3$
   b) $-9 + \frac{4}{7}tw + 4.3t^2w^2 - 8w^3 + w^4$

7. a) -92
   b) 1
   c) -0.064
   d) -64
   e) $y^7$
   f) $x^3$
   g) $\frac{1}{x^{20}}$
   h) $\frac{13}{a}$
   i) -0.512
   j) $81a^8b^{12}$
   k) 64
   l) $\frac{u^3}{w^3}$
   m) $a^6b^0$
   n) 1
   o) $\frac{5x^2}{y^{10}}$
   p) $\frac{u^6}{w^{12}v^9}$
   q) $72x^4y^5$
   r) $\frac{27}{64}x^3y^3$

8. a) 1
   b) $\frac{8}{27}$
   c) 9

9. a) $4.56 \times 10^7$
   b) $5.23 \times 10^6$
10. a) 3578
   b) 0.000043

11. a) $2.37396 \times 10^6$
    b) $3.75 \times 10^{-6}$

12. a) 14
    b) $\frac{11}{15}$
    c) $8\sqrt{5}$
    d) $\frac{\sqrt{13}}{3}$

**Unit 12**

1. 3, 17, 1
2. 2, 7, 14
3. 234.55 g, 625.45 g
4. 6.3 L
5. 13 km/h
6. 0.2 h, 0.286 h
7. 5%
8. 22.5%
9. $61.11
10. $27,960
11. $29.85, $169.15
12. $23,450, $445,550
13. $20,000, $120,000
14. $2662.56
15. $41.05
16. $33170.73
17. $3500, $2000
18. 3, 9, 10, 30
19. 1.2 L

**Unit 13**

1. a) 12
   b) 9
   c) 8

2. a) \( -8y \)
b) \( \frac{5}{8} x \)
c) \(-9xy^2 + 4x^2 - y^3\)

3. \(9x^4 - x^3 - 8x + 10\)

4. \(4x^2 + 7x - 18\)

5. a) \(11a^3 - 4a^2 + 9a + 2\)
b) \(5x^2 - 4x + 11\)

6. a) \(24x^3y^3\)
b) \(12a^6 - 24a^3\)
c) \(14x^2y^6 + 7x^3y^3 - 21xy^3\)
d) \(12x^2 - 31x + 20\)
e) \(-2a^4 + a^3 + 13a^2 - 15a\)

7. a) \(8t^7 - 20t^4\)
b) \(3x^2 - 17x + 10\)
c) \(36a^2 - 25\)
d) \(9w^2 - 6w + 1\)
e) \(25u^2 + 5u + \frac{1}{4}\)
f) \(36x^2 - 4xy + \frac{1}{9}y^2\)
g) \(\frac{1}{25}z^2 - \frac{1}{16}\)

8. a) \(56x^3\)
b) \(-9\left(\frac{a^2}{b^3}\right)\)
c) \(4y + 1 - \frac{3}{7y}\)
d) \(3(2a + 1)\)

9. a) \(3x + 2\), Remainder = 2
b) \(2x^2 - 7x + 14\), Remainder = 2

**Unit 14**

1. \(2 \cdot 2 \cdot 3 \cdot 5\)

2. a) \(5x\)
b) \(3ab\)
c) \(y + 4\)
d) \(\frac{1}{4}x\)
e) \(-4y\)

3. a) \((5x - 1)(5x + 4)\)
b) \((6b - a)(8ab + 1)\)
c) $25uv - 6vw$

d) $(x + y)(x - y)^2$

e) $5(y + 2)(y - 2)$

f) $(1 + 7w)(1 - 7w)$

g) $(9u + 11)(9u - 11)$

h) $(5a + 6b)(5a - 6b)$

i) $(2y^3 + 0.3)(2y^3 - 0.3)$

4. a) $(x + 4)(x + 5)$

b) $(x - 4)(x - 6)$

c) $(x + 3)(x - 6)$

d) $2(x - 2)(x + 7)$

e) $(x - 3)(4x + 5)$

f) $(5y - 6)(y + 3)$

g) $(6b - a)(4ab + 1)$

h) $17uv - 6vs$

5. a) $3(2x + 5)(x - 4) = 0$

b) $2(3x - 4)(x + 2)$

6. a) $(3x + 5)^2$

b) $3(2y - 3)^2$

c) $2(3t^2 - 2)^2$

7. a) $0, -\frac{7}{23}$

b) $\pm \frac{7}{9}$

c) $-9, 17$

8. a) $-6, 7$

b) $\frac{4}{7}, 5$

c) $\frac{1}{4}, \frac{3}{4}$

9. $-9$

10. $4, -6$

11. $7m, 9m$

12. $6m, 8m$

Unit 15

1. $(2, -1): \text{ IV, } (-4, 3): \text{ II, } (-1, -3): \text{ III, } (3, 2): \text{ I}$
2.

a) 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 3x$</th>
<th>$(x, y)$</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>(0, 0)</td>
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<td>1</td>
<td>3</td>
<td>(1, 3)</td>
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</tbody>
</table>

b) 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 7x - 3$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3</td>
<td>(0, -3)</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>(1, 4)</td>
</tr>
</tbody>
</table>

c) 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -\frac{1}{3}x + 2$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>(0, 2)</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>(3, 1)</td>
</tr>
</tbody>
</table>

Third point may vary.

3.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = \frac{1}{3}x - 4$</th>
<th>$(x, y)$</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>-4</td>
<td>(0, -4)</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
<td>(3, -3)</td>
</tr>
</tbody>
</table>

4.  
a) $y = -1$

b) $y = -5$

c) $y = -2$

5. $m = -6$

6. $m = 8$

7.  
a)

8.  
a) $m = -7$

   $b = -11$ or $(0, -11)$

b) $m = \frac{3}{5}$

   $b = \frac{2}{5}$ or $(0, \frac{2}{5})$

c) $m = -35$

   $b = 10$ or $(0, 10)$

9.  
a)
10. (3, 0), (0, -9).
11. a) \[
\begin{array}{c|c|c}
\text{x} & y = 4x - 8 & (x, y) \\
0 & -8 & (0, -8) \\
2 & 0 & (2, 0) \\
\end{array}
\]

b) \[
\begin{array}{c|c|c}
\text{x} & y = \frac{1}{2}x + 3 & (x, y) \\
0 & 3 & (0, 3) \\
6 & 0 & (6, 0) \\
\end{array}
\]
12. a) \[
y = 2x + 6
\]

b) \[
y = \frac{-3}{4}x + 3
\]
13. a) \[y = -4x - 3\]

b) \[y = \frac{3}{5}x - 10\]
14. a) \[y = -9x + 29\]

b) \[y = 2x + 6\]

c) \[y = -\frac{2}{5}x + 5\]
# Index

| A | Absolute value | 89 |
|   | AC method for factoring trinomials | 221 |
|   | Acute angle | 157 |
|   | Acute triangle | 159 |
|   | Adding signed numbers | 90 |
|   | Add or subtract polynomials | 204 |
|   | Addition property of equality | 114 |
|   | Additive identity property | 84 |
|   | Additive inverse | 90 |
|   | Additive inverse property | 84 |
|   | Adjacent | 161 |
|   | Algebra | 35 |
|   | Angle | 157 |
|   | Angle of elevation | 167 |
|   | Angle of depression | 167 |
|   | Algebraic expression | 35, 99 |
|   | Area | 55 |
|   | Area unit conversion | 72 |
|   | Ascending order | 99, 176 |
|   | Associative property | 84, 85 |

| B | Bar or column graph | 23 |
|   | Base | 42 |
|   | Basic rules of exponents | 103 |
|   | Binomial | 99 |
|   | Business problems | 194 |

| C | Cartesian graphing | 232 |
|   | Circle | 51, 53, 55 |
|   | Circle (or pie) graph | 25 |
|   | Circumference | 51 |
|   | Closure property of addition | 84 |
|   | Closure property of multiplication | 85 |
|   | Coefficient | 35, 99 |
|   | Commission | 194 |
|   | Common factor | 215 |
|   | Complementary angles | 157 |
|   | Composite number | 3 |
|   | Compound interest | 132, 194 |
|   | Commutative property | 84, 85 |
|   | Concentration mixture problems | 191 |
|   | Conditional equation | 118 |
|   | Cone | 57, 60, 61 |
|   | Constant | 35, 99 |
|   | Consecutive integers | 122 |
|   | Contradiction equation | 118 |
|   | Coordinate | 232 |
|   | Coordinate plane | 232 |
|   | Corresponding angles | 149 |
|   | Corresponding sides | 149 |
|   | Cosecant | 162 |
|   | Cotangent | 162 |
|   | Cube | 57 |
|   | Cylinder | 57, 60 |

<p>| D | Decagon | 52 |
|   | Decimal place | 8 |
|   | Decimal number | 8 |
|   | Decimal to fraction | 10 |
|   | Decimal to percent | 10 |
|   | Degree of a polynomial | 100, 176 |
|   | Degree of a term | 100 |
|   | Descending order | 99, 176 |
|   | Diameter | 51 |
|   | Difference of squares | 206 |
|   | Discount | 132, 194 |
|   | Distance | 132 |
|   | Distributive property | 85 |
|   | Dividing monomials | 103, 207 |
|   | Dividing polynomials | 207 |
|   | Dividing signed numbers | 92 |
|   | Division property of equality | 114 |
| E | Identity equation | 118 |
| -- | Imperial and metric unit conversion | 76 |
| -- | Imperial conversion | 75 |
| -- | Imperial equivalents | 74 |
| -- | Imperial system units | 74 |
| -- | Improper fraction | 7 |
| -- | Incomplete quadratic equation | 223 |
| -- | Integers | 2, 83 |
| -- | Intelligence quotient (I.Q.) | 132 |
| -- | Irrational number | 83 |
| -- | Isosceles triangle | 159 |
| F | Factor | 3 |
| -- | Factoring | 215 |
| -- | Factoring $x^2 + bx + c$ | 218 |
| -- | Factoring $ax^2 + bx + c$ | 219, 220 |
| -- | Factoring difference of squares | 217 |
| -- | Factoring polynomials by grouping | 216 |
| -- | Factoring special products | 222 |
| -- | Factoring trinomials | 218 |
| -- | Finding an equation of a line | 339 |
| -- | First-degree equation | 233 |
| -- | FOIL method | 105 |
| -- | Four quadrants | 232 |
| -- | Fraction | 6 |
| -- | Fraction to percent | 10 |
| G | Lateral area | 59 |
| -- | Least common denominator (LCD) | 11 |
| -- | Least common multiple (LCM) | 11 |
| -- | Like fractions | 7 |
| -- | Like terms | 35, 99, 101 |
| -- | Line graph | 24 |
| -- | Linear equation | 233 |
| -- | Long division of polynomials | 208 |
| H | Markup | 132, 194 |
| -- | Mean | 21 |
| -- | Median | 22 |
| -- | Metric conversion | 70 |
| -- | Metric prefixes | 69 |
| -- | Metric system | 69 |
| -- | Mixed fraction | 6 |
| -- | Mixed Problems | 123, 196 |
| -- | Mode | 22 |
| -- | Monomial | 99 |
| -- | Motion problems | 193 |
| -- | Multiplication property of equality | 114 |
| -- | Multiplying binomials | 105, 205 |
| -- | Multiply monomials | 103, 205 |
| -- | Multiplying polynomials | 205 |
| -- | Multiplicative identity property | 85 |
| -- | Multiplicative inverse property | 85 |
| -- | Multi-step equation | 115 |</p>
<table>
<thead>
<tr>
<th><strong>N</strong></th>
<th><strong>P</strong></th>
<th><strong>O</strong></th>
<th><strong>Q</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural numbers</td>
<td>83</td>
<td>Radius</td>
<td>51</td>
</tr>
<tr>
<td>Negative exponent</td>
<td>177, 178</td>
<td>Parallelogram</td>
<td>50, 53, 55</td>
</tr>
<tr>
<td>Number line</td>
<td>2</td>
<td>Rate</td>
<td>144</td>
</tr>
<tr>
<td>Number problems</td>
<td>120</td>
<td>Pentagon</td>
<td>52</td>
</tr>
<tr>
<td><strong>O</strong></td>
<td><strong>P</strong></td>
<td><strong>O</strong></td>
<td><strong>Q</strong></td>
</tr>
<tr>
<td>Obtuse angle</td>
<td>157</td>
<td>Percent</td>
<td>10, 147</td>
</tr>
<tr>
<td>Obtuse triangle</td>
<td>159</td>
<td>Percent decrease</td>
<td>132, 148, 194</td>
</tr>
<tr>
<td>Octagon</td>
<td>52</td>
<td>Percent increase</td>
<td>132, 148, 194</td>
</tr>
<tr>
<td>Odd numbers</td>
<td>2</td>
<td>Percent proportion method</td>
<td>147</td>
</tr>
<tr>
<td>One exponent</td>
<td>175, 177</td>
<td>Percent to decimal</td>
<td>10</td>
</tr>
<tr>
<td>Opposite side</td>
<td>161</td>
<td>Percent to fraction</td>
<td>10</td>
</tr>
<tr>
<td>Opposite inverse</td>
<td>90</td>
<td>Perfect square</td>
<td>182</td>
</tr>
<tr>
<td>Order of operations</td>
<td>44, 89, 183</td>
<td>Perimeter</td>
<td>52</td>
</tr>
<tr>
<td>Ordered pair</td>
<td>232</td>
<td>Pitch</td>
<td>143</td>
</tr>
<tr>
<td>Original price</td>
<td>132</td>
<td>Place value</td>
<td>2</td>
</tr>
<tr>
<td><strong>P</strong></td>
<td><strong>Q</strong></td>
<td><strong>R</strong></td>
<td><strong>R</strong></td>
</tr>
<tr>
<td>Power</td>
<td>177, 178</td>
<td>Quadratic equation</td>
<td>223</td>
</tr>
<tr>
<td>Power of a product</td>
<td>177, 178</td>
<td>Quadrilateral</td>
<td>50</td>
</tr>
<tr>
<td>Power of a quotient</td>
<td>177, 178</td>
<td>Quotient of like bases</td>
<td>103</td>
</tr>
<tr>
<td>Prime factor</td>
<td>215</td>
<td>Quotient rule</td>
<td>177</td>
</tr>
<tr>
<td>Prime factorization</td>
<td>4</td>
<td>Radius</td>
<td>51</td>
</tr>
<tr>
<td>Prime number</td>
<td>3</td>
<td>Range</td>
<td>21</td>
</tr>
<tr>
<td>Procedure for solving word problems</td>
<td>120</td>
<td>Rate</td>
<td>13</td>
</tr>
<tr>
<td>Product of like bases</td>
<td>103</td>
<td>Ratio</td>
<td>13, 143</td>
</tr>
<tr>
<td>Product rule</td>
<td>177</td>
<td>Rational number</td>
<td>83</td>
</tr>
<tr>
<td>Proper fraction</td>
<td>6</td>
<td>Real numbers</td>
<td>83</td>
</tr>
<tr>
<td>Properties for solving equations</td>
<td>114</td>
<td>Real number line</td>
<td>88</td>
</tr>
<tr>
<td>Properties of addition</td>
<td>84</td>
<td>Real number system</td>
<td>83</td>
</tr>
<tr>
<td>Properties of exponents</td>
<td>177</td>
<td>Rectangle</td>
<td>50, 53, 55</td>
</tr>
<tr>
<td>Properties of equality</td>
<td>114</td>
<td>Rectangular solid</td>
<td>57</td>
</tr>
<tr>
<td>Properties of multiplication</td>
<td>85</td>
<td>Reflex angle</td>
<td>157</td>
</tr>
<tr>
<td>Properties of zero</td>
<td>5, 92</td>
<td>Regular polygon</td>
<td>50</td>
</tr>
<tr>
<td>Proportion</td>
<td>13, 145</td>
<td>Relationship between ( mL, g ) and ( cm^3 )</td>
<td>73</td>
</tr>
<tr>
<td>Protractor</td>
<td>158</td>
<td>Remove parentheses</td>
<td>102</td>
</tr>
<tr>
<td>Pyramid</td>
<td>57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>Page</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td>------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rhombus</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right angle</td>
<td>157</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right triangle</td>
<td>136, 159</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rounding</td>
<td>29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sale price</td>
<td>132</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales tax</td>
<td>194</td>
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<td>Signs of multiplication</td>
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<td>Signed number</td>
<td>88</td>
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<td>Simple interest</td>
<td>132, 194</td>
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<td>Similar triangles</td>
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<td>Sine</td>
<td>162, 163</td>
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<td>Special binomial products</td>
<td>206</td>
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<td>157</td>
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<td>Surface area</td>
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<td>Table</td>
<td>23</td>
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<td>Tangent</td>
<td>162, 163</td>
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<td>Term</td>
<td>35, 99</td>
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<td>148</td>
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<td>Triangle</td>
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<td>118</td>
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<td>71</td>
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<td>y – intercept</td>
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<td>Zero exponent</td>
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<td>223</td>
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